

# Parencletic Networks applied to credit card fraud detection

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The results presented in this talk are part of some **joint works** with:

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- **Massimiliano Zanin** (CSIC-IFISC, Spain)

They have been developed in **collaboration** with:

- National Institute of Cybersecurity - **INCIBE** (León, Spain),
- Rey Juan Carlos University - **URJC** (Madrid, Spain)
- Technological Institute of Data, Networks and Cybersecurity - **DCNC** (Madrid, Spain)



- ① Setting the problem: credit card fraud detection
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- ③ A real application to credit card fraud detection
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  - 3.3. Some results
- ④ Conclusions and related works

# I. Setting the problem



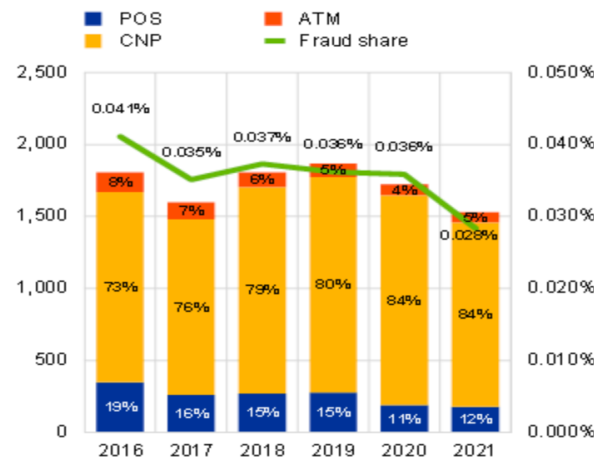
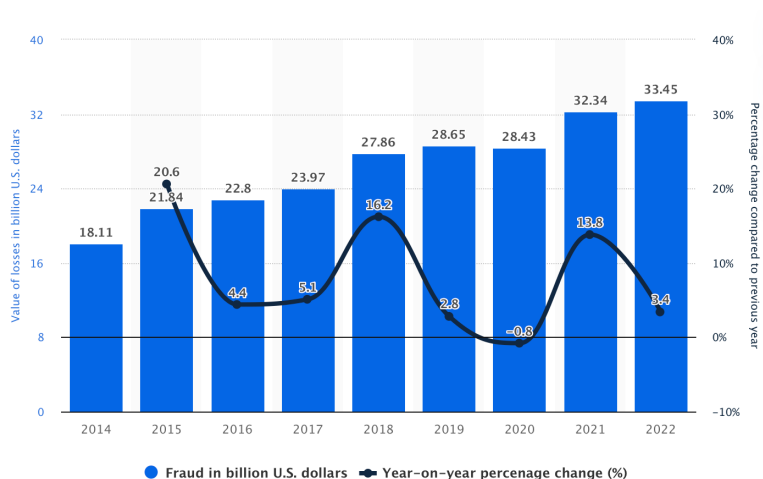
The exponential growth in **e-commerce** has resulted in an increasing number of **credit card frauds** and therefore significant financial costs.





# I. Setting the problem

The exponential growth in **e-commerce** has resulted in an increasing number of **credit card frauds** and therefore significant financial costs.



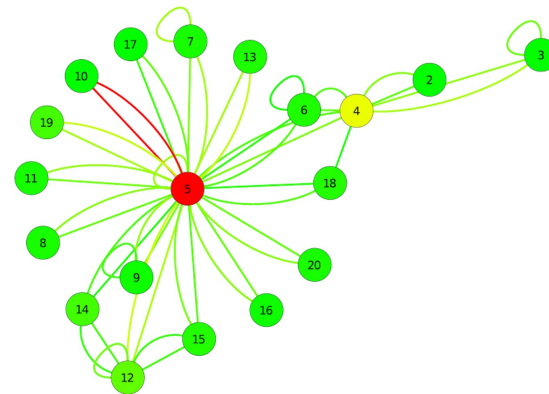
EUROPEAN CENTRAL BANK

Techniques used in order to detect such fraudulent credit card transactions include:

- **Classic learning** (Rule-based and known fraud patterns).
- **Machine learning.**

## Question

Can we use **complex networks** to detect **credit card frauds**?







**Parenclitic networks** allow representing time independent, scalar data sets as complex networks, as follows:

- We start from a dataset  $D$  with a **pre-labelled subset** of subjects  $S \subset D$ .
- For every subject  $x \in D$ , we will construct a complex network  $PN(x)$  that measures the **divergence** between  $x$  and the test set  $S$ .

In fact the term **parenclitic** comes from the word **παρεγκλισις**, the Greek term for '**deviation**', originally used by the Greek philosopher **Epicurus** to designate the spontaneous and unpredictable swerving of free-falling atoms.



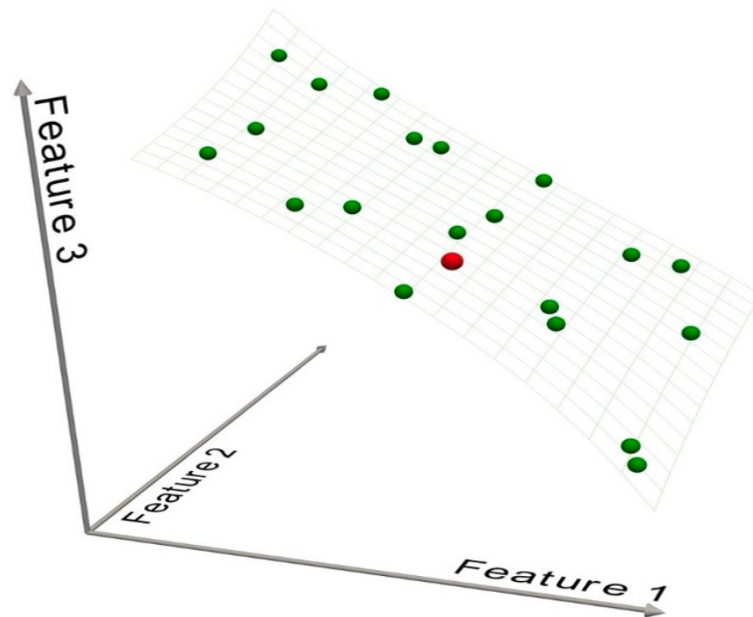
## 2. Parenclitic networks' Analysis



Given a dataset  $D = \{t_1, \dots, t_n\}$ :

- each subject  $t_i \in D$  is a feature vector  $t_i = (f_{i1}, \dots, f_{id})$ ,
- we have fixed a **pre-labelled subset** of subjects  $S \subset D$  (same subjects).

We construct a function  $H : \mathbb{R}^d \longrightarrow \mathbb{R}$  and  $\delta > 0$  such that if  $|H(t_i)| \leq \delta$ , then  $t_i \in S$ .



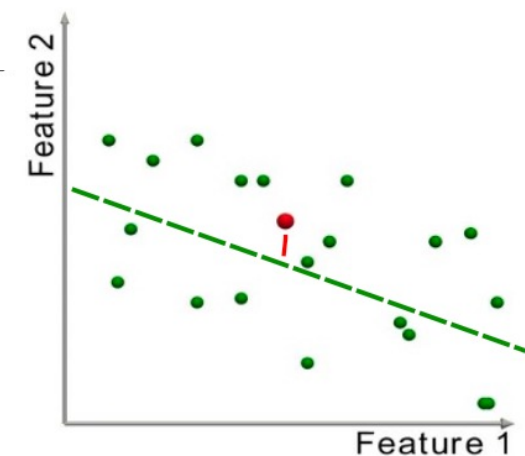
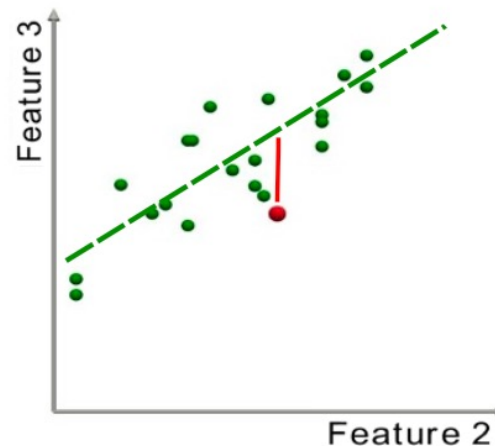
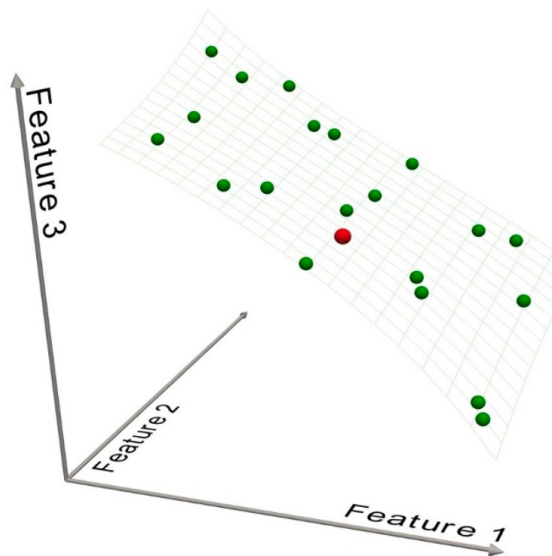


## 2. Parenclitic networks' Analysis



In general, **computing** such function  $H : \mathbb{R}^d \longrightarrow \mathbb{R}$  and  $\delta > 0$  is **not possible**, since it is a **high dimensional** problem, so we **project** the problem into  $\mathbb{R}^2$ .

For every  $1 \leq i, j \leq d$ , we consider the **projection**  $\pi_{ij} : \mathbb{R}^d \longrightarrow \mathbb{R}^2$  and construct a function  $H_{ij} : \mathbb{R} \longrightarrow \mathbb{R}$  which is **approximately** the  **$(i, j)$ -projection** of  $H$ .

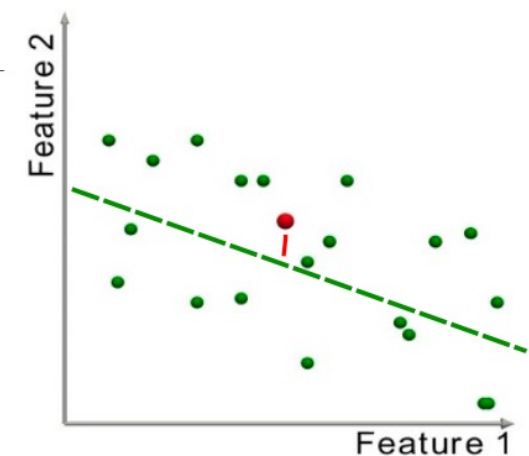
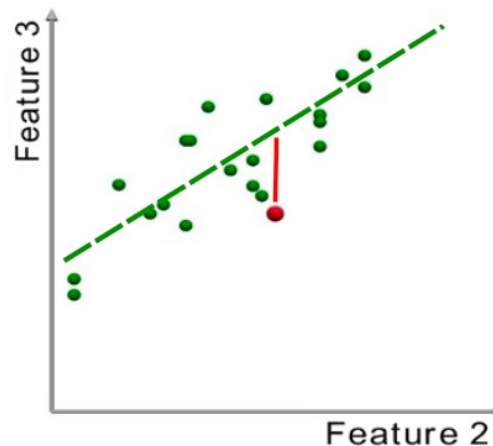
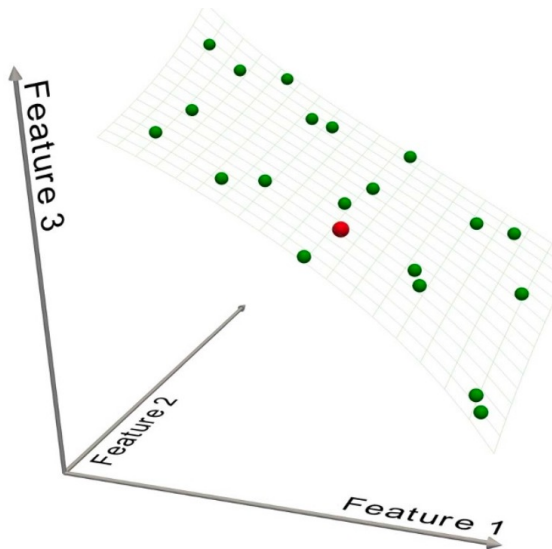


## 2. Parenclitic networks' Analysis



In practice, functions  $H_{ij} : \mathbb{R} \rightarrow \mathbb{R}$  are constructed **separately** from  $H$ , and they can be computed by any classic **curve-fitting technique**. For example, later we will use **linear regression** formalism and consider

$$H_{ij}(t_i) = a_{ij}t_i + b_{ij}.$$

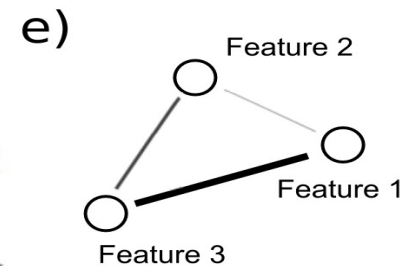
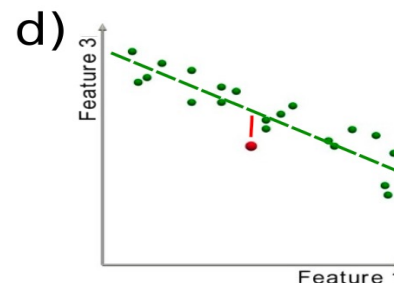
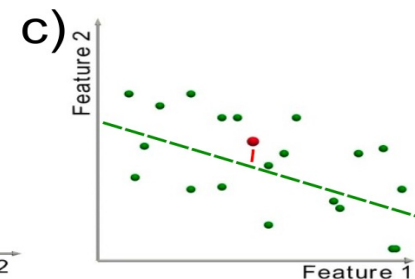
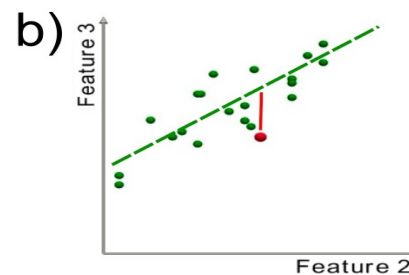
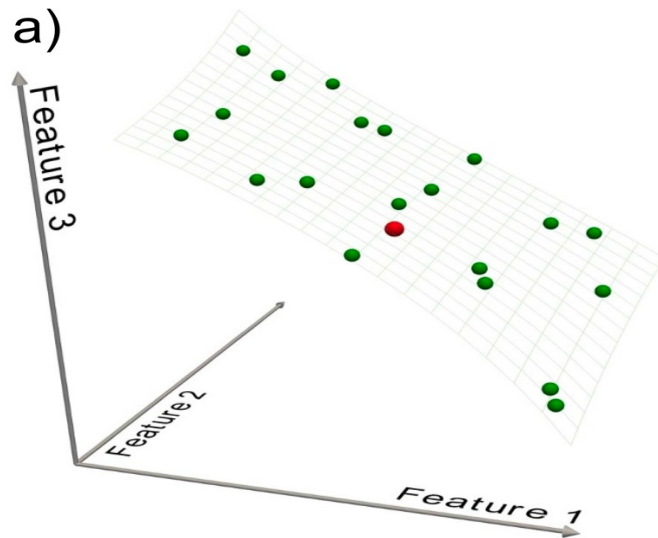




### Definition 2.1.

For every subject  $t_i = (f_{i1}, \dots, f_{id}) \in D$ , its (weighted) parenclitic network  $PN(t_i) = (V, E)$ , given by

- $V = \{1, \dots, d\}$  (i.e. nodes are the features).
- $w_{jk} = d(\pi_{jk}(t_i), H_{jk}(f_{ij}))$  (i.e. link weights are deviations from the expected features).

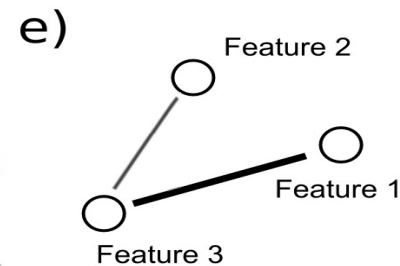
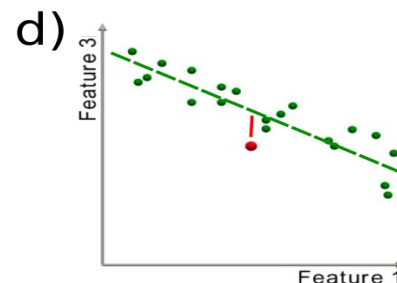
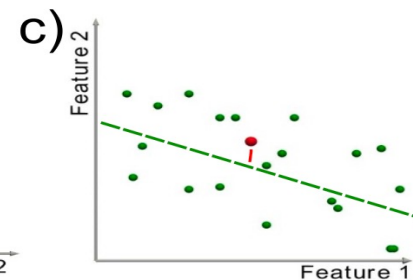
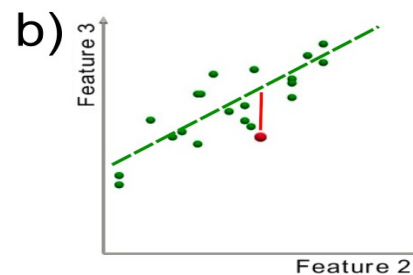
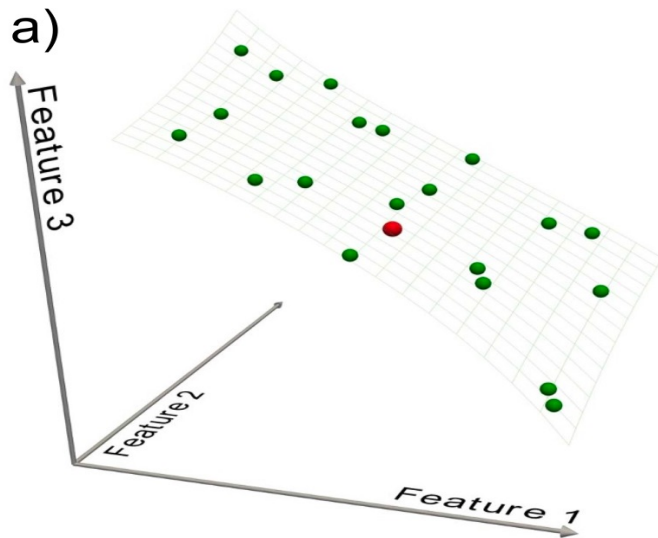




### Definition 2.2.

Given a threshold  $\theta \geq 0$  and a subject  $t_i = (f_{i1}, \dots, f_{id}) \in D$ , its **parenclitic network**  $PN(t_i, \theta) = (V, E)$  is given by

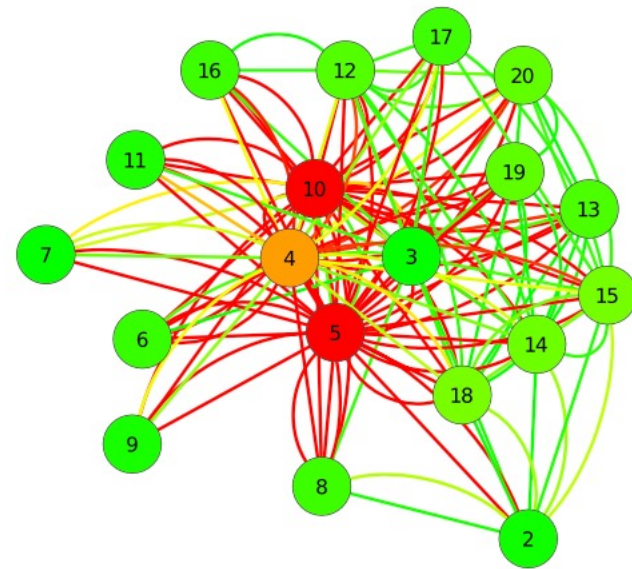
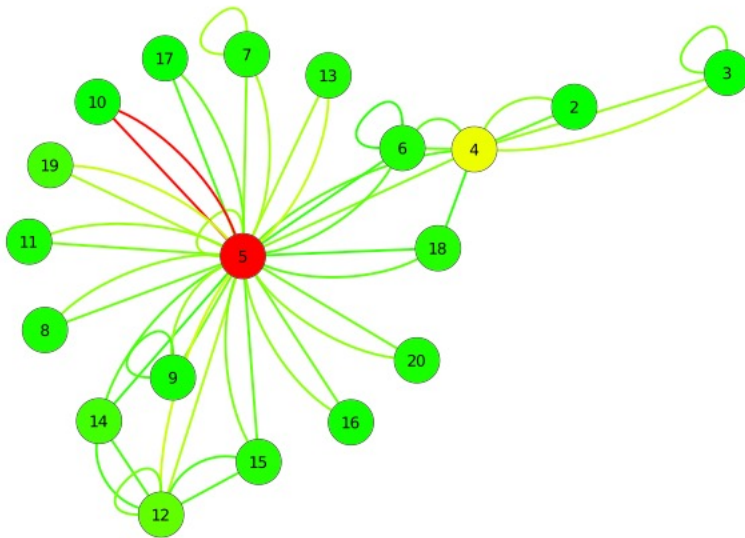
- $V = \{1, \dots, d\}$  (i.e. **nodes** are the **features**).
- $\{j, k\} \in E$  if  $d(\pi_{jk}(t_i), H_{jk}(f_{ij})) \geq \theta$  (i.e. **links** are pairs of features with **abnormal correlation**).



## 2. Parenclitic networks' Analysis



- If  $\theta = 0$ , then  $PN(t_i, \theta)$  is a **complete graph** for all subject  $t_i \in D$ .
- If  $\theta$  is **big enough**, then  $PN(t_i, \theta)$  is an **empty graph** for all subject  $t_i \in D$ .
- for  $\theta$  **in-between**,  $PN(t_i, \theta)$  **differentiates**  $S$  and  $D \setminus S$ .



## 2. Parenclitic networks' Analysis



- Originally, **parenclitic networks** were considered for **identifying critical genes** in **genomics**.
- In particular, it was used to spot **critical genes** of the plant **Arabidopsis thaliana** responsible of the plant **response** under **osmotic stress** (Zanin2014).

### Question

Can we use such **parenclitic networks** in order to work on **supervised classification problem**?

Yes, it has been used in biomedical problems (see, for example Whitwell2018 and Zhang2022).



#### Question

Can we use **parenclitic networks** for **credit card fraud detection**?

- We consider the problem of **credit card fraud detection** as a **supervised classification problem**.
- We start with a **dataset** such that each **record** is a (past) **credit card transaction** labeled as *“fraudulent”* or *“licit”*.
- We **enrich** the features of each record/transaction  $t_i$ , by **adding** its corresponding **parenclitic network**  $PN(t_i, \theta)$ .
- **Threshold**  $\theta$  can be considered as a **variable** of the problem or as an **external parameter**.



### 3. Application to fraud detection

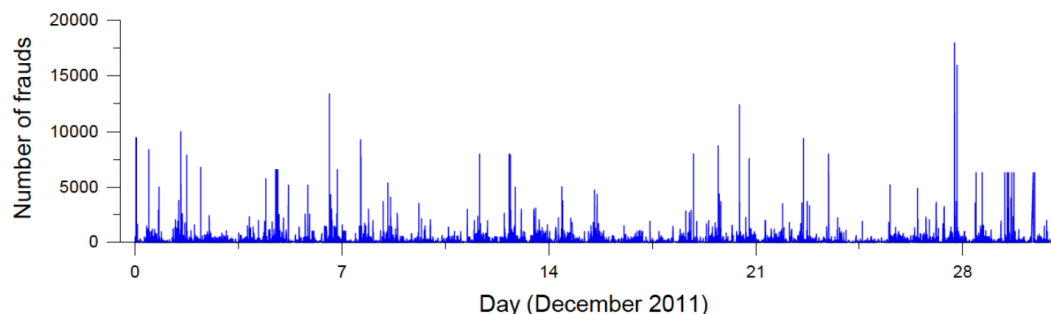


We have test the proposed method in two scenarios:

- **Synthetic datasets** based on random models (Zanin2016).
- A **real dataset** obtained from a **Global Spanish Bank**.

#### Description of the dataset

- It includes all **credit** and **debit card** transactions of clients of a major Spanish bank from January 2011 to December 2012.
- Includes all **in-person** and **virtual** (on-line) card transactions.
- Each month, an average of **15 million operations** were realized by **7 million cards**.
- Around **40,000 transaction per moth** were reported as **fraud**.



### 3. Application to fraud detection



For each transaction, the **raw features** considered were:

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In addition to this, the following features were **constructed**:

- **Time since last transaction**: Time since the last transaction of the same card.
- **Last transaction size**: Size of the previous transaction by the same card.
- **Average transaction size**: Average size of the transactions executed by the card in the last month.
- **Average time between transactions**: Average time, in seconds, between consecutive transactions of the same card.
- **Same shop**: 1 if the shop is the same of the last transaction.
- **Fraud rate**: Average rate of illegal operations, for all cards, in the last 50.000 transactions.



By using previous 8 features of each transaction  $t_i$ , it is observed that the **parenclitic network**  $PN(t_i, \theta)$  of a **fraudulent transaction**:

- it has a **star-like** structure,
- it shows a **high clustering** coefficient (number of triangles),
- it has **high** efficiency,
- it exhibit a **low** Information Content.



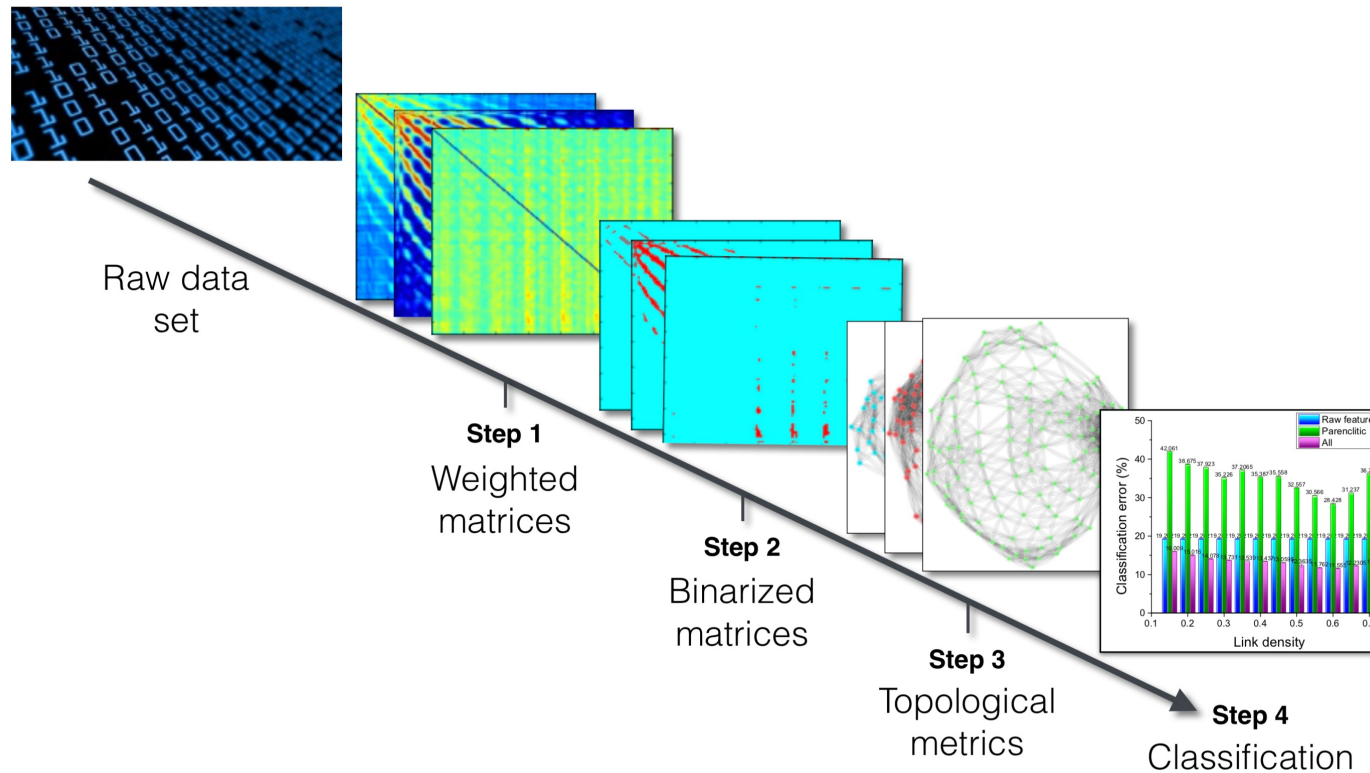
Finally, for every transaction  $t_i$ , its **parenclitic network**  $PN(t_i, \theta)$  have been added (for a **fixed  $\theta$** ). For this step, there are **two alternatives**:

- Adding **directly**  $PN(t_i, \theta)$ .
- Adding only several **structural parameters** of  $PN(t_i, \theta)$ .

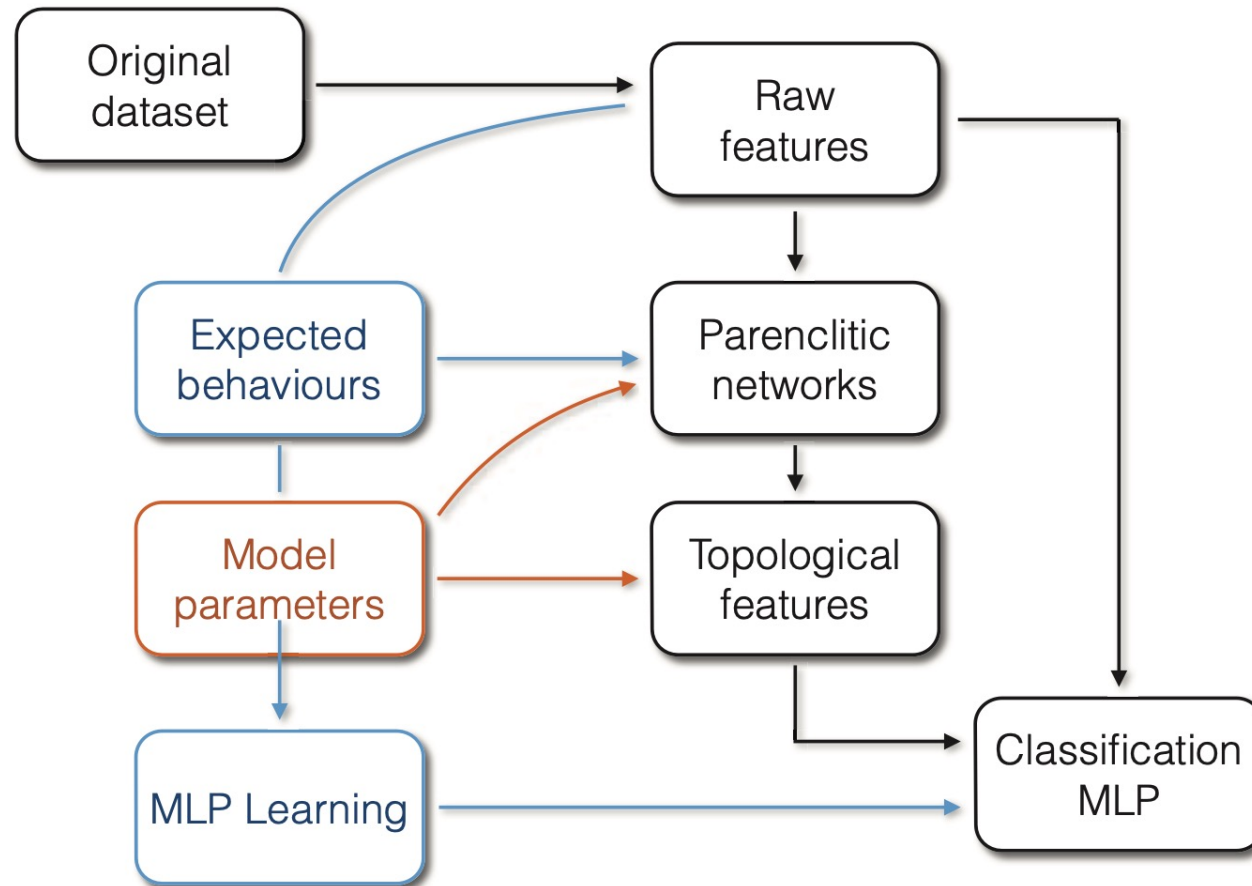
The final **structural parameters** considered were:

- Maximum node degree.
- Entropy of the degree distribution.
- Assortativity coefficient.
- Clustering coefficient.
- Average Geodesic distance.
- Efficiency.
- Information Content.

# 3. Application to fraud detection



### 3. Application to fraud detection





### 3. Application to fraud detection



In order to analyze the contribution of parenclitic networks, we consider three different scenarios:

- **Classic approach:** Taking only the (8) non-parenclitic features.
- **Pure-parenclitic approach:** Taking only the (7) parenclitic features.
- **Combined approach:** Taking the (8) non-parenclitic features and the (7) parenclitic features.



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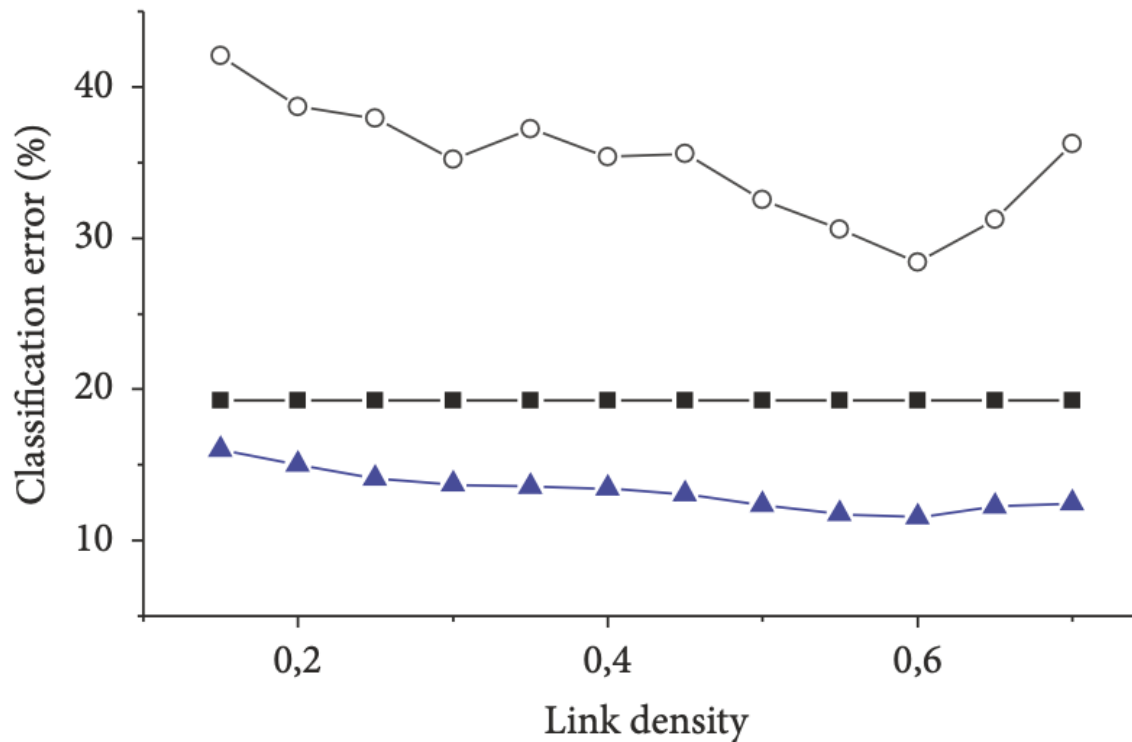
For the **classification model**, we have tested several classic method including Random Forest, SVM and Logistic Regression, but the (best) results presented used **Multi-Layer Perceptrons** with **sigmoid activation function**

$$f(x) = \frac{1}{1 + \exp(-x)}.$$

### 3. Application to fraud detection



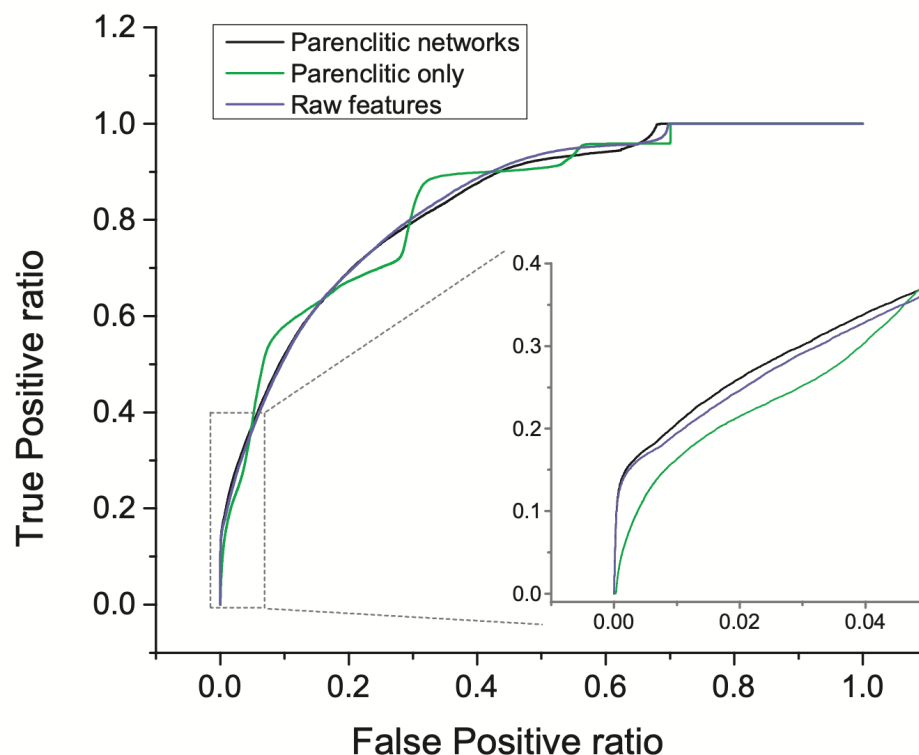
The evolution of the **classification error** as a function of the considered **link density** (related to  $\theta$ ), for the **classic** (■), **pure-parenclitic** (△) and **combined** approach (○) is the following:



### 3. Application to fraud detection



Similarly, the **Receiver Operating Characteristic** (ROC) that depicts the TPR against the FPR at various threshold settings for the **classic** (in blue), **pure-parenclitic** (in green) and **combined** approach (in black) is the following:





- Parenclitic networks' Analysis construct a network for each entry in a dataset that measures the divergence of the relations between its features.
- They give complementary information about relationships useful for supervised classification problems.
- In particular, in credit card fraud detection, a combined approach gives better detection rates.



- Parenclitic networks' Analysis construct a network for each entry in a dataset that measures the divergence of the relations between its features.
- They give complementary information about relationships useful for supervised classification problems.
- In particular, in credit card fraud detection, a combined approach gives better detection rates.
- We are also considering directly  $PN(t_i, \theta)$  in the classification model (GNN).
- Higher order networks parenclitic networks are being introduced for multi-features interactions.



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- [2] M. ZANIN, ET AL. *“Parenclitic networks: uncovering new functions in biological data”*, Sci. Rep. 4, 5112 (2014).
- [3] M. ZANIN, M. ROMANCE, S. MORAL, AND R. CRIADO. *“Credit card fraud detection through parenclitic network analysis”*, Complexity, 2018(1), 5764370 (2018).
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- [5] H. ZHANG, ET AL. *“Prognosis and Survival Modelling in Cirrhosis Using Parenclitic Networks”*, Front. Netw. Physiol. 2:833119 (2022).



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