Understanding Opportunities for Learning Mathematics

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Design (1962)

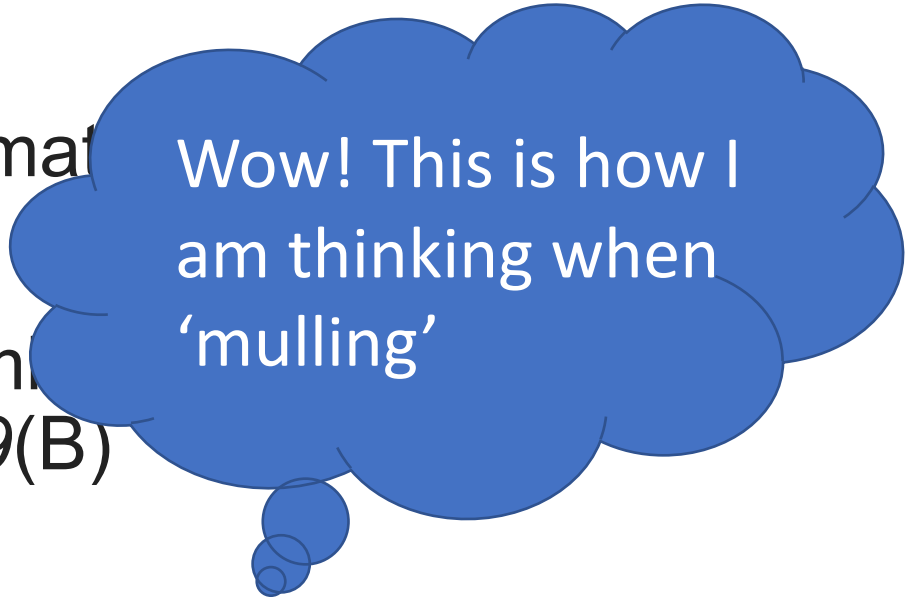
How could a set of examples, and the associated pedagogy, and the classroom climate, be constructed so that ‘discovery’ can be exciting, validated, meaningful and maybe even central to a lesson? (2000)

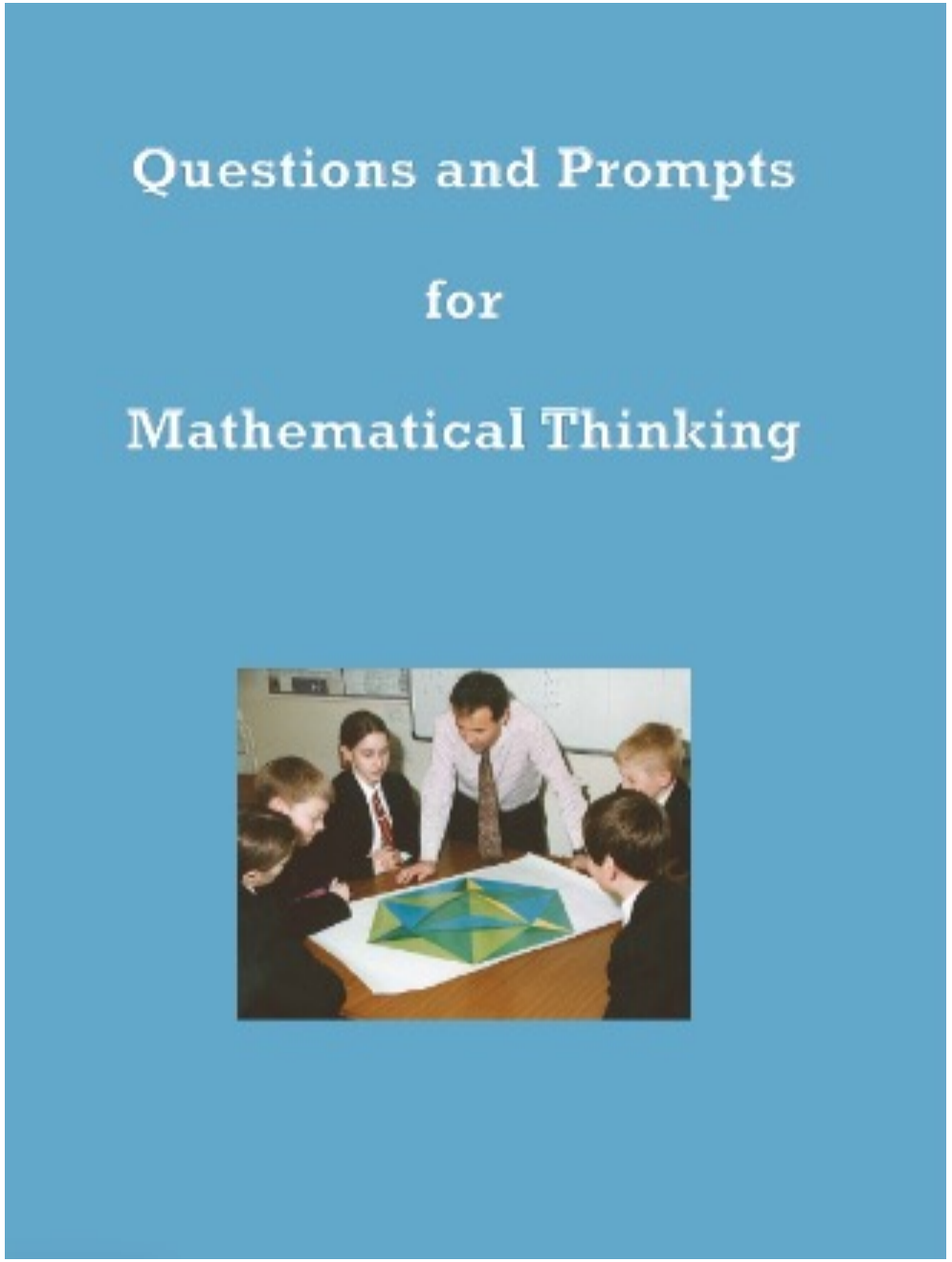
Polya: How to solve it

Mason, J., Burton, L., & Stacey, K. (1985). *Mathematically*. Addison-Wesley.

Sierpinska (1994) Understanding in Mathemat

Dyrszlag, Z. (1984) Sposoby Kontroli rozumi matematycznych. *Oswiata i Wychowanie*, *9*(B)





Watson & Mason, 1998

Exemplifying Specialising	Completing Deleting Correcting	Comparing Sorting Organising	Changing Varying Reversing Altering	Generalising Conjecturing	Explaining Justifying Verifying Convincing Refuting

List B grouped

Under each heading we generated appropriate generic questions.

Exemplifying, Specialising	Completing, Deleting, Correcting	Comparing, Sorting, Organising
Give me one or more examples of ...	What must be {added / removed / altered}	What is the same and different about ...?
Describe, Demonstrate, Tell, Show, Choose, Draw, Find, Locate, an example of ...	in order to {allow / ensure / contradict} ...?	Sort or organise the following according to ...
Is ... an example of ...?	What can be {added / removed / altered} without affecting ... ?	Is it or is it not ...?
What makes ...an example?	Tell me what is wrong with ...	
Find a counter-example of ...	What needs to be changed so that ...?	
Are there any special examples of ...?		

Changing, Varying, Reversing, Altering	Generalising, Conjecturing	Explaining, Justifying, Verifying, Convincing, Refuting
Alter an aspect of something to see effect. What if ...? If this is the answer to a similar question, what was the question? Do .. in two (or more) ways. What is quickest, easiest, ...? Change ... in response to imposed constraints.	Of what is this a special case? What happens in general? Is it always, sometimes, never ...? Describe all possible ... as succinctly as you can. What can change and what has to stay the same so that ... is still true?	Explain why ...,. Give a reason ... (using or not using ...) How can we be sure that ...? Tell me what is wrong with ... Is it ever false that ...? (always true that...?) ... How is ... used in ...? Explain role or use of ... Convince me that ...

How could a set of examples, and the associated pedagogy, and the classroom climate, be constructed so that ‘discovery’ can be exciting, validated, meaningful and maybe even central to a lesson? (2000)

What is available for me to see, hear, read, do, say and learn in this lesson?

Exemplification

Teacher led

Learner generated

Variatio est mater studiorum (2000) Marton & Trigwell

What varies? What stays the same? How do these elements relate?

The new car syndrome

Yizhu Liu (ICME 2004) Compare –2.5 and |–2.5|

Tuckey (1904): Multiply each of the terms in the top row by each of the terms in the bottom row in pairs:

$x - 1$	$x + 1$	$x + 2$	$x + 3$
$x - 1$	$x + 1$	$x + 2$	$x + 3$

Tuckey (1904)

Draw the graphs of :

(1) $y = x^2$. (2) $y = -x^2$. (3) $y = 2x^2$.

(4) $y = x^2 + 2{\cdot}5$. (5) $y = (x-1)^2$. (6) $y = (x+2)^2 + 1$.

(7) $y = x^2 + 4x + 6$. (8) $y = x^2 - 3x + 1$.

(9) Write out a general statement of the difference between the graphs of $y = x^2$ and of $y = \pm a\{(x-b)^2 + c\}$.

For this exercise A = (-2,-1). Mark A on a coordinate grid. For each point P in (a) to (h) below calculate $Dt(P, A)$ and mark P on the grid:

(a) $P = (1, -1)$

(b) $P = (-2, -4)$

(c) $P = (-1, -3)$

(d) $P = (0, -2)$

(e) $P = (\frac{1}{2}, -1\frac{1}{2})$

(f) $P = (-1\frac{1}{2}, -3\frac{1}{2})$

(g) $P = (0, 0)$

(h) $P = (-2, 2)$

Overheard

“When I am told a generality I make some examples;
when I am shown an example I construct a generality”

Examples for, of or in

elements of objects

classes of objects

techniques

physical objects

symbolic objects

questions

calculations

representations

properties

manifestations

The power of a word

‘Variation’ and ‘examples’

Tools to make sense of mathematical understanding

- naming what they do anyway
- a vocabulary for designing tasks
- a tool for exploring or expressing the scope of learners’ knowledge

- Arthur said: I see functions as input-output machines, which receive some input and give an appropriate output.
- Ruth said: I see function as a mapping of each element of one set to exactly one element of a second set.
- Ian said: Functions for me represent relations between variables.
- Naomi said: A function shows how one variable changes in relation to another variable.
- Liz said: I see functions as expressions to calculate y-values from given x-values.

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