L/2/01

Contents

Measurements

A measurement on a $d \times d$ density matrix D is given by a sequence $P_1, \ldots, P_k$ of $d \times d$ matrices such that:

(a) $P_i \succeq 0$,

(b) $\sum P_i = I$.

They are referred to by the acronym POVM.

you don't want to know

The outcome of this measurement is a random variable. We observe i in $\{1, \dots, m\}$ with probability

$$\langle P_i, D \rangle.$$

Note that

$$\sum_i \langle P_i, D \rangle = \langle \sum P_i, D \rangle = \langle I, D \rangle = \operatorname{tr}(D) = 1,$$

In most cases, our quantum system has dimension d and $P_i = e_i e_i^\dagger$ $(i=1,\dots,d)$.

measure w.r.t. the standard basis

Then

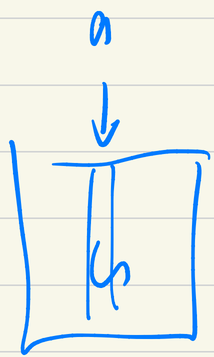
$$\langle P_i, D\rangle = tr(e_i e_i^\dagger D) = tr(e_i^\dagger D e_i) = e_i^\dagger D e_i = D_{i,i}.$$

Since D is a density, $D_{i,i} \geq 0$ and $\sum_i D_{i,i} = 1$.

If $a \in V(X)$ & $D_a := e_a e_a^T$ then

$$(U D_a U^*)_{i,i} = \left(U e_a (U e_a)^*\right)_{i,i} = |(U e_a)_i|^2$$

$$= |U_{i,a}|^2 .$$



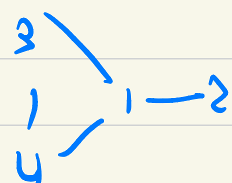
Functions & Vectors

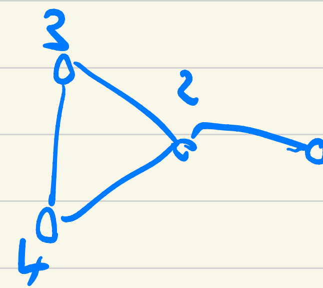
graph X

vertices V

edges E ← unordered pairs of vertices

NO LABELS!





$\mathbb{R}^n$ – functions from $\{1, \dots, n\}$ to $\mathbb{R}$

$$\begin{array}{cc} 1 & \tfrac{1}{2} \\ 2 & \tfrac{1}{3} \\ 3 & -\pi \\ 4 & 2 \end{array}$$

$f \in \mathbb{R}^V$, $\quad V = V(X)$

$$(Af)(i) = \sum_{j \sim i} f(j)$$

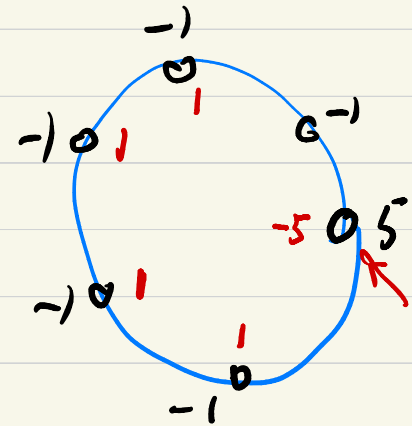
adjacency operator

K_n

regular, valency $n-1$

all-ones $\rightarrow n-1^{(1)}$

fn sums to zero $\rightarrow -1^{(n-1)}$



$Af = -f$

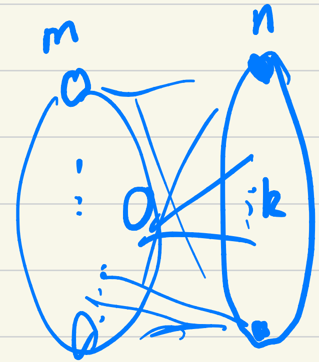
$\frac{v}{1-\boxed{\frac{k}{\tau}}}$

$A(K_n) = J - I$

For K_n $\quad \theta = n-1, \quad \tau = -1 \quad \alpha \le \frac{n}{1 - \frac{n-1}{-1}} = 1$ ratio bound for K_n

regular bipartite graphs

colour classes



ratio

$$\frac{2n}{1-\frac{k}{(-k)}} = m$$

regular: constant fns are e/vectors with e/value k

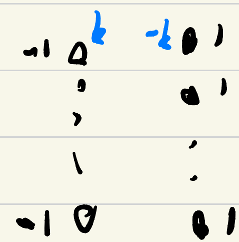
1 on one colour class

−1 " the other

e/value −k

mk nk

$m = n$

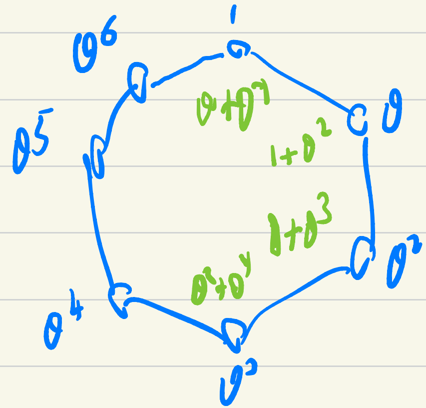


functions summing to 0 on each colour class

→ eigenvalue 0

cycles

C_7



$\theta : \theta^7 = 1 \to \theta + \theta^{-1}$

$Af = ?$

θ

$\theta^{-1} + \theta = \theta^{-1} + \theta$ ⟵ eigenvalue

$1 + \theta^2 = (\theta^{-1} + \theta)\theta$

$\theta + \theta^3 = (\theta^{-1} + \theta)\theta^2$

$\vdots$