Improving Students' Learning of Mathematics:

Some Insights from Several Decades of Research

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Thanks for the Invitation

AN IMPORTANT ASSUMPTION:

Improving Students' Learning

Of

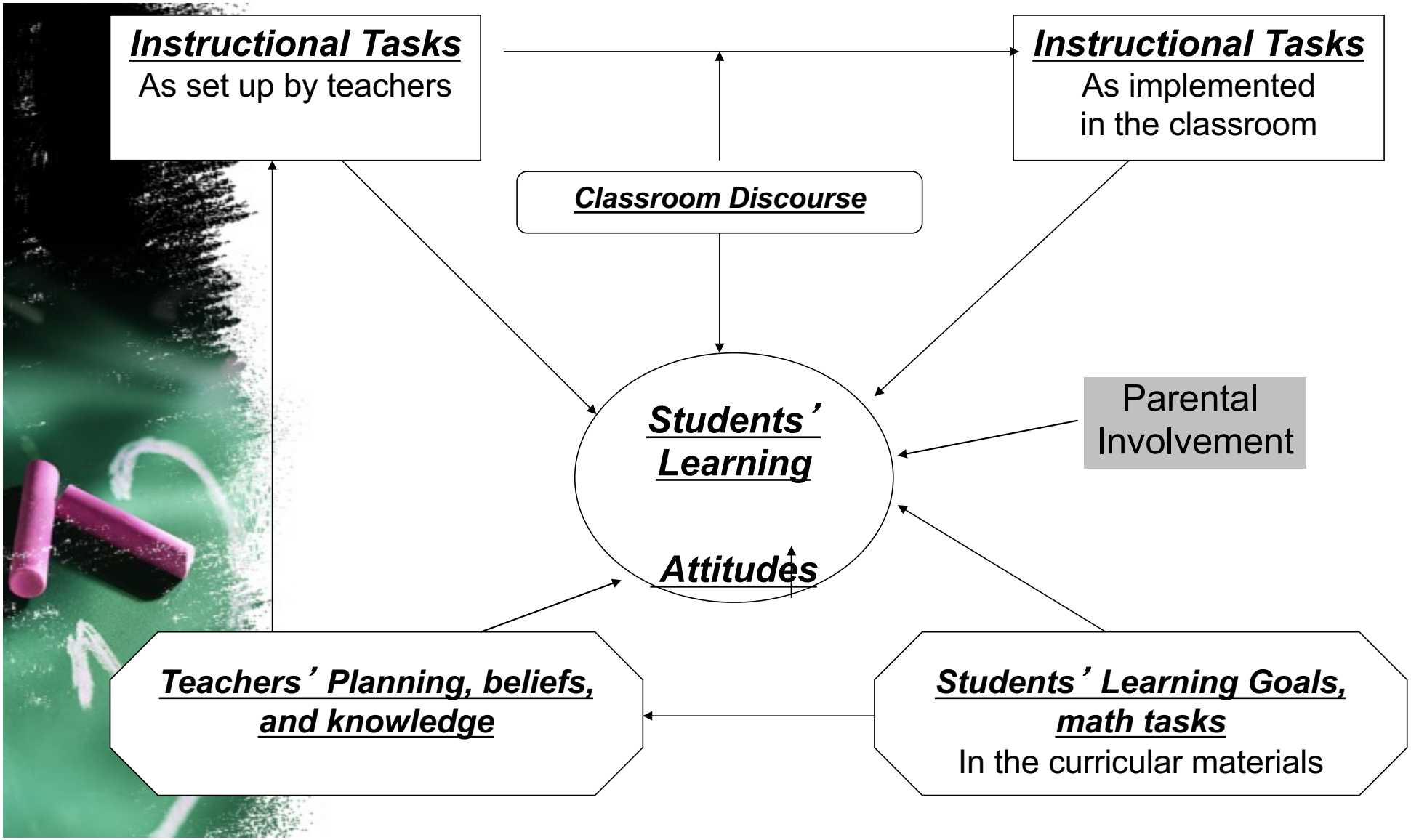
MATHEMATICS

Three Lines of Research

First, A Project on Cross-national Comparative Studies

Second, A Project on Curriculum

Third, A Project on Mathematical Problem Posing


Instructional Tasks
As set up by teachers
Instructional Tasks
As implemented
in the classroom
Classroom Discourse
Students' Learning
Attitudes
Parental Involvement
Teachers' Planning, beliefs, and knowledge
Students' Learning Goals, math tasks
In the curricular materials

U.S. and Chinese Students' Performance on Four Types of Tasks

(Cai, 2000, 2001)

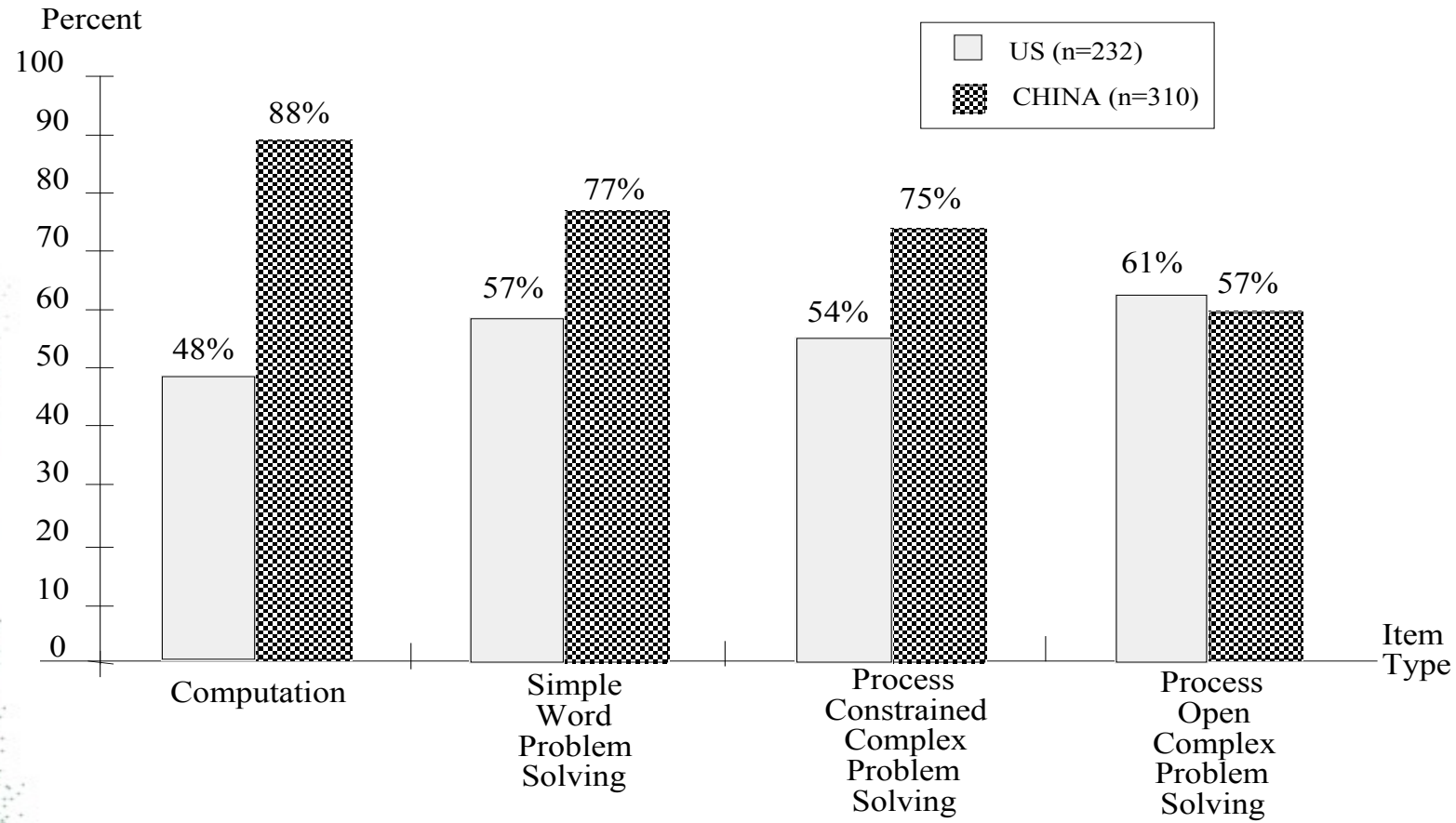
- 13 Computation Tasks
- 18 Simple Word PS Tasks
- 6 Process Constrained PS Tasks
- 6 Process Open PS Tasks

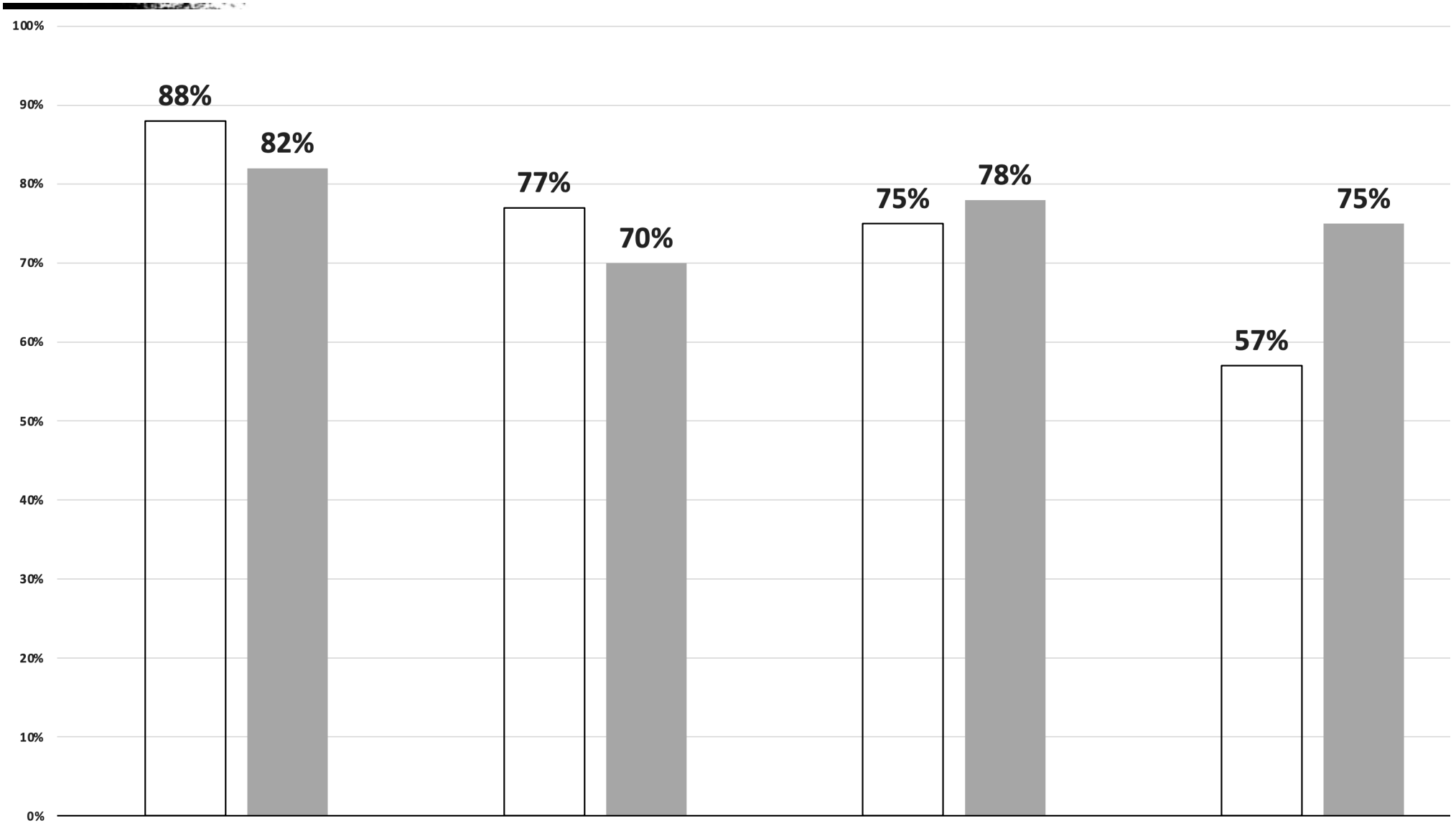
Translation Equivalence

English-back Translation

were used to ensure translation equivalence

U.S. and Chinese Students Performance on Four Types of Tasks




100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
88%
82%
77%
70%
75%
78%
57%
75%

Some Research Findings

(Cai & Merlino, 2011)

- A total of 1316 high school students
- Different programs:

  285 Non-college preparation mathematics

  858 college preparation math (traditional)

  173 college preparation math (NSF-Funded)

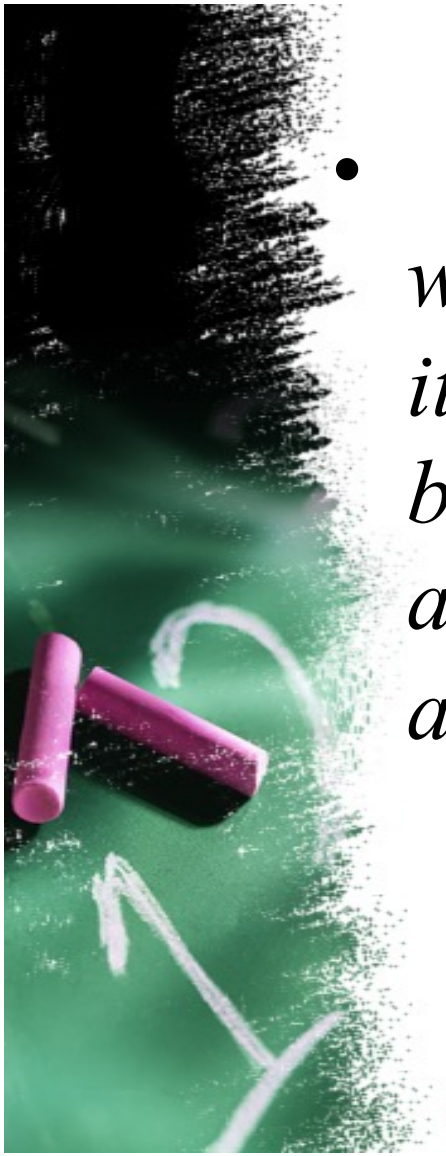
Survey Instrument

We are interested in learning how you think and feel about mathematics. Please take a few minutes to think about the following questions and write how you truly feel. There are no right or wrong answers.

- If Math were a **food**, it would be_because_____
- If Math were a **color**, it would be_because_____
- If Math were an **animal**, it would be_because

- *“Purple is my favorite color. It’s my birth stone color plus it brings passionate. That’s how I feel about math.”*

- *"Math is like steak because math is a full, expansive subject. However, like a steak there are tough bits of gristle scattered throughout obstacles you must work around. The full meal is satisfying, but the process of eating is somewhat unusually strenuous."*
- *"Vegetables are good for you, and so is mathematics for daily things. It is needed in life. Some people like it, and some people don't, but you still need it to live a healthy life."*



- *“I would say a mosquito, because whatever you do to try and get away from it, it always comes back. It’s annoying because you hate taking math every year, and whatever you try to do to stop it, it always fails.”*

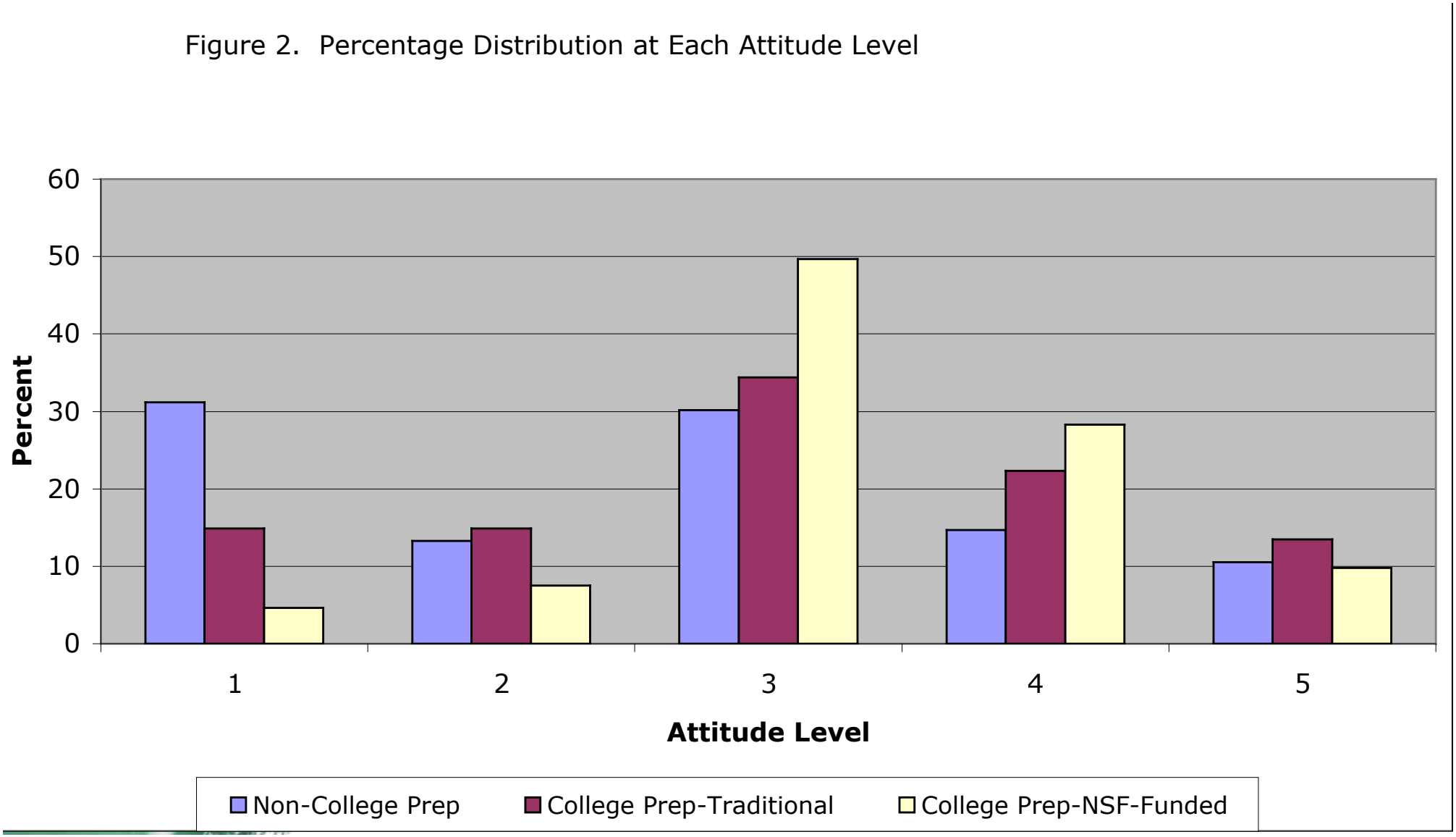
- *“It is like gum. You chew gum and use it to freshen up your breath, but in the end, it’s worthless and doesn’t have any nutrition or vitamins. Math is used in school to determine your intelligence, but there is no need for it later.”*

- Quantitative Analysis: Holistic scoring (1 - 5)

1 Point	Very Negative
2 points	Moderately Negative
3 points	Neutral or Ambivalent
4 points	Moderately Positive
5 points	Very Positive.

- Qualitative Analysis: Reveal what kinds of metaphors students used and why

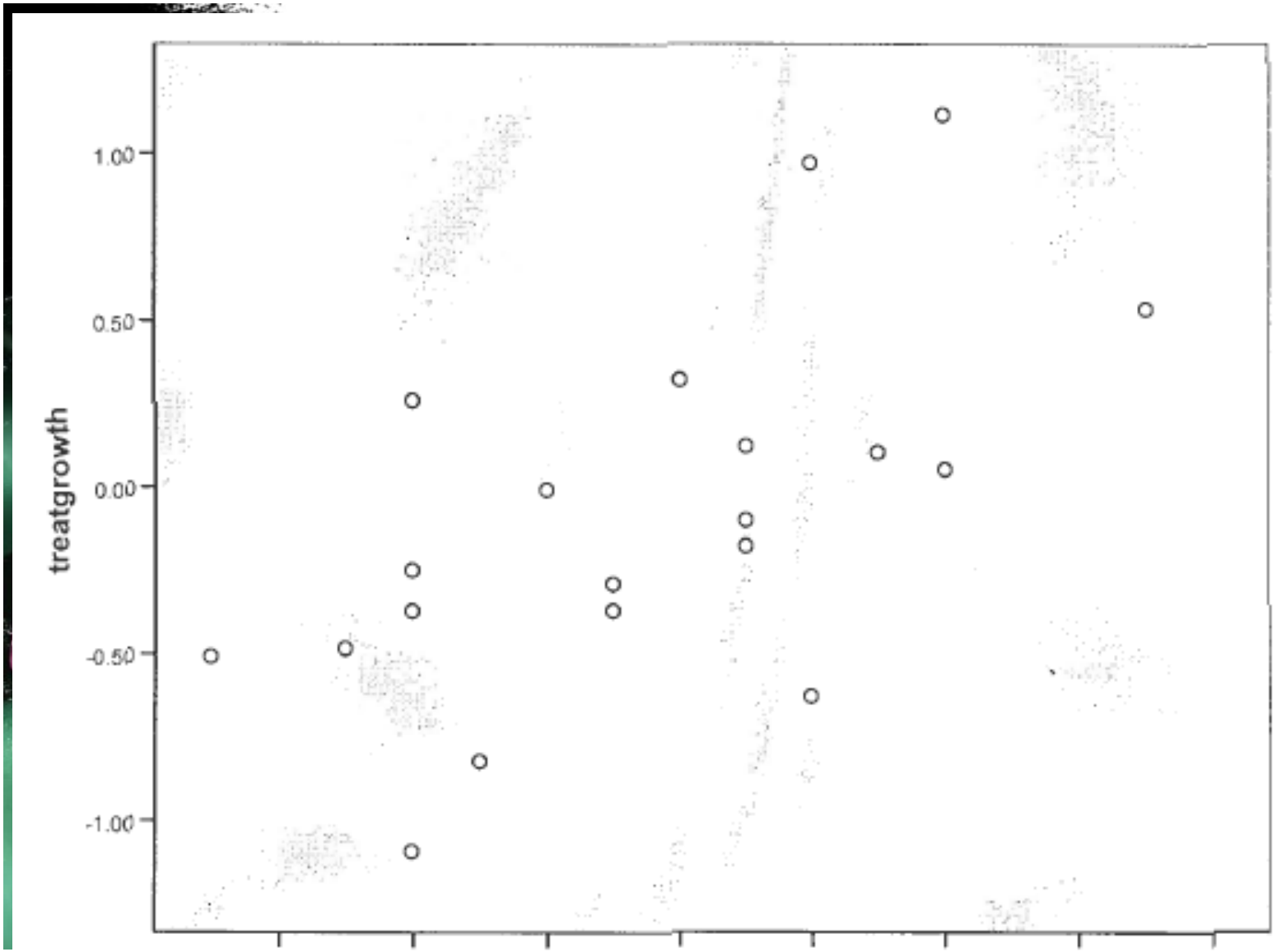
Figure 2. Percentage Distribution at Each Attitude Level



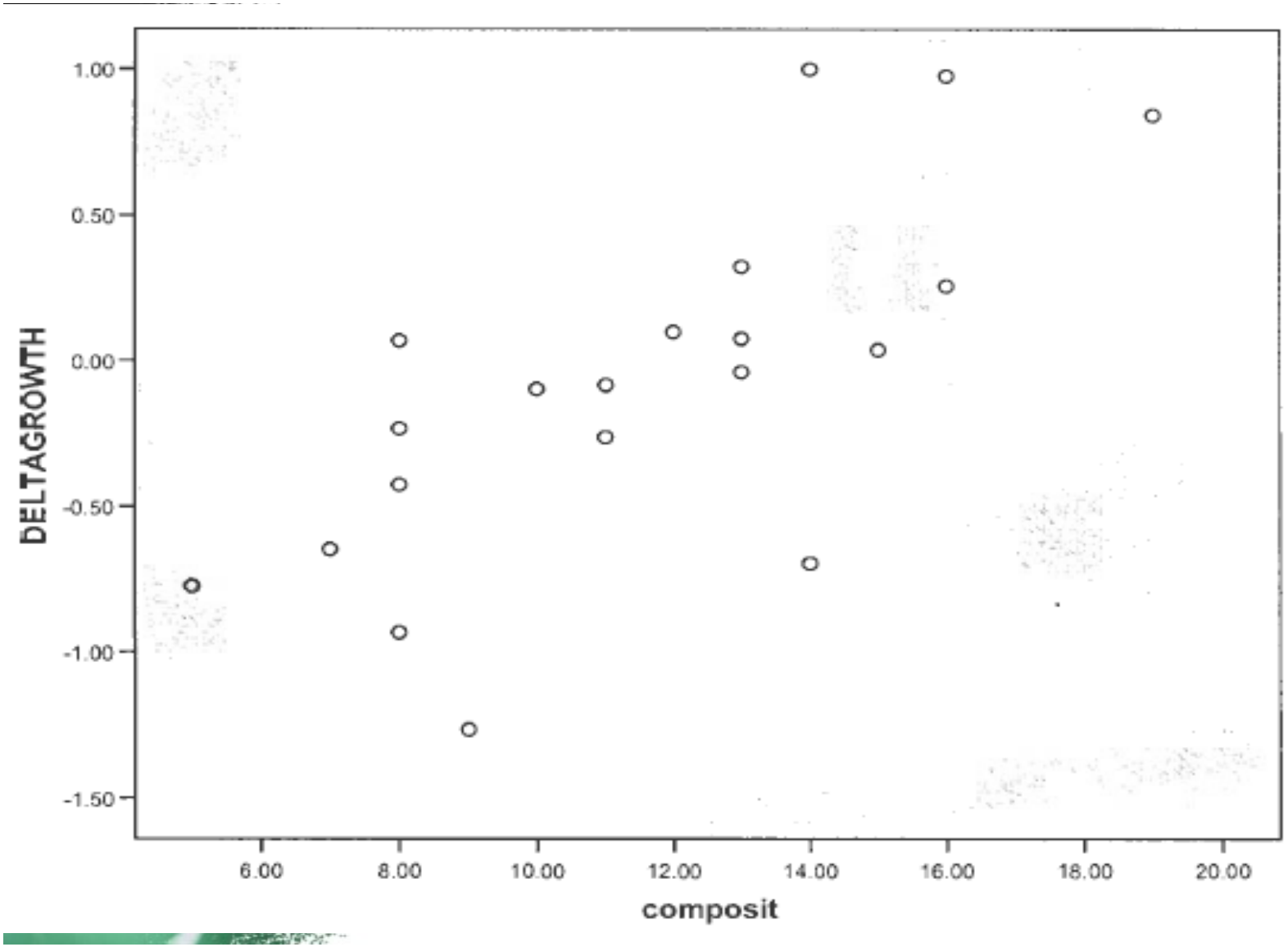
Background Information in the Ten School Districts in GPSMP

(Kramer, Cai, & Merlino, 2015)

	School District	Curriculum (20 Middle School)	Approximate # of Students (Middle School)	Curriculum (High School)	Approximate # of Students (12 High School)
1	District A (PA)	CMP	4000	CPMP	5000
2	District B (PA)	MiC	2000	CPMP	2000
3	District C (PA)	MiC	1000	IMP	2000
4	District D (PA)	MiC	1000	IMP	2000
5	District E (NJ)	CMP	500	CPMP	500
6	District F (PA)	MiC	1000	IMP	1000
7	District G (NJ)	CMP	1000	CPMP	1000
8	District H (NJ)	CMP	1000	IMP	2000
9	District I (PA)	CMP	1000	IMP	2000
10	District J (PA)	CMP	1000	CPMP	2000



Scatter-plot of "Treatment Growth" (zmath04-zmath98 in PA; zmath04-zmath99 in NJ)



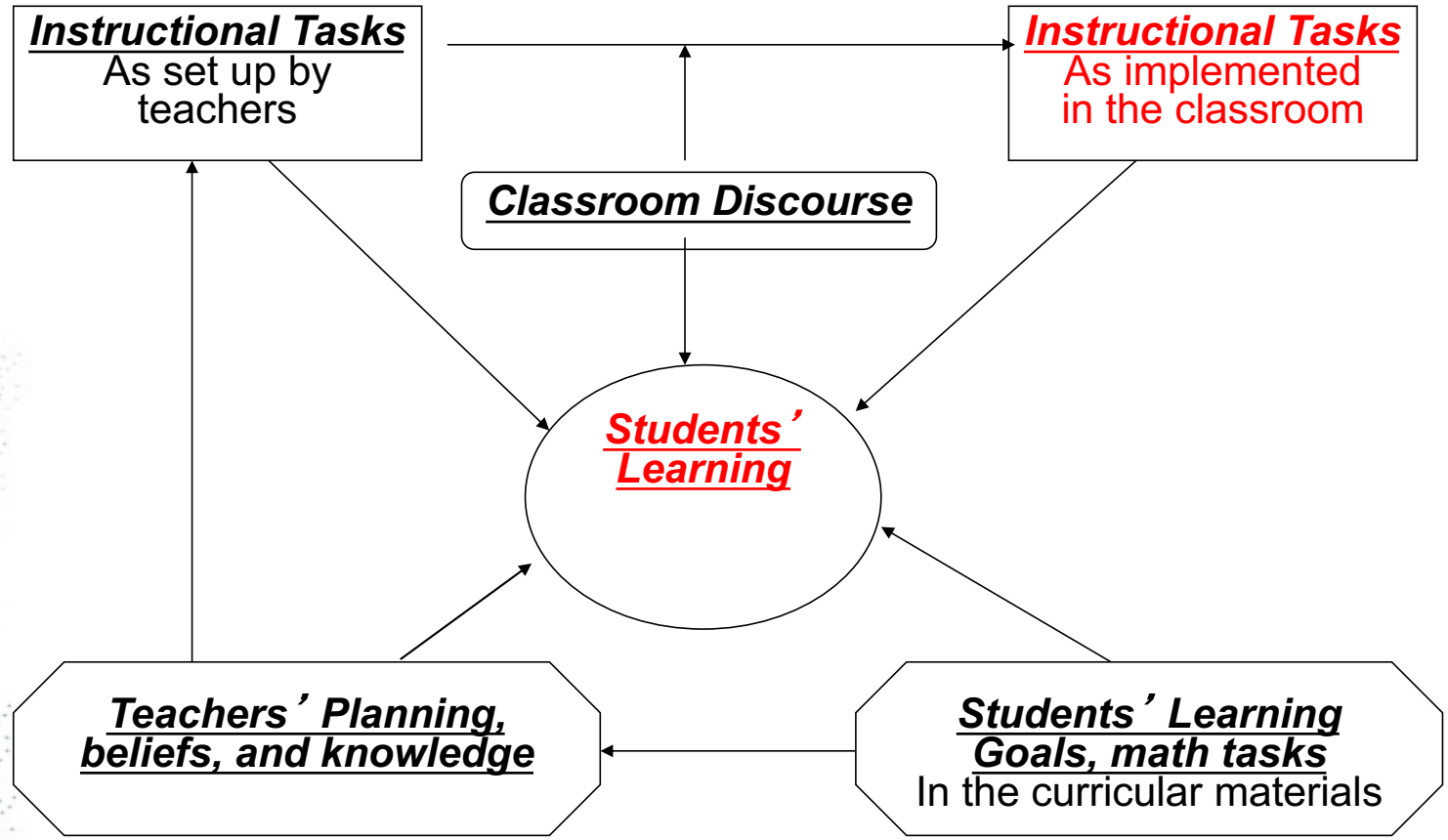
Scatter plot of "Treatment Growth" - "Control Growth"

Longitudinal Investigation of the Effect of Curriculum on Algebra Learning (LieCal Project)

Profile of Schools

(Cai et al., 2011, 2013; Moyer et al., 2018)

Achievement Level	CMP	Non-CMP
High Achieving	2	2
Average Achieving	3	3
Low Achieving	2	2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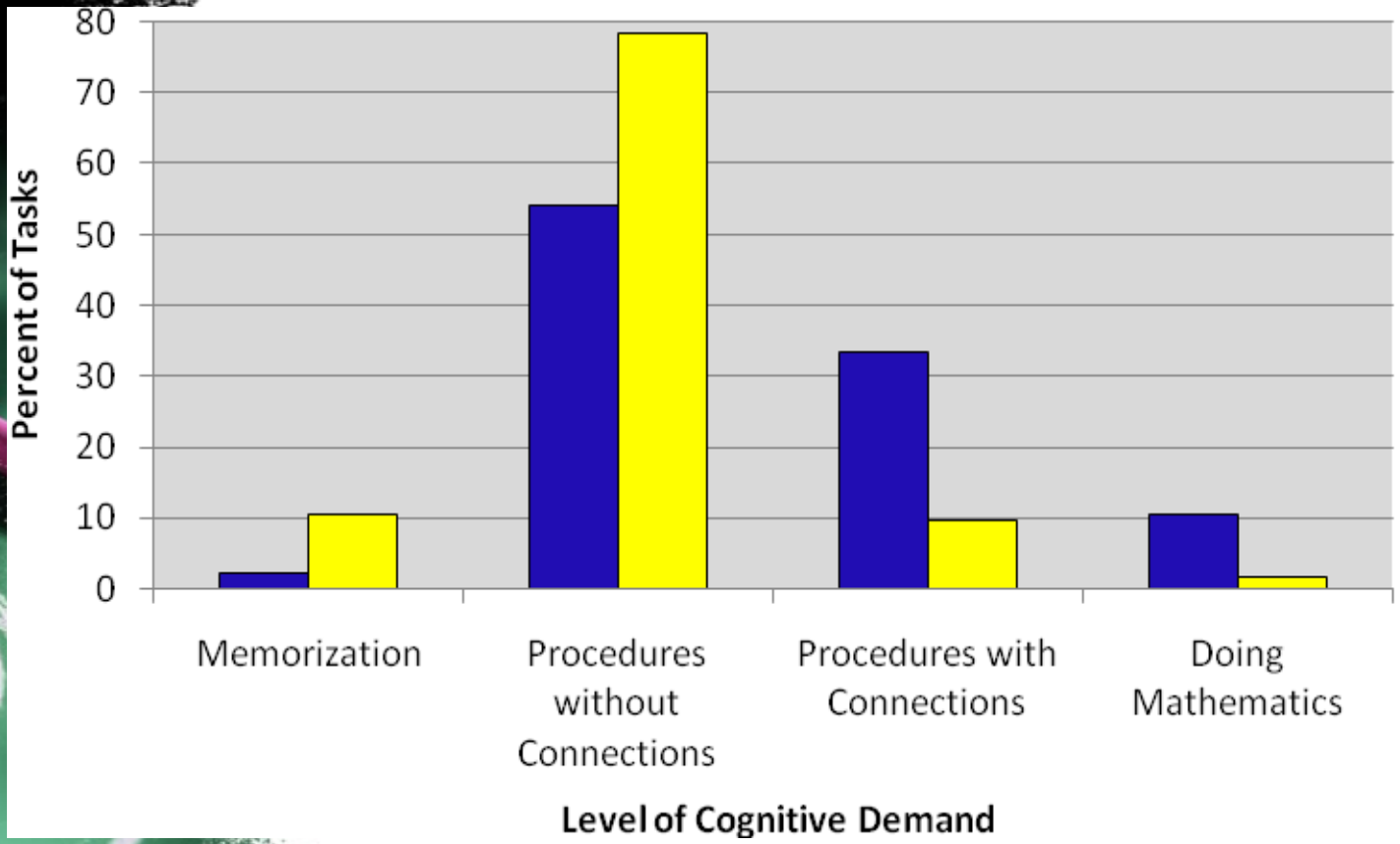
Percentages of Implemented Tasks with Different Levels of Cognitive Demand in CMP and Non-CMP Classrooms



	Problem Solving	Computation	Equation Solving
Without control for cognitive demand	CMP	---	---
With control for cognitive demand	---	Non-CMP	---
With added growth rate from cognitive demand	Non-CMP by 52%	Non-CMP by 91%	Non-CMP by 63%

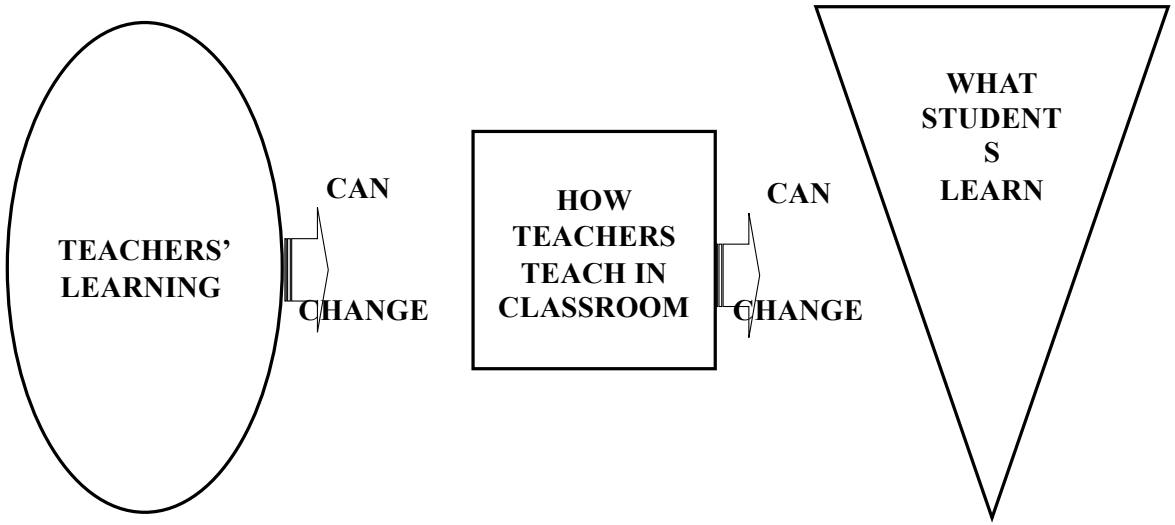
LieCal Project History (PBL)

6th	7th	8th	9th	10th	11th	12th

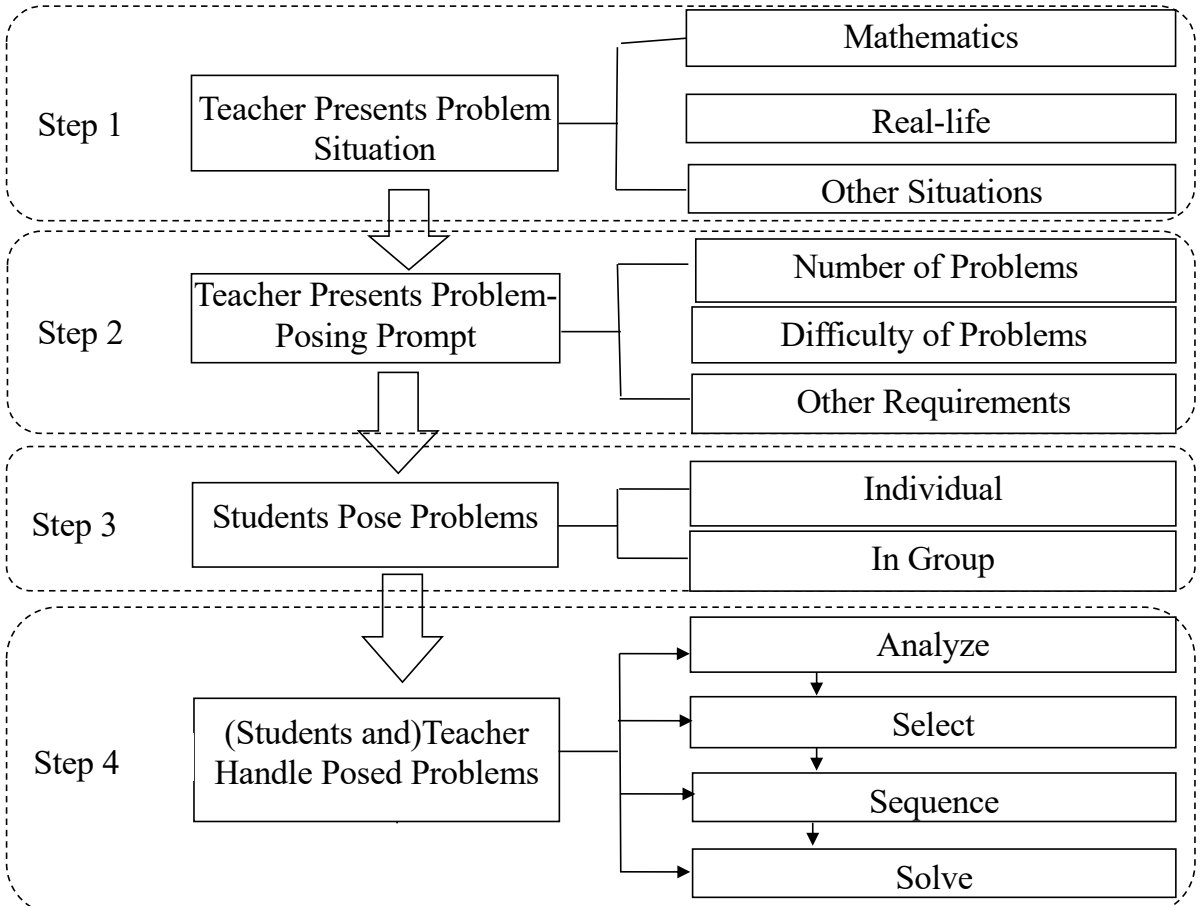
Supporting Teachers to Teach Mathematics Through Problem Posing: An Early-Stage Longitudinal Study (in Middle Grades)

Problem-Posing Based Learning (P-PBL) Project

Investigating *longitudinally* how teachers teach and learn to teach mathematics through problem posing and the impact of these processes on students' learning.



Teaching mathematics through PP (Cai, 2022)



Odd Number Pattern

1

3 5

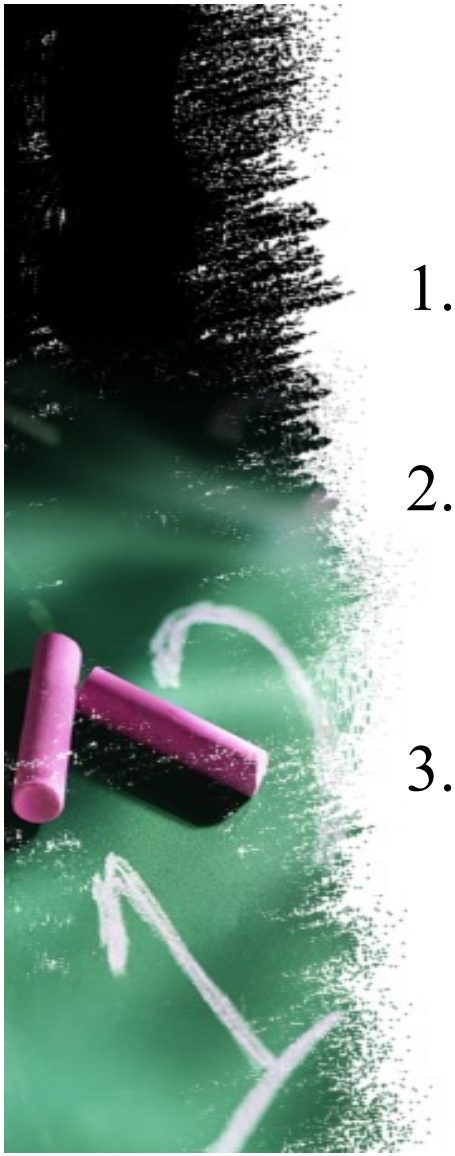
7 9 11

13 15 17 19

21 23 25 27 29

…… The pattern continues.

Cai et al., 2015; Cai in preparation



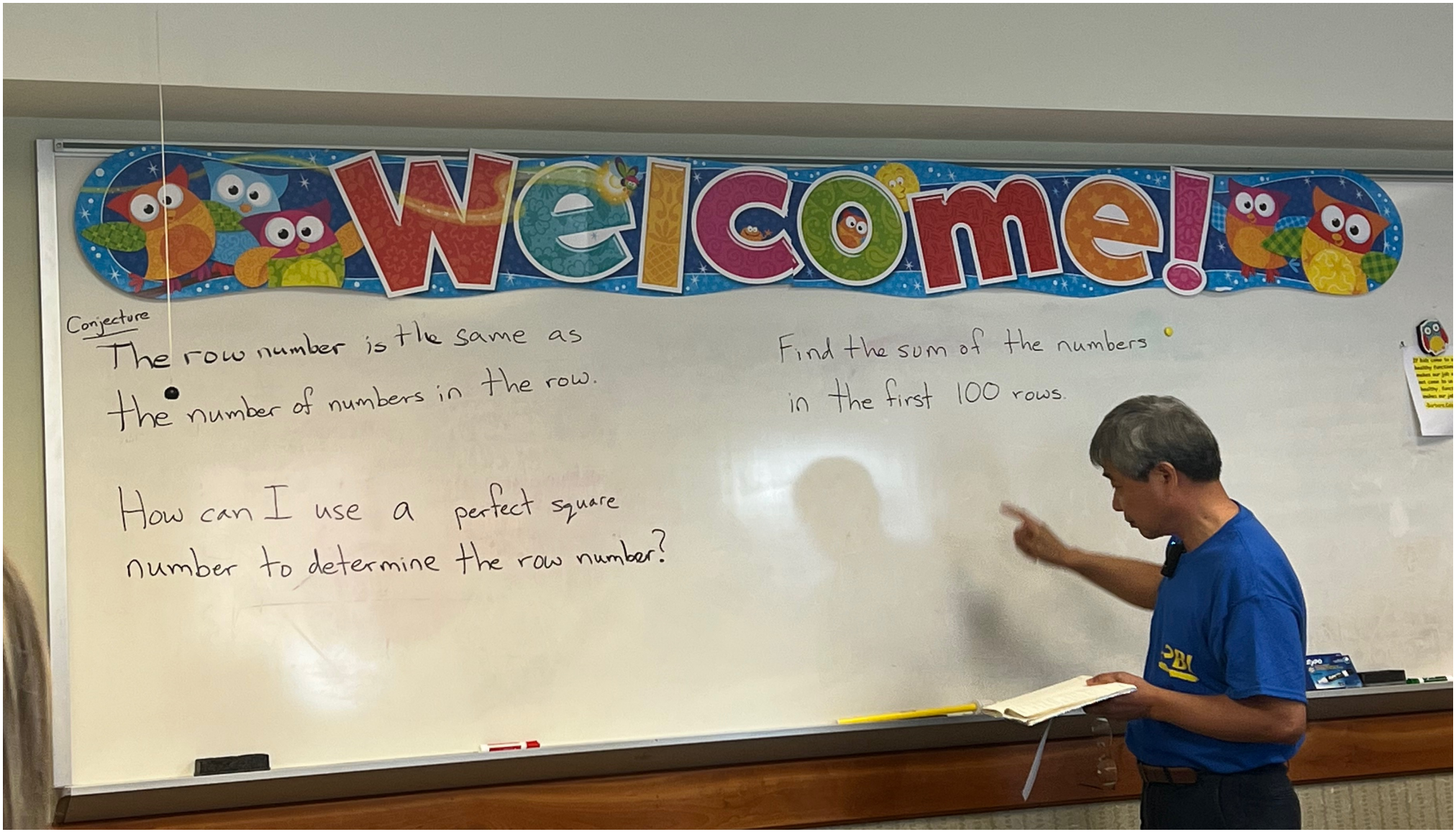
Three Prompts

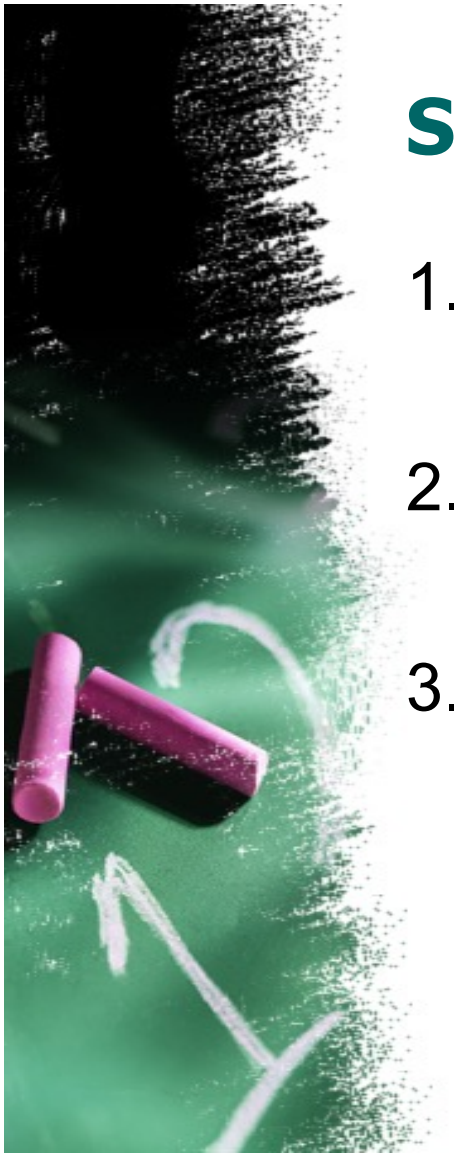
1. Pose three different mathematical problems that could be solved based on this pattern
2. Pose one easy mathematical problem, one moderately difficult mathematical problem, and one difficult mathematical problem
3. Make three different mathematical conjectures about the pattern

- Can you tell me the next row of numbers? [list]
- How many values in the 17th row?
- List the values in the 10th row? 11th.
- Determine the center value (number) (of the row) of row 9.
- can you find all the numbers in row "n": (pattern)
- How many rows would it take to reach a triple digit value? (number)
- Find the sum of the values (number) in row 15.

G 1

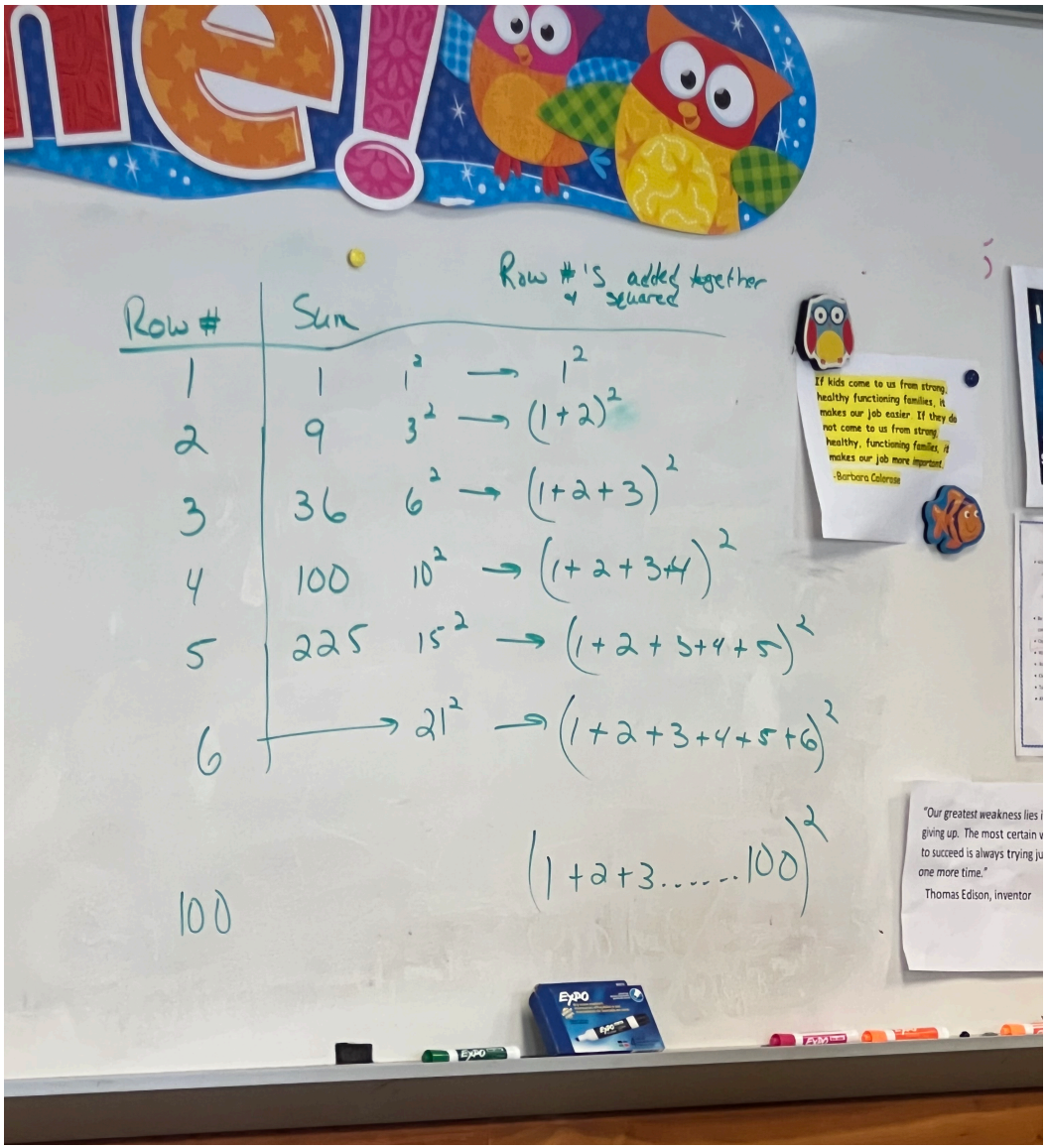
- The next row will contain 8 numbers.
- The sum of the values is a row is = to n^3. where "n" is the raw.
- Row number = number of numbers.
- Difference between first and last number of each row increases by 2 as you move down the table, resulting in the rule $2n-2$ where "n" is the row number.
- row 100 would have a difference of 198 between the first + last number.

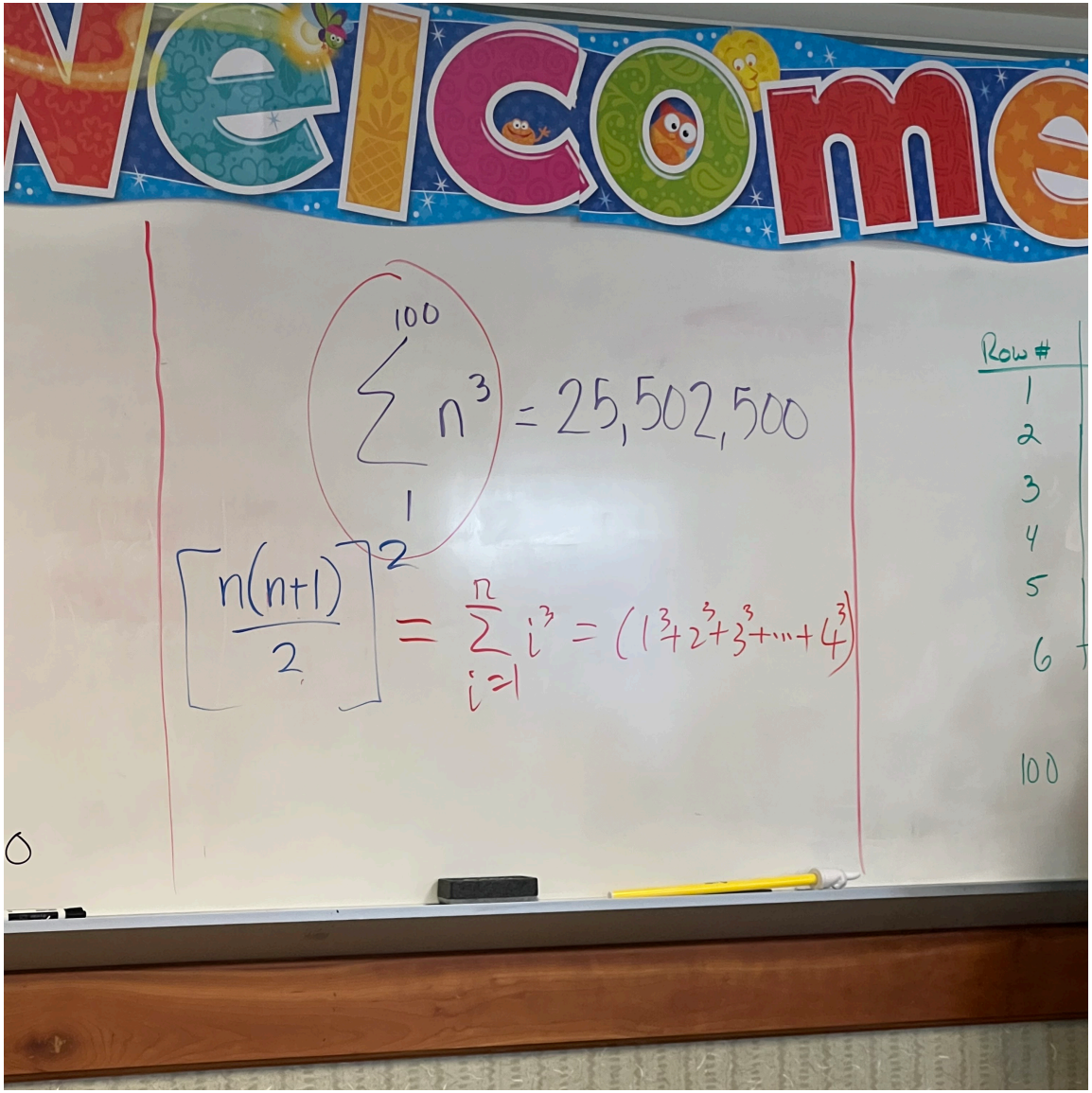

Welcome!
Conjecture
The row number is the same as the number of numbers in the row.
How can I use a perfect square number to determine the row number?
Find the sum of the numbers in the first 100 rows.



Selected Three Problems/Conjectures

1. The row number is the same as the number of numbers in the row.
2. How can I use a perfect square to determine the row number?
3. Find the sum of the numbers in the first 100 rows.


Row #'s added together squared
Row #
Sum
1
1
$1^2 \rightarrow 1^2$
2
9
$3^2 \rightarrow (1+2)^2$
3
36
$6^2 \rightarrow (1+2+3)^2$
4
100
$10^2 \rightarrow (1+2+3+4)^2$
5
225
$15^2 \rightarrow (1+2+3+4+5)^2$
6
$\rightarrow 21^2 \rightarrow (1+2+3+4+5+6)^2$
100
$(1+2+3.....100)^2$
If kids come to us from strong, healthy functioning families, it makes our job easier. If they do not come to us from strong, healthy, functioning families, it makes our job more important.
-Barbara Coloroso
"Our greatest weakness lies in giving up. The most certain way to succeed is always trying just one more time."
Thomas Edison, inventor
EXPO


Welcome
$\sum_{1}^{100} n^3 = 25,502,500$
$\left[\frac{n(n+1)}{2}\right]^2 = \sum_{i=1}^{n} i^3 = (1^3+2^3+3^3+\cdots+4^3)$
Row #
1
2
3
4
5
6
100

What is the sum of the numbers in the first n rows?

1

3 5

7 9 11

13 15 17 19

21 23 25 27 29

… …

What is the sum of the numbers in the first n rows?

1	*=1*
3 5	*=8*
7 9 11	*=27*
13 15 17 19	*=64*
21 23 25 27 29	*=125*
... ...	... ...

What is the sum of the numbers in the first n rows?

$1 = 1$

$3 \quad 5 = 8$

$7 \quad 9 \quad 11 = 27$

$13 \quad 15 \quad 17 \quad 19 = 64$

$21 \quad 23 \quad 25 \quad 27 \quad 29 = 125$

... ...

The sum = $1^3 + 2^3 + 3^3 + 4^3 + \ldots + (n-1)^3 + n^3$

What is the sum of the numbers in the first n rows?

$$1$$

$$3 \quad 5$$

$$7 \quad 9 \quad 11$$

$$13 \quad 15 \quad 17 \quad 19$$

$$21 \quad 23 \quad 25 \quad 27 \quad 29$$

$$\ldots \quad \ldots$$

$$[1 + 3 + 5 + \ldots + (2m-1) = m^2]$$

$$= (1 + 2 + 3 + 4 + \ldots + n)^2$$

An unexpected finding：

$$1^3 + 2^3 + 3^3 + 4^3 + \ldots + (n-1)^3 + n^3$$

$$= (1 + 2 + 3 + 4 + \ldots + n)^2$$

$$= [n(n+1)/2]^2$$

Note: $1 + 2 + 3 + 4 + \ldots + n = n(n+1)/2$

Thanks!

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