New Irreducible Representations of Leavitt Path Algebras

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Main Result

For a **directed graph $\mathbf{\Gamma}$** and any field $\mathbb{F}$, there is an $\mathbb{F}$-algebra denoted $\mathbf{L}_{\mathbb{F}}(\mathbf{\Gamma})$. This work will focus on representations over $L_{\mathbb{F}}(\Gamma)$. Previously, known irreducible representations were **Chen modules** or projective modules generated by sinks.

Theorem (Özaydın, P.)

If Γ has intersecting cycles, then $L_{\mathbb{F}}(\Gamma)$ has infinitely many isomorphism types of irreducible representations that are not Chen.

Overview

1. Introduction
2. Preliminaries
   - $\mathbb{F}\Gamma$
   - $L_{\mathbb{F}}(\Gamma)$
3. Reduction of problem
4. Symbolic Dynamics
5. $\partial\Gamma$ and functions
   - Chen Modules
6. New simple $L_{\mathbb{F}}(\Gamma)$ modules
   - Step functions over $\partial\Gamma$
   - A sobering example
7. References

1. Introduction
2. Preliminaries
   - $\mathbb{F}\Gamma$
   - $L_{\mathbb{F}}(\Gamma)$
3. Reduction of problem
4. Symbolic Dynamics
5. $\partial\Gamma$ and functions
   - Chen Modules
6. New simple $L_{\mathbb{F}}(\Gamma)$ modules
   - Step functions over $\partial\Gamma$
   - A sobering example
7. References

Let Γ be a **digraph** consisting of a set of vertices V, a set of arrows E, and $s, t : E \longrightarrow V$ the source and target maps respectively. A vertex with $s^{-1}(v) = \emptyset$ is called a **sink**. Vertices $v \in V$ are **paths** of length 0, and $p = e_1 e_2 \cdots e_n$ where $e_i \in E$ such that $t(e_i) = s(e_{i+1})$ are paths of length $n > 0$. We will extend the maps s, t to all paths. A **cycle** is a simple closed path. We will assume that both V and E are finite, making Γ a finite digraph.

The **path algebra** of Γ over a field $\mathbb{F}$, denoted by $\mathbb{F}\Gamma$, is the algebra of formal linear combinations of paths of Γ, where multiplication of paths is given by concatenation when such concatenation makes sense - if not, the multiplication yields 0. This is then extended linearly.

One can also define $\mathbb{F}\Gamma$ as the $\mathbb{F}$-algebra defined by generators $V \sqcup E$ and relations:

$$vu = \delta_{u,v} v, \;\; s(e)e = e = e\, t(e) \text{ for } v, u \in V \; e \in E$$

where $\delta_{u.v}$ is the Kronecker delta.

The **Leavitt Path Algebra** (LPA) of Γ is obtained from $\mathbb{F}\Gamma$ by adding new arrows (generators) and then taking a quotient by new relations. The additional arrows are $E^* = \{e^* | e \in E\}$ where $s(e^*) = t(e)$ and $t(e^*) = s(e)$, and are called dual arrows. Now $V \sqcup E \sqcup E^*$ generate a path algebra on the doubled graph, which is then quotiented by the following (Cuntz, Krieger) relations:

(CK1) $e^* f = \delta_{e,f} t(e)$ for all $e, f \in E$

(CK2) $v = \sum_{s(e)=v} e\, e^*$ for every nonsink $v \in V$

Consequently, $L_{\mathbb{F}}(\Gamma)$ is spanned by pq^* where p, q are paths in Γ with $t(p) = t(q)$ and $q^* := e_n^* \cdots e_1^*$ if $q = e_1 \cdots e_n$. We define $v^* = v$ and $(e^*)^* = e$.

Thus, $L_{\mathbb{F}}(\Gamma)$ is a $*-$algebra.

$L_{\mathbb{F}}(\Gamma)$ is NOT a path algebra, but the obvious homomophism from $\mathbb{F}\Gamma$ to $L_{\mathbb{F}}(\Gamma)$ is injective (Goodearl 2007).

1. Introduction
2. Preliminaries
   - $\mathbb{F}\Gamma$
   - $L_{\mathbb{F}}(\Gamma)$
3. Reduction of problem
4. Symbolic Dynamics
5. $\partial\Gamma$ and functions
   - Chen Modules
6. New simple $L_{\mathbb{F}}(\Gamma)$ modules
   - Step functions over $\partial\Gamma$
   - A sobering example
7. References

New simple $L_{\mathbb{F}}(\Gamma)$ modules

We are interested in understanding simple $L_{\mathbb{F}}(\Gamma)$ modules. The complete classification is perhaps wild (i.e., impossible). We provide infinitely many new non isomorphic simple modules when Γ has intersecting cycles. When the cycles are disjoint, all simple modules are Chen (Ara, Rangaswamy 2014).

We first reduce the problem to that of a strongly connected digraph Γ.

Fix a module M over $L_{\mathbb{F}}(\Gamma)$.

$V_M := support(M) = \{v \in V | Mv \neq 0\}$ is the support of M.

Γ_M is the full subgraph on V_M .

We note that the action of $L_{\mathbb{F}}(\Gamma)$ is identical to that of $L_{\mathbb{F}}(\Gamma_M)$ because

$$L_{\mathbb{F}}(\Gamma_M) \cong L_{\mathbb{F}}(\Gamma) \Big/ (V \setminus V_M).$$

Considering M as an $L_{\mathbb{F}}(\Gamma_M)$ module, we may assume $\Gamma = \Gamma_M$.

Vertex u is a **predecessor** of v, denoted $u \rightsquigarrow v$, if there is a path p of Γ such that $s(p) = u$ and $t(p) = v$. This defines a preorder on V. If both u and v are predecessors of each other, then we say that u and v are equivalent. For example, if $p = e_1 \cdots e_n$ is a cycle, then $\{s(e_i)\}_{i=1}^{n}$ are all equivalent.

Lemma

For a simple $L_{\mathbb{F}}(\Gamma)$ module M, there is a $w \in V$ such that $supp(M) = \{u \rightsquigarrow w | u \in V\}$.

Let $[w]$ be the set of vertices equivalent to w and $\Gamma_{[w]}$ the full subgraph on $[w]$.

The category of $L_{\mathbb{F}}(\Gamma_{[w]})$ modules is equivalent to the full subcategory of $L_{\mathbb{F}}(\Gamma)$ modules N such that N is generated by Nw, that is $N = NwL_{\mathbb{F}}(\Gamma)$. Since $M = MwL_{\mathbb{F}}(\Gamma)$, we may restrict attention to $L_{\mathbb{F}}(\Gamma_{[w]})$.

After the previous reductions of the algebra, we will now assume Γ is strongly connected. There are three possibilities for Γ:

- Γ is a single vertex
- Γ is a cycle
- Γ contains multiple cycles

For the first case, $L_{\mathbb{F}}(\Gamma) \cong \mathbb{F}$, and any simple module M is a one dimensional vector space over $\mathbb{F}$. This vertex corresponds to a sink w in the original digraph, and the module over $L_{\mathbb{F}}(\Gamma)$ is the projective simple module $wL_{\mathbb{F}}(\Gamma)$. All projective simple modules over LPAs are of this form.

In the second case, $L_{\mathbb{F}}(\Gamma)$ is Morita equivalent to $\mathbb{F}[x, x^{-1}]$. Since this is a PID (all ideals are generated by one element) we know all simple modules over it. A simple module M over $L_{\mathbb{F}}(\Gamma)$ is a Chen module, or a twisted one.

Our work is focused on the third case where we produce non-Chen modules.

1. Introduction
2. Preliminaries
   - $\mathbb{F}\Gamma$
   - $L_{\mathbb{F}}(\Gamma)$
3. Reduction of problem
4. Symbolic Dynamics
5. $\partial\Gamma$ and functions
   - Chen Modules
6. New simple $L_{\mathbb{F}}(\Gamma)$ modules
   - Step functions over $\partial\Gamma$
   - A sobering example
7. References

Symbolic Dynamics

We start with a finite set of symbols $\mathcal{A}$ that we call an *alphabet*. Elements of the alphabet are called *letters* or *symbols*. Finite sequences of letters are called *words*.

One can construct bi-infinite (or infinite) sequences of letters indexed by $\mathbb{Z}$ (or $\mathbb{N}$). For our purposes, we will focus on the infinite case. The *full* $\mathcal{A}-$*shift* is denoted by

$$\mathcal{A}^{\mathbb{N}} = \{x = (x_i)_{i \in \mathbb{N}} \,|\, x_i \in \mathcal{A} \text{ for all } i \in \mathbb{N}\}$$

This set is given the product topology.

The *shift map* $\sigma : \mathcal{A}^{\mathbb{N}} \longrightarrow \mathcal{A}^{\mathbb{N}}$ maps any point $x \in \mathcal{A}^{\mathbb{N}}$ to the point $y = \sigma(x)$ such that $y_i = x_{i+1}$ for all $i \in \mathbb{N}$. These ingredients (a full shift and a shift map) comprise a *dynamical system*.

A *shift space* $X_{\mathcal{F}} \subseteq \mathcal{A}^{\mathbb{N}}$ is a closed, shift invariant subset of a full shift space. Here, shift invariant means that $\sigma(X_{\mathcal{F}}) \subseteq X_{\mathcal{F}}$. Equivalently, $X_{\mathcal{F}}$ is defined by a set $\mathcal{F}$ of *forbidden words* (no forbidden word can appear as a substring for any point in $X_{\mathcal{F}}$).
One kind of shift space we will be interested in is a *Shift of Finite Type (SFT)*, where $\mathcal{F}$ is a finite set. We will also be interested in Shift spaces that are *irreducible* which means that for any two allowed words u and w, there is an allowed word v such that uvw is an allowed word.

Examples of shift spaces

Let $E^{\mathbb{N}}$ be a full shift space, where E is the arrows of the directed (finite) graph Γ.
Let $\mathcal{F} = \{ef \mid e, f \in E \text{ and } t(e) \neq s(f)\}$.
$X_{\mathcal{F}} = \partial\Gamma = \{e_1e_2e_3 \cdots \mid e_i \in E,\ t(e_i) = s(e_{i+1})\ \forall i \in \mathbb{N}\}$ is the set of infinite paths in Γ. We place a topology on $\partial\Gamma$ which is the restriction of the product topology on $E^{\mathbb{N}}$ (with the discrete topology on E). Given a path p in Γ, our basic open sets are $p\partial\Gamma$, the set of infinite paths in Γ that start with the path p. We have $\partial\Gamma$ as a closed subset of $E^{\mathbb{N}}$, hence a compact metrizable space.
We will introduce some notation here: for any $L_{\mathbb{F}}(\Gamma)$ module M and for any $m \in M$, we have the set

$$\partial\Gamma_m := \{e_1e_2e_3 \cdots \in \partial\Gamma \mid me_1e_2 \cdots e_n \neq 0\ \forall n \in \mathbb{N}\}.$$

Leaning into modules

We will define an action of $V \sqcup E \sqcup E^*$ on the one-point compactification of $\partial\Gamma$ (the extra point will be denoted as ∞). We will act on the left, for reasons which will become clear.

$$\text{for } p \in V \sqcup E \sqcup E^*, \alpha = e_1 e_2 e_3 \cdots \in \partial\Gamma \qquad p \cdot \alpha = \begin{cases} p\alpha & pe_1 \neq 0 \\ \infty & \text{else} \end{cases}$$

$$\text{for } p \in V \sqcup E \sqcup E^* \qquad p \cdot \infty = \infty$$

The action of $\sum_E e$ is very similar to that of σ of a shift space.

1. Introduction
2. Preliminaries
   - $\mathbb{F}\Gamma$
   - $L_{\mathbb{F}}(\Gamma)$
3. Reduction of problem
4. Symbolic Dynamics
5. $\partial\Gamma$ and functions
   - Chen Modules
6. New simple $L_{\mathbb{F}}(\Gamma)$ modules
   - Step functions over $\partial\Gamma$
   - A sobering example
7. References

$\partial\Gamma$ and functions

We will define an action of $L_{\mathbb{F}}(\Gamma)$ on functions f from $\partial\Gamma \sqcup \{\infty\}$ to $\mathbb{F}$ that vanish at ∞ as follows:

$$\text{for } p \in V \sqcup E \sqcup E^*, \alpha \in \partial\Gamma, \quad (f \cdot p)(\alpha) = f(p \cdot \alpha)$$

This extends with composition and linearity to all functions that vanish at ∞.

Two infinite paths are tail equivalent ($\sim$) if they agree from some point on, which means

$$e_1 e_2 e_3 \cdots \sim f_1 f_2 f_3 \cdots \text{if there is } m, k \in \mathbb{N} \text{ such that } \forall n \in \mathbb{N}, e_{m+n} = f_{k+n}.$$

A quick example is : Given $\Gamma = e \circlearrowright \bullet \circlearrowleft f$, all of the following infinite paths are tail equivalent.

$$efefeeeeee \cdots \sim eeeee \cdots \sim fffeeeee \cdots$$

We will denote the tail equivalence class of an infinite path α by $[\alpha]$. Notice that α and $p\alpha$ have the same tail equivalence class.

One kind of simple $L_{\mathbb{F}}(\Gamma)$ modules appear as submodules of the space of functions on $\partial\Gamma \sqcup \{\infty\}$ that vanish at infinity are called Chen Modules. These simple modules will be generated by indicator functions of a single infinite path.

$$\mathbb{1}_\alpha : \partial\Gamma \longrightarrow \mathbb{F}$$

$$\mathbb{1}_\alpha(x) = \begin{cases} 1 & x = \alpha \\ 0 & x \neq \alpha \end{cases}$$

A quick examination of the action yields that a generic element of this module will be a linear combination of indicator functions of infinite paths that are all tail equivalent to α.

1 Introduction

2 Preliminaries
- $\mathbb{F}\Gamma$
- $L_{\mathbb{F}}(\Gamma)$

3 Reduction of problem

4 Symbolic Dynamics

5 $\partial\Gamma$ and functions
- Chen Modules

6 New simple $L_{\mathbb{F}}(\Gamma)$ modules
- Step functions over $\partial\Gamma$
- A sobering example

7 References

Our new simple $L_{\mathbb{F}}(\Gamma)$ modules appear as submodules of the space of **step functions** on $\partial\Gamma$, that is, linear combinations of indicator functions of closed sets.

$$\mathbb{1}_S : \partial\Gamma \longrightarrow \mathbb{F}$$
$$\mathbb{1}_S(x) = \begin{cases} 1 & x \in S \\ 0 & x \notin S \end{cases}$$

Lemma

If $f \in \mathbb{F}^{\partial\Gamma}$, then $\partial\Gamma_f = \overline{\partial\Gamma \setminus f^{-1}(0)}$.

Lemma

If $f \in \mathbb{F}^{\partial\Gamma}$ is an indicator function $\mathbb{1}_S$, then $\partial\Gamma_f = \overline{S}$.

Lemma

If $f \in \mathbb{F}^{\partial\Gamma}$ is a step function $\sum_i \lambda_i \mathbb{1}_{S_i}$, then $f = \sum_i \lambda_i \mathbb{1}_{S_i \cap \partial\Gamma_f}$

When we have indicator functions of closed sets, we also have indicator functions of *locally closed sets*, which are intersections of open sets and closed sets. We also have indicator functions of *constructible sets*, which are finite unions of locally closed sets. This is detailed in the following lemma.

Lemma

The following are equivalent:

1. $f = \sum_{i=1}^{n} \lambda_i \mathbb{1}_{S_i}$, *where* S_i *are nonempty, closed sets.*
2. $f = \sum_{i=1}^{m} \mu_i \mathbb{1}_{B_i}$, *where* b_i *are nonempty, disjoint, locally closed sets.*
3. f *has finite image and* $\{f^{-1}(\mu_i)\}$ *are constructible for each* $\mu_i \in \mathbb{F}$.

Proof

(1) $\implies$ (2) We have
$f = \sum_{i=1}^{n} \lambda_i \mathbb{1}_{S_i}$, where S_i are nonempty closed sets. Let $S_i^1 := S_i$, and $S_i^0 := \partial\Gamma \setminus S_i$. Consider each set $A_{\mathbf{a}}$, nonempty locally closed sets which are disjoint, where:

$$A_{\mathbf{a}} = \bigcap_{\substack{\mathbf{a}=\{a_1,\dots,a_n\}\in\{0,1\}^n \\ \bigcap_{i=1}^{n} S_i^{a_k} \neq \varnothing}} S_i^{a_k}$$

Since the intersection of locally closed sets is a locally closed set, we have successfully rewritten $f = \sum_{i=1}^{n} \sum_{a_i=1} \lambda_i \mathbb{1}_{A_{\mathbf{a}}}$ as a linear combination of indicator functions of nonempty, disjoint, locally closed sets.

Proof

(2) $\implies$ (3) We have $f = \sum_{i=1}^{m} \mu_i \mathbb{1}_{B_i}$, where B_i are nonempty, disjoint, locally closed sets. For any μ_i in the image of f, notice $f^{-1}(\mu_i)$ is a finite union of B_i, which makes $f^{-1}(\mu_i)$ a constructable set. Thus $f = \sum_{i=1}^{k} \mu_i \mathbb{1}_{f^{-1}(\lambda_i)}$, a linear combination of indicator functions of disjoint constructable sets.

Proof

(3) $\implies$ (1) We have
$f = \sum_{i=1}^{k} \mu_i \mathbb{1}_{f^{-1}(\mu_i)}$, where $\{f^{-1}(\mu_i)\}$ are constructable disjoint sets. The $\{f^{-1}(\mu_i)\}$ are disjoint since $f^{-1}(\mu_i)$ is disjoint from $f^{-1}(\mu_j)$ for $i \neq j$ Consider that we can write $\{f^{-1}(\mu_i)\}$ as a finite union of locally closed sets. Since the intersection of two locally closed sets is a locally closed set, note that for locally closed sets A and B, we have $\mathbb{1}_{A\cup B} = \mathbb{1}_A + \mathbb{1}_B - \mathbb{1}_{A\cap B}$. For any locally closed set $U \cap F$ where U is open and F is closed, we have $\mathbb{1}_{U\cap F} = \mathbb{1}_F - \mathbb{1}_{U^c\cap F}$. Iterating these formulas, we can rewrite f as a linear combination of indicator functions of closed sets.

Lemma

If $f = \sum_i \lambda_i \mathbb{1}_{S_i} \neq 0$ is linear combination of indicator functions with S_i distinct nonempty closed subsets of $\partial\Gamma$, then there exists a path p such that $fp = \lambda \mathbb{1}_S$ with S closed, $S \neq \varnothing$.

Proof.

We can rewrite f as $\sum_j \lambda_j \mathbb{1}_{S_j \cap \partial\Gamma_f}$, a linear combination of indicator functions of nonempty closed sets. If there is more than one summand, choose an $\alpha \in \partial\Gamma_f$ that is not in all $S_j \cap \partial\Gamma_f$. Without loss of generality, $\alpha \notin S_1 \cap \partial\Gamma_f$. Choose an initial segment of α, say $e_1 \cdots e_n$ which does not extend to anything in $S_1 \cap \partial\Gamma_f$. When you act on $\sum_j \lambda_j \mathbb{1}_{S_j \cap \partial\Gamma_f}$ by $e_1 \cdots e_n$, you get something nonzero ($\alpha \in \partial\Gamma_f$) with fewer summands. By repeating this argument, you can act on f with a path and get a single indicator function. $\square$

Lemma

Each simple submodule of the module of all step functions is generated by an indicator function.

Then the question remains of which indicator functions generate simple modules. It will be helpful to change our view of the algebra we are working with once more.

Definition

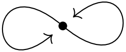
Given a directed graph Γ without sinks, define another directed graph Γ', the maximal out-split of Γ. The vertices of Γ' are the arrows in Γ, and its arrows are paths of length two in Γ. The first arrow of this path corresponds to the source vertex, the second arrow corresponds to the target vertex.

Lemma

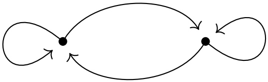
The LPAs of Γ and Γ' are isomorphic.

We obtain $\Gamma^{(n)}$ by iterating this process n times.

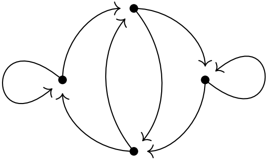
Γ :



Γ' :



Γ'' :



Lemma

If $\mathbb{1}_S \neq 0$ for S all paths on a strongly connected subgraph, then for any allowed path p in that subgraph, $(\mathbb{1}_S)p = (\mathbb{1}_S)t(p)$.

Proof.

The set of infinite paths that start with p and those that start with $t(p)$ are in $1-1$ correspondence. □

Lemma

If $(\mathbb{1}_S)v \neq 0$ for S all paths on a strongly connected subgraph and for some $v \in \Gamma$, then there is an element x of $\mathbb{F}\Gamma$ such that $(\mathbb{1}_S)x = \mathbb{1}_S$.

Proof.

Let x be a sum of paths that start at v and end with different vertices in the strongly connected subgraph that defines S, for all vertices in that subgraph. □

Theorem

All $S \subset \partial\Gamma$ where S consists of all paths on a strongly connected subgraph of some maximal outsplit of Γ define indicator functions $\mathbb{1}_S$ whose associated cyclic modules are simple.

Proof.

Consider $\mathbb{1}_S \sum_i \lambda_i p_i q_i^*$, where $\sum_i \lambda_i p_i q_i^* \in L(\Gamma)$ such that $\mathbb{1}_S \sum_i \lambda_i p_i q_i^* \neq 0$. We can find a path r longer than any of the q_i such that $\mathbb{1}_S \sum_i \lambda_i p_i q_i^* r \neq 0$. By this and a previous lemma, we obtain $\lambda \mathbb{1}_S t(r) \neq 0$. By the previous lemma, there is an element $x \in \mathbb{F}\Gamma$ (scaled appropriately) that will send $\lambda \mathbb{1}_S t(r)$ to $\mathbb{1}_S$. Thus, we can go from any generic element of the cyclic module back to the generator $\mathbb{1}_S$ via the action of $L(\Gamma)$, so the module is simple. □

Theorem (Özaydın, P.)

If S is the set of all infinite paths on some strongly connected full subgraph $\Gamma^{(n)}$ for some $n \in \mathbb{N}$, then the module generated by the indicator function of S is simple.

This gives us infinitely many non-isomorphic examples of simple modules. When M is simple, we have:

$$\partial\Gamma_M := \bigcup_{0 \neq m \in M} \partial\Gamma_m \qquad \partial\Gamma_M/_{\sim} = \partial\Gamma_m/_{\sim}$$

This set is the (tail equivalent) infinite paths along which M survives. For any Chen module, this will always be in a single tail equivalence class. However, for any Γ that is strongly connected with multiple cycles, there are infinity many S on some maximal outsplit whose indicator function survives on many paths that are not tail equivalent.

Here are some quick examples: For $\Gamma = e \circlearrowright \bullet \circlearrowleft f$:

- $S = \partial\Gamma$ is irreducible. Notice that the indicator function $\mathbb{1}_{\partial\Gamma}$ survives along all infinite paths in Γ.
- $S = \{eee\cdots\}$, of all paths that avoid f. This is the same as the strongly connected subgraph of the maximal outsplit with only the vertex f. This is also a Chen module, which shows how rational Chen modules (Chen modules where there is an infinite path in the tail equivalence class that is periodic) can be thought of as defined by the set S. In general, Chen modules are generated by indicator functions of singletons.
- S is all paths on a strongly connected subgraph avoiding a particular arrow of the second iteration of the maximal outsplit (let us say that arrow corresponds to p in the original graph). In this case, notice that $ppp\cdots$ is in $\partial\Gamma_{\mathbb{1}_{\partial\Gamma}}$, but not in $\partial\Gamma_{\mathbb{1}_S}$, demonstrating that the modules generated by these indicator functions would be different. It is also easy to show that there would be infinity many non-tail equivalent infinite paths that avoid p, showing this module is not a Chen module.

A natural (if a bit hopeful) question to ask is if all simple modules over $L_{\mathbb{F}}(\Gamma)$ are generated by step functions on $\partial\Gamma$. **No**. Here is an example: For $\Gamma = e \circlearrowright \bullet \circlearrowleft f$: consider the action of the generators on Laurent polynomials with powers not divisible by 3 over a field with characteristic not equal to 2:

$$x^k e = \begin{cases} x^{k/2} & k \text{ is even} \\ x^{(k-3)/2} & k \text{ is odd} \end{cases}$$

$$x^k f = \begin{cases} x^{k/2} & k \text{ is even} \\ -x^{(k-3)/2} & k \text{ is odd} \end{cases}$$

$$x^k e^* = \tfrac{1}{2}(x^{2k} + x^{2k+3}), \qquad x^k f^* = \tfrac{1}{2}(x^{2k} - x^{2k+3})$$

We can see that $\partial\Gamma_x = \partial\Gamma$. However, there is no fixed nonzero Laurent polynomial that is invariant under the action of $V \sqcup E$ (equivalently, there is no $1-$dimensional submodule for the path algebra), so this is not isomorphic to the module generated by $\mathbb{1}_{\partial\Gamma}$.

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THANK YOU!