# Beyond Instantaneous Partnerships

## Re-Examining the Force of Infection Equation in Compartmental HIV Transmission Models

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2024 Feb 13

Colloquium on Mathematics for Public Health

# Outline

- **Motivation:** modelling HIV & sex work in Eswatini
- **Instantaneous partnerships:** why, how, & issues
- **Effective Partnerships Adjustment:** a new approach
- **Experiment:** comparing approaches
- **Appendix:** mathy details

# Motivation

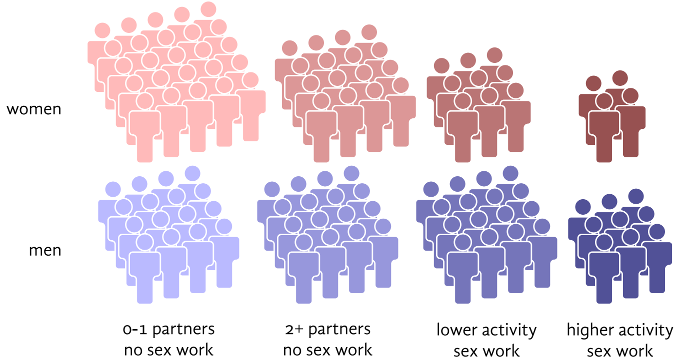
# Modelling HIV & Sex Work in Eswatini

**Research Question:**

*What unmet needs drive HIV transmission in Eswatini?*

**Model Structure:**

8 risk groups



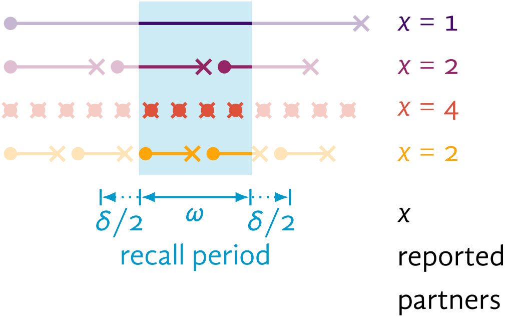
# Modelling HIV & Sex Work in Eswatini

**Model Structure:**

4 partnership types

# Quantifying Partnerships from Survey Data

*How many sexual partners (x) did you have in the past 12 months (ω)?*



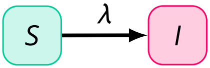

- Effective recall period: $\omega' = \omega + \delta$
- Partnership change **rate**: $Q = \dfrac{x}{\omega + \delta}$
- Current partner **number**: $K = Q\,\delta$

# Instantaneous Partnerships

# Rationale for Instantaneous Partnerships

**Problem:** compartments are *homogeneous* & *memoryless*

→ cannot track sex acts before *vs* after transmission




**Solution:** estimate *cumulative probability of transmission per partnership* ($B$)

→ multiply by average *partnership change rate* ($Q$)

$$\lambda = \sum Q \, B \, \frac{I}{N}$$

force of infection → $\lambda$

infection prevalence → $\frac{I}{N}$

partnership change rate → $Q$

transmission probability per partnership → $B$

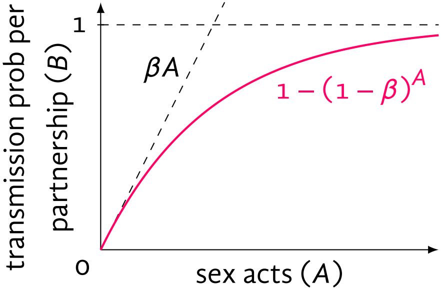
# Probability of Transmission per Partnership ($B$)


transmission probability per partnership → $B = 1 - (1 - \beta)^A$

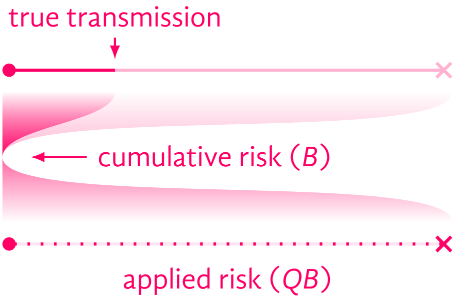
transmission probability per act → $\beta$

sex acts per partnership → $A$

$(1-\beta)^A$: probability of escaping infection



# Issue 1: Transmission is Instantaneous




- Dynamic risk within partnerships not *anticipated*
- Instant onward transmission via *same partnership*

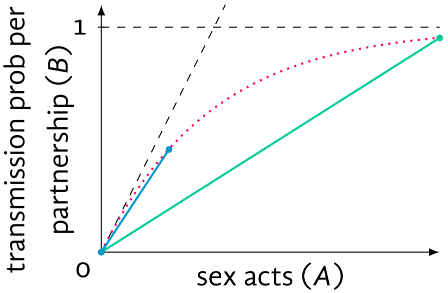
# Issue 2: Trade-Off when Adjusting for Inert Sex Acts

**“Inert Sex Acts”:**

after transmission, within the same partnership

**Adjustment** may consider:

- 1 partnership, full duration → frontload inert
- 1 partnership, 1 year → ignore inert
- all partnerships, 1 year → ignore inert



# Issue 2: Trade-Off when Adjusting for Inert Sex Acts

probability of transmission per partnership → $B = 1 - (1 - \beta)^A$

incidence rate

$$\lambda = \sum Q\,B\,\frac{I}{N}$$

- partnership-duration — $A$: acts per partnership, $Q$: change rate
- partnership-year — $A$: acts per year, $Q$: partners per year

$$\lambda = 1 - \prod \left(1 - B\frac{I}{N}\right)^Q$$

- all partnerships per year — $A$: acts per year, $Q$: partners per year

incidence proportion

implicit $\Delta_t = 1$ year

# Effective Partnerships Adjustment

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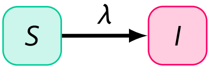
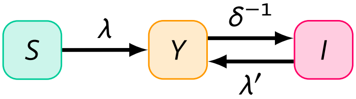
**Problem:** compartments are *homogeneous* & *memoryless*

$\longrightarrow$ cannot track sex acts before *vs* after transmission




**Solution:** track who recently *acquired* or *transmitted*

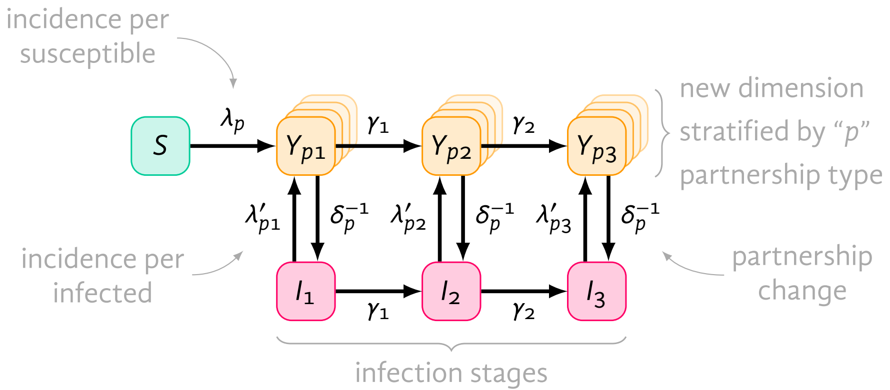
$\longrightarrow$ new "holding state" compartment ($Y$)




**Details:**

- $Y$ have 1 fewer partners for incidence $\lambda$ (mixing unchanged)
- $Y$ exit to $I$ when partners change ($\delta^{-1}$)

# Effective Partnerships Adjustment: Major Model Changes




**No "inert sex acts" adjustment** → use sex *frequency* per partner & *number* of partners

# Experiment

# Comparing FOI Approaches: Overview

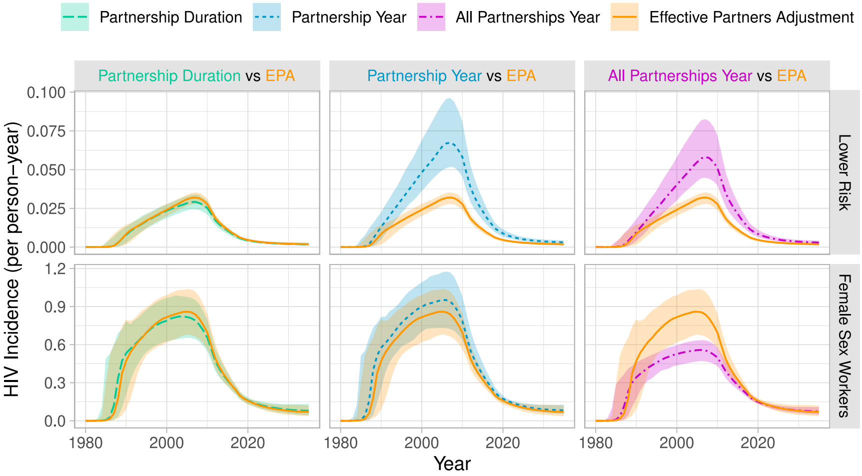
**FOI Approaches Compared:**

- Instantaneous, adjusting for:
  - Partnership Duration
  - Partnership Year
  - All Partnerships per Year
- Effective Partnerships Adjustment

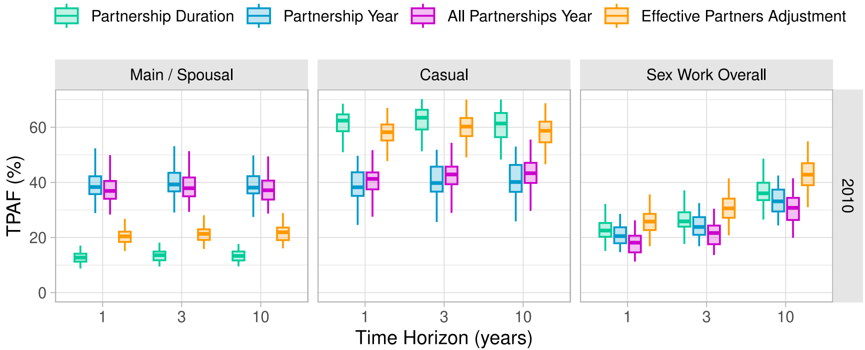
**Experiments:**

1. Equal parameters
   → compare dynamics
2. Recalibrated parameters
   → compare attributable fractions

# Comparing FOI Approaches: Dynamics with Equal Parameters



# Comparing FOI Approaches: Attributable Fraction after Recalibration



# Comparing FOI Approaches: Summary

**Partnership-Year** & **All Partnerships per Year** Adjustments:

- ignore inert sex acts → **overestimate transmission in longer partnerships**

**Partnership-Duration** Adjustment:

- frontload inert sex acts → **slightly underestimate transmission in longer partnerships**

**Effective Partnerships** Adjustment:

- track inert sex acts explicitly → **"just right" attribution of transmission?**

*What unmet needs drive HIV transmission in Eswatini?* → depends on FOI approach!

# Appendix

# Thanks

**Toronto**

Sharmistha Mishra
Huiting Ma
Linwei Wang
Korryn Bodner
Alex Whitlock
Siyi Wang
Kristy Yiu
Samantha Lo
Ekta Mishra
Michael Escobar
Rupert Kaul

**External**

Stefan Baral
Bheki Sithole
Lungile Khumalo
Sheree Schwartz
Laura Muzart
Sindy Matse
Zandile Mnisi
Marie Claude Boily
Leigh Johnson
*Survey Respondents*
*Program Implementers*

**Funding & Support**

University of Toronto
MAP Centre for Urban Health Solutions
St. Michael's Unity Health Toronto
NSERC CRSNG
Ontario
NIH
Digital Research Alliance of Canada

# Notation Summary

**Base Definitions:**

- $F$: sex frequency per partnership
- $K$: number of current partnerships
- $\delta$: partnership duration
- $A = F\delta$: sex acts per partnership
- $Q = K/\delta$: partnership change rate

**"Partnership-Year" Definitions:**

- $\delta_1 = \min(\delta, 1)$ → "up to 1 year duration"
- $Q_1 = K/\delta_1$ → "at least once per year"
- $A_1 = F$ → "sex acts per year"

# Effective Partnerships Adjustment: Force of Infection Equation

probability of transmission per sex act

effective partnerships among susceptible: $S_i K_{pi}$

sex frequency per partnership

effective partnerships among infected: $I_j K_{pj} + \sum_{\bar{p} \neq p} Y_{\bar{p}j} K_{pj} + Y_{pj}(K_{pj} - 1)$

population-level incidence →

$$\Lambda_{pij} = F_p\, \beta_{pij} M_{pij} \frac{M_{S,pi}}{M_{pi}} \frac{M_{I,pj}}{M_{pj}}$$

total mixing

incidence per susceptible →

$$\lambda_{pi} = \textstyle\sum_j \Lambda_{pij} \,/\, S_i$$

per infected →

$$\lambda'_{pj} = \textstyle\sum_i \Lambda_{pij} \,/\, I_j$$

# Effective Partnerships Adjustment: What About Multiple Transmissions?

- $Y_{pj}$ reflects % group $j$ who cannot transmit to 1 type-$p$ partner
- $Y_{pj}$ can be $> 100\%$ if number of partnerships $K_{pj} > 1$
- if $Y_{pj} > 100\%$, then $I_j$ must be *negative*, provided:

$$Y_{pj} \leq \left(I_j + \sum_{\bar{p}} Y_{\bar{p}j}\right) K_{pj} \qquad (*)$$

  i.e. cannot "remove" more partnerships than group $j$ has

- As $(*)$ approaches equality, effective partnerships among infected $M_{I,pj} \to 0$
  i.e. no transmission if all partnerships "removed"

# Double Checking the Force of Infection Equation

"Let's go. In and out. 20 minute adventure."

6 months later