Stability of polyhedra

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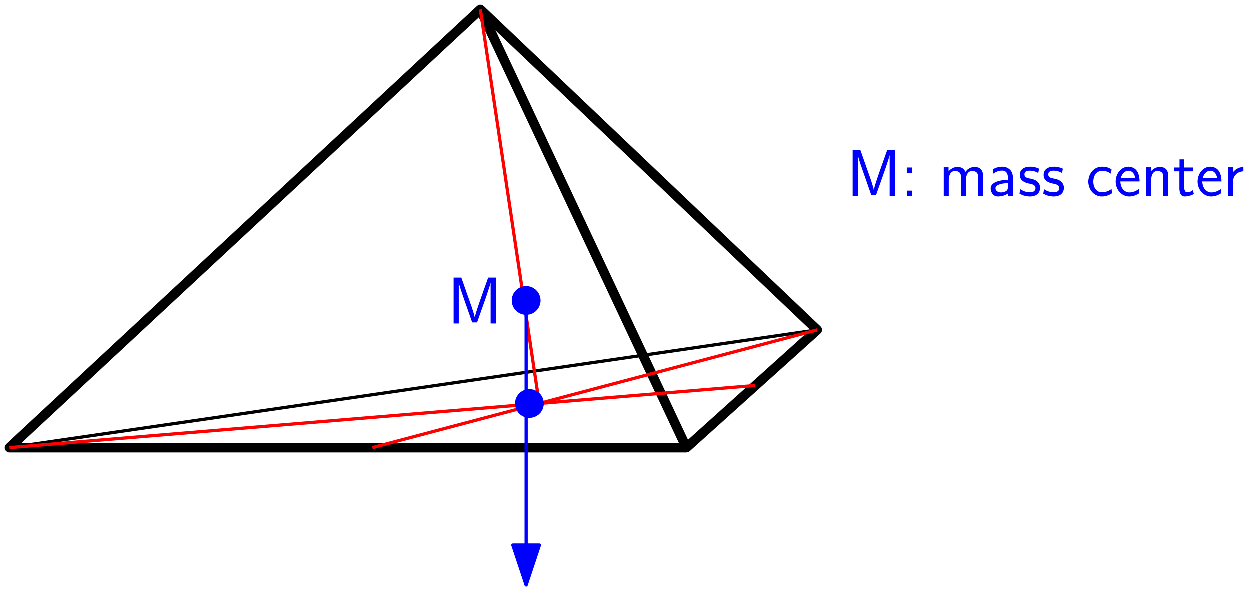
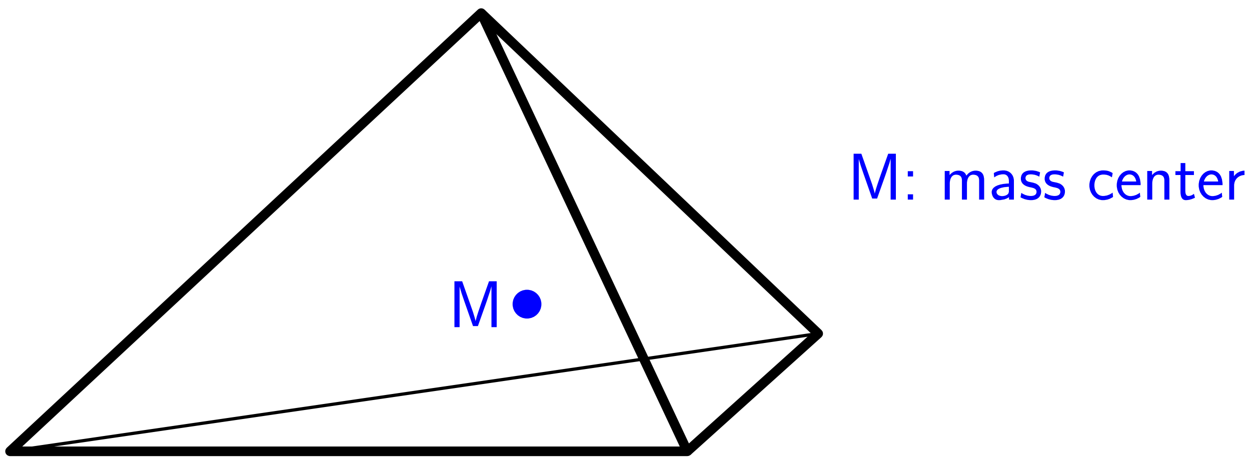
Definition:
Stable face: projection of the mass center onto the plane of a particular face is not outside of the face.



Thm: Every tetrahedron is stable on at least two faces.

Goldberg (1967) ??, J. Conway - R.Dawson (1984), Proof:

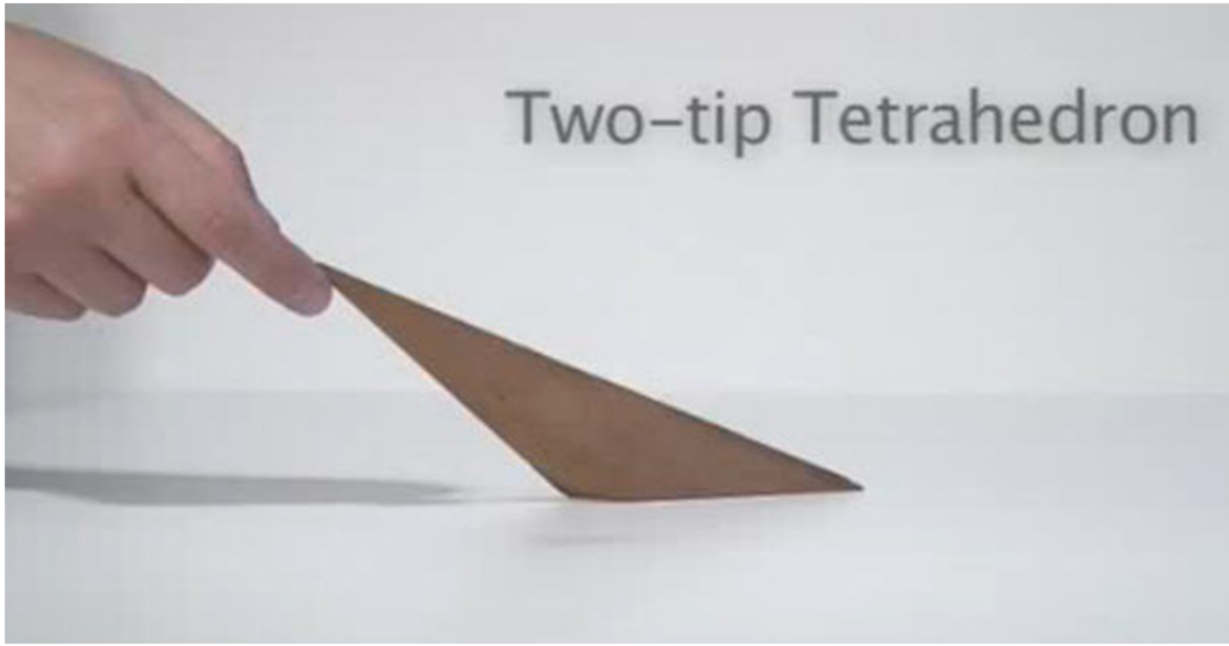


Let the faces be indexed by $1, 2, 3, 4$ so that

$$d_1 \leq d_2 \leq d_3 \leq d_4$$

,
where d_i is the distance between M and the plane of the ith face.

A. Heppes (1967)



Gömböc (2006) : a mono monostatic body by G. Domokos and P. Várkonyi

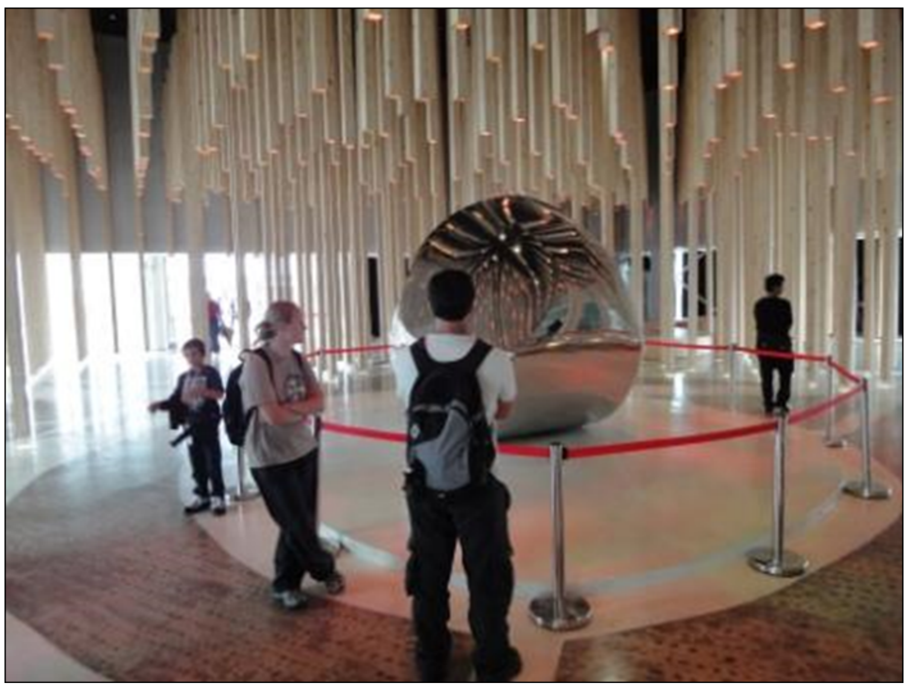


Photo:2010

Skeletal densities:

body density:	δ_V
face density:	δ_F
edge density:	δ_E

Comments on M. Goldberg (1967):
‘ Every tetrahedron has at least 2 stable faces paper:
R. Dawson mentions incompleteness in 84, refers to J. Conway.

A.B. (2011):
Every tetrahedron with:

uniform body density δV
uniform face density δF
uniform edge density δ**E**

has at least two stable faces.

Volume:	V
Surface area:	F
Total edge length:	E

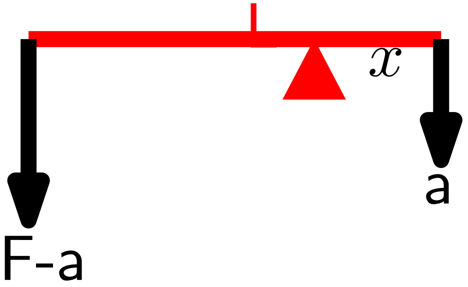
body density:	δ_V
face density:	δ_F
edge density:	δ_E

faces:	$1, \ldots 4$
face and its area:	a_i
face perimeter:	p_i

$$d(M_V, a_i) = \frac{3}{4}\frac{V}{a_i}$$

$$d(M_F, a_i) = \frac{V}{a_i}\frac{F-a_i}{F}$$

$$d(M_E, a_i) = \frac{3}{2}\frac{V}{a_i}\frac{E-p_i}{E}$$



$$x = \frac{F-a}{F}L$$

$$d(M, a_i) = \frac{\delta_V V \frac{3}{4}\frac{V}{a_i} + \delta_F F \frac{V}{a_i}\frac{F-a_i}{F} + \delta_E E \frac{3}{2}\frac{V}{a_i}\frac{E-p_i}{E}}{\delta_V V + \delta_F F + \delta_E E}$$

Questions from 1967:

- Are there polyhedra with exactly one stable face?
- If yes what is the smallest possible face number of such polyhedra?
- Is it 4?

A 19 faceted polyhedron which has exactly one stable face.

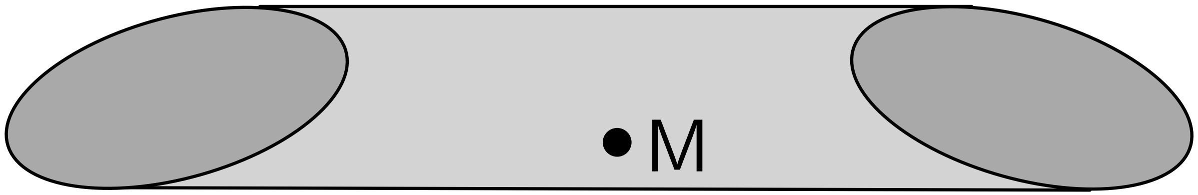
Gömböc (2006) : a mono monostatic body by G. Domokos and P. Várkonyi

Photo:2010

Through the media people were told that:

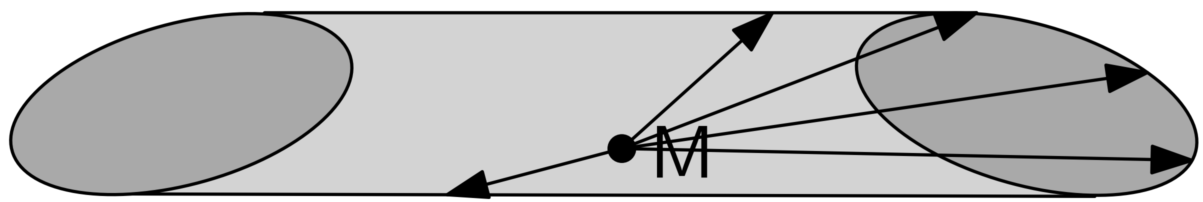
A 3D shape made out of homogeneous material, which rolls back to the same position, just like the loaded toy called 'stand up kid'.

G. Domokos and P. Varkonyi



Sliced solid tube.

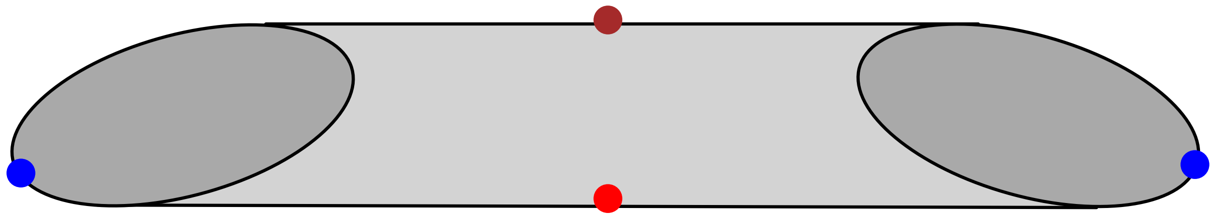
Is the sliced tube just as good as the Gömböc?



Types of stabilities:

The distance function measured from the mass center has:

Stable equilibrium	Local minima
Unstable equilibrium	Local maxima
Additional balance points	Saddle point



Sliced solid tube.

# of stable equilibria :	s
# of unstable equilibria :	u
# of other balanced equilibria:	t

Euler type formula holds: $s + u - t = 2$

$$1 + 2 - 1 = 2$$

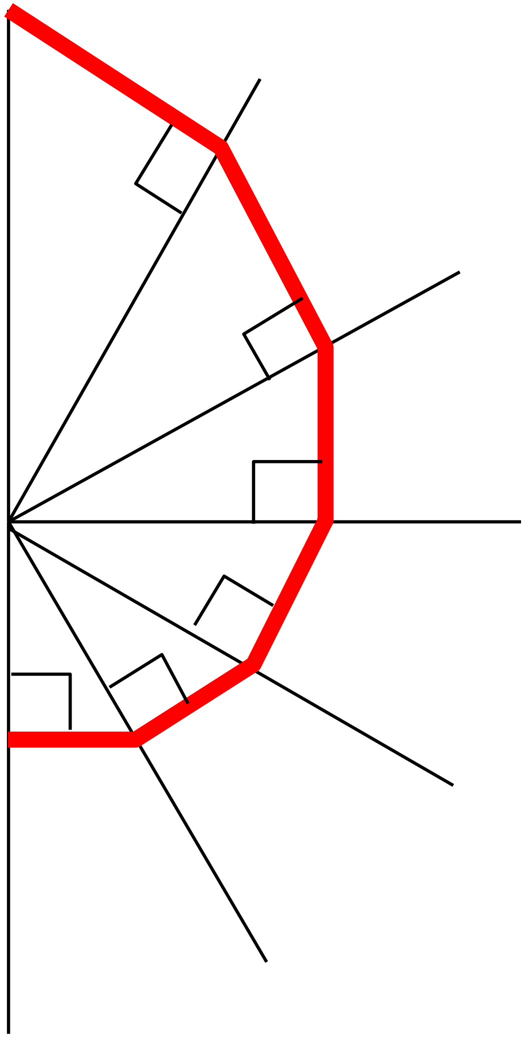
Arnolds question: Is there a shape for which one satisfies the Euler type formula with

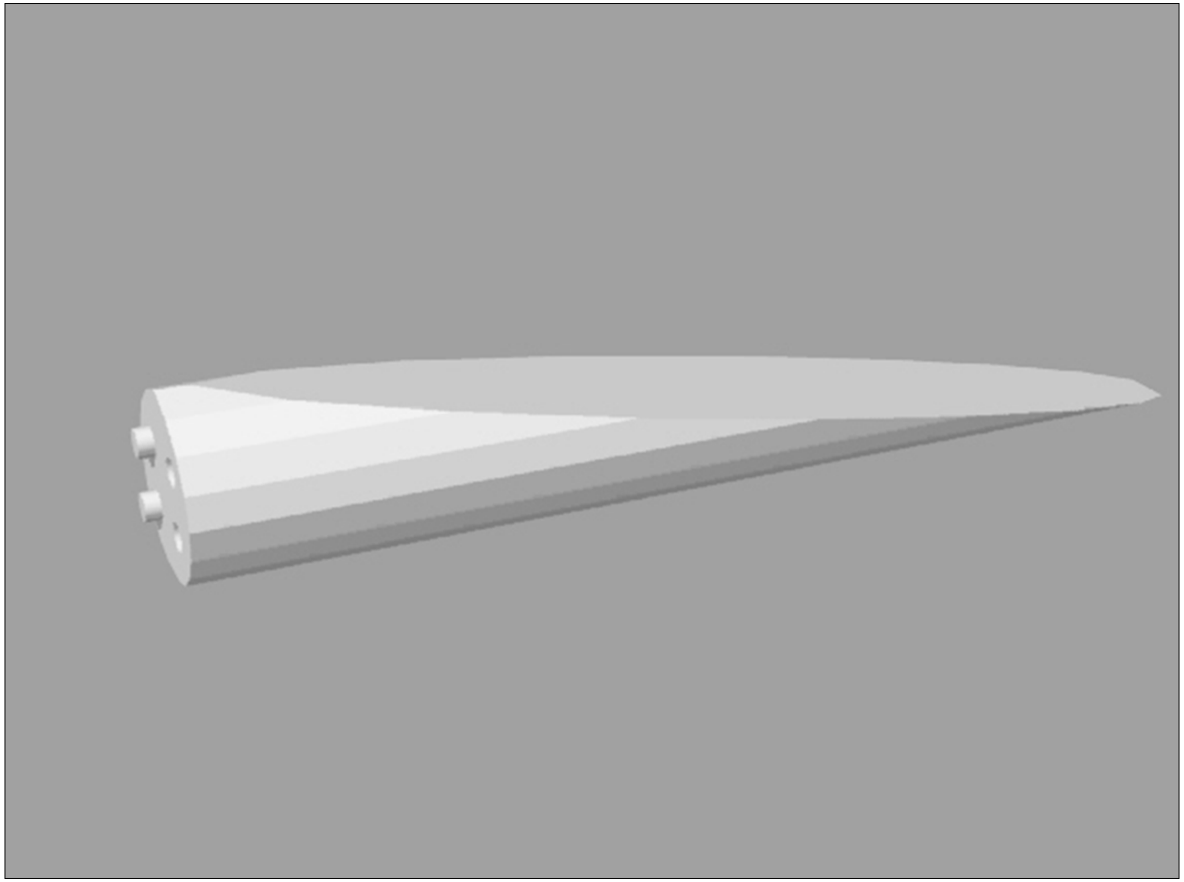
$$1 + 1 - 0 = 2 \ ?$$

Gömböc:
a mono monostatic body
by
G. Domokos & P. Várkonyi

# of stable equilibria :	s =1
# of unstable equilibria :	u =1
# of other balanced equilibria:	t = 0

A 19 faceted polyhedron which has exactly one stable face.





Spiral of N segments $N = 6$

It was believed that:
19 is the smallest face number of uni stable polyhedra.

A.B. (2011)
There is a uni stable polyhedron with 18 faces.
One can modify this polyhedron by adding 3 faces so that the stable face has arbitrary small diameter.