

The Quantum Nature of Energy and Entropy in Human Cognition

Towards a Non-classical Thermodynamic Theory of Human Culture

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Workshop

Applications of Topology to Quantum Theory and Behavioral Economics
Fields Institute, Toronto, Canada

Centre for Quantum Social and Cognitive Science

The Centre for Quantum Social and Cognitive Science (CQSCS) at Memorial University brings together top researchers working in the field of applying quantum mechanics formalism to the social sciences.



Building on the work of the Institute for Quantum Social and Cognitive Science (IQSCS) at the University of Leicester, United Kingdom, the centre support researchers and broadly disseminates knowledge through events and publications.

► [Webpage of Centre CQSCS](#)

Research associated with this centre has to date established an excellent research record, with publications in well-established journals such as *PNAS* and other important mainstream journals in both psychology and economics. Several monographs have also been published with Cambridge University Press.

Current activities

The CQSCS is currently planning for the 2022-2023 year, which will include research talks, filming at university by the American Physical Society and a workshop at the Fields Institute, University of Toronto.

- 1 Introduction
- 2 Quantization of energy and Bose–Einstein statistics in large texts
- 3 Entanglement and entropy in the combination of concepts
- 4 Towards a thermodynamic theory of human cognition

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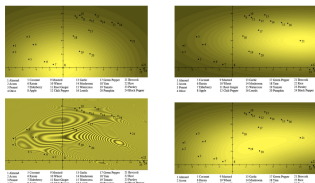
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- One of the most interesting aspects of human cognition concerns how people combine, exchange and form new concepts through **language**.
- These processes are also important in **applied disciplines**, e.g., computational linguistics and information retrieval.
- In the last decades, growing empirical evidence has revealed that cognitive phenomena cannot generally be represented by means of **classical structures**.
- On the other side, **quantum structures** have been successful to represent these classically problematical phenomena.

Reason

Aspects as uncertainty, contextuality, emergence and superposition are **not peculiar** of microscopic entities, as believed in the early days of quantum theory, but they also appear in cognitive phenomena.



1.2 The Brussels approach

- The Brussels team have dedicated two decades to the investigation of the epistemological and mathematical differences between classical and quantum theories, in physics and cognitive domains.
- This research has led to the development of a **theoretical perspective** for human cognition in which concepts are regarded as **entities that can be in different states and interact with each other and with contexts in a non-deterministic way.**

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- In this perspective, concepts behave as quantum entities, e.g., electrons, photons, etc.
- Further, we have elaborated an **interpretation of quantum theory** in which quantum entities themselves do not behave as physical objects but, rather, as concepts.

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Recent results

We have recently found that two additional genuine quantum aspects appear in human language, namely, **entanglement** and **Bose–Einstein indistinguishability**.

Content of the talk

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2.1 Identity and indistinguishability in physics

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- I In classical physics, identical entities **can be distinguished**, and obey the **Maxwell–Boltzmann statistics** when large numbers of these entities are considered.

- II In quantum physics, identical entities are **indistinguishable**. Entities with integer spin, also called **bosons**, e.g., photons, are characterised by wave functions that are **symmetric** with respect to entity exchange. At a statistical level, bosons obey the **Bose–Einstein statistics**.



2.2 The phenomenon of Bose–Einstein condensation in physics

- III In quantum physics, identical and indistinguishable entities with semi-integer spin, also called **fermions**, e.g., electrons, are characterised by wave functions that are **anti-symmetric** with respect to entity exchange. At a statistical level, fermions obey the **Fermi–Dirac statistics**.



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- Indistinguishability is responsible of the **violation of statistical independence**, i.e. the probability of an entity to occupy a given energy state depends on another entity occupying that state, an aspect that emerges when the so–called **de Broglie wavelength** reaches a critical value.

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- Indistinguishability is responsible of the **violation of statistical independence**, i.e. the probability of an entity to occupy a given energy state depends on another entity occupying that state, an aspect that emerges when the so–called **de Broglie wavelength** reaches a critical value.
- Despite their common indistinguishability, bosons and fermions behave differently:
 - II fermions obey **Pauli exclusion principle**, hence any two of them cannot occupy the same state;
 - III at temperatures close to absolute zero, a gas of bosons tend to occupy the lowest energy state, a phenomenon known as **Bose–Einstein condensation**.

2.3 Quantum-type indistinguishability in language

We have identified an **analogous statistical behaviour** in human language, namely, if one considers a large story-telling text and attributes energy levels to the words appearing in the text, according to their numbers of appearance, then the component words exhibit **genuinely quantum aspects**, namely,

- superposition and entanglement
- overlapping de Broglie wave functions
- Bose-Einstein indistinguishability
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Non-classicality of energy in language

We stress a first **important point**: once the notion of **energy** is introduced and quantified in language, this variable systematically behaves non-classically, i.e. it is **quantized** and gives rise to a **Bose-Einstein**, rather than a Maxwell-Boltzmann, distribution.

2.4 Quantifying energy in the linguistic realm

- Consider a story-telling text, together with its meaning content, e.g., the Winnie the Pooh story “In Which Piglet Meets a Haffalump” (Milne 1926).
- However, the analysis has been repeated with several texts, including short and long stories, e.g., novels, and we always found the results presented here.



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- Each different word w_i , $i = 1, \dots, n$, appearing in the text can be associated with a conceptual entity in a given state whose energy level E_i is defined by the number of times $N(E_i)$ the word appears in the text.



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- Each different word w_i , $i = 1, \dots, n$, appearing in the text can be associated with a conceptual entity in a given state whose energy level E_i is defined by the number of times $N(E_i)$ the word appears in the text.
- We order different words according to the increasing energy level or, equivalently, according to their **decreasing order of appearance in the text**, and set, for a given word w_i , $E_i = i$, $i = 0, 1, \dots, n - 1$.
- Thus, the most frequent word is given the lowest energy level E_0 (**ground state energy**).



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- As in physics, the numbers N and E can be retrieved from data, i.e. word counts in the case of a text.
- We however stress a **difference** between physics and language with respect to measures of energy: in physics, energy is a derived quantity, whereas it is a fundamental quantity in language, where the conceptual equivalent of physical space cannot be **uniquely identified**.

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Example

Consider the concept combination *Eleven Animals*.

- 1 In the case of **eleven physical animals**, there are always differences between each one of the eleven animals, because as **objects** present in the physical world, they have an individuality and, as individuals with spatially localized physical bodies, none of them is really identical to the others.
- ii On the other side, at a conceptual level, each one of the eleven animals is completely **identical with** and **indistinguishable from** each other of the eleven animals.

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- 4 If animals are instead indistinguishable, we expect a non-classical statistics to apply.

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- If we exchange one of the words *Cat* with the other word *Cat*, **absolutely nothing** changes in the meaning content of the text.
- Thus, at a conceptual level, a text contains a **perfect symmetry** for the exchange of words in the same state, exactly as in the case of identical and indistinguishable bosons.

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- Thus, at a conceptual level, a text contains a **perfect symmetry** for the exchange of words in the same state, exactly as in the case of identical and indistinguishable bosons.
- The two distributions, Bose–Einstein and Maxwell–Boltzmann, behave differently with respect to **statistical dependence**.
- Indeed, consider, e.g., two entities S_1 and S_2 which can be in two different states p_1 and p_2 , respectively.

2.6 Bose–Einstein statistics in human language

- From a Maxwell–Boltzmann point of view, four different configurations may occur: (i) S_1 and S_2 are in the state p_1 , (ii) S_1 and S_2 are in the state p_2 , (iii) S_1 is in the state p_1 and S_2 in the state p_2 , and (iv) S_1 is in the state p_2 and S_2 in the state p_1 .

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- Each of these configurations occurs with probability $1/3$, and this is **incompatible** with an independent behaviour of the individual entities.
- Even assuming an **epistemic-only indistinguishability** of (iii) and (iv) in the Maxwell–Boltzmann case, one would get three configurations with probabilities $1/4$, $1/4$, $1/2$, which is again different from $1/3$, $1/3$, $1/3$.

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- Hence, Bose–Einstein statistics assigns a **higher probability** to configurations (i) and (ii), i.e. it predicts that **entities bundle together in the same state, more than one would expect** (see, e.g., a Bose–Einstein condensate).

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- Coming to a text and its meaning, it is reasonable to expect that the (micro–)states are mostly assigned to the parts of the text that carry meaning, often even to the whole text, and the (micro–)states assigned to individual words in the text, i.e. the product states, are the minority, since these individual words have often lost their individuality at the expense of the pure entangled state of the whole text.

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- Finally, coming to the mathematical representation, Bose–Einstein statistics would entail the $N(E_i)$ s to satisfy

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- Maxwell–Boltzmann statistics would instead entail the $N(E_i)$ s to satisfy

$$N(E_i) = \frac{1}{Ce^{E_i/D}}$$

where C and D are again constants to be determined from data.

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- We have determined the constants A , B , C and D , and compared the ensuing Bose–Einstein and Maxwell–Boltzmann distributions with data.

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- The results show that Bose–Einstein statistics is in **remarkably good fit with empirical data**, whereas a **large deviation** is observed in the data from Maxwell–Boltzmann statistics.
- Hence, a collection of words in a story-telling text behaves as a suitable gas of bosons – we have introduced the term **cogniton** as the fundamental quantum of cognition.

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- But, the analogy with quantum physics is even **more impressive**: indeed, the presence of **meaning** makes in language the same effect that **coherence** makes in quantum physics, namely, a gas of cognitons behaves as a **Bose–Einstein condensate** in which most of the cognitons tend to occupy the lowest energy state.

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Result

If we introduce and quantify energy in language, then this behaves ‘macroscopically’ in a **non-classical way**.

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- A **Bell-type** test consists of a composite bipartite physical entity S_{12} , prepared in a given initial state and such that two individual entities S_1 and S_2 can be recognised as parts of S_{12} .
- Then, four coincidence experiments XY are performed on S_{12} which consist in performing experiments X with outcomes X_i on S_1 , with $X = A, A'$ and $i = 1, 2$, and experiments Y with outcomes Y_j , with $Y = B, B'$ and $j = 1, 2$, on S_2 .
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- If the experiment outcomes can only be ± 1 , then the expected values of XY become the **correlation functions** $E(XY)$, with $X = A, A'$ and $Y = B, B'$.
- In this case, the correlation functions can be represented in a classical probabilistic formalism if and only if they satisfy the **Clouser-Horne-Shimony-Holt** (CHSH) version of Bell's inequalities

$$-2 \leq \Delta_{CHSH} \leq +2$$

where Δ_{CHSH} is the **CHSH factor** defined as

$$\Delta_{CHSH} = E(A'B') + E(A'B) + E(AB') - E(AB)$$

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- If one considers the spectral decomposition of ρ , i.e. $\rho = \sum_{i=1}^n p_i |\psi_i\rangle\langle\psi_i|$, where $p_i \geq 0$, $i = 1, \dots, n$, $\sum_{i=1}^n p_i = 1$ and $\{|\psi_i\rangle\}_{i=1, \dots, n}$ is an ON basis in \mathcal{H} , then the von Neumann entropy becomes

$$S(\rho) = -\sum_{i=1}^n p_i \log_2 p_i$$

3.3 Entanglement, density states and entropy

- One then easily proves that $S(\rho) \geq 0$, and $S(\rho) = 0$ if and only if ρ represents a pure state, that is, $\rho = |\psi\rangle\langle\psi|$, for some unit vector $|\psi\rangle \in \mathcal{H}$.

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- The von Neumann entropy is $S(\rho_{12}) = 0$.
- The states of the component entities S_1 and S_2 are represented by density operators ρ_1 and ρ_2 obtained through the partial trace operation with respect to S_2 and S_1 , respectively, i.e. $\rho_1 = \text{Tr}_2\rho_{12}$ and $\rho_2 = \text{Tr}_1\rho_{12}$.
- One easily proves that ρ_1 and ρ_2 represent **non-pure**, or **density, states**.
- In this case, one finds that $S(\rho_1) = S(\rho_2) > 0$ – for a maximally entangled state in a 2-dimensional Hilbert space gives $S(\rho_1) = S(\rho_2) = \log_2 2$.

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- This behaviour of the entropy of a composite bipartite physical entity in quantum theory does not occur with classical entropy, hence it is typically considered as a **measure of entanglement**.

3.3 Entanglement and entropy in human language

Non-classicality of entropy in language

We stress a second **important point**: once the notion of **entropy** is introduced and quantified in language, this variable systematically behaves non-classically, i.e. the composition process **reduces** the entropy of an entity.

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- We intend to prove that, once a notion of entropy is introduced for a combination of concepts, this behaves as the von Neumann entropy of a quantum entity.
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- To this end, we need to put into perspective the empirical and theoretical studies we have made in regard to the violation of Bell's inequalities in concept combinations.
- We have performed various empirical tests on Bell's inequalities and the possible presence of entanglement using different combination of two concepts.
- In particular, we have performed two tests involving human participants, three documents retrieval tests on the web, using the corpuses of documents Google Books, Contemporary American English (COCA) and News on Web (NOW), and one image retrieval test using the search engine Google Images.

3.4 Identification of entanglement in *The Animal Acts*

- In these tests, we have used the concept combination *The Animal Acts* which we have considered as a composite bipartite conceptual entity made up of the component entities *Animal* and *Acts*, where “acts” refers to the sound, or noise, produced by an animal.
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Tests	Probabilities of experiment AB				Entropy S	CHSH factor Δ_{CHSH}
	$p(HG)$	$p(HW)$	$p(BG)$	$p(BW)$		
2011 cognitive test	0.049	0.630	0.259	0.062	0.177	2.4197
Google Books test	0	0.6526	0.3474	0	0.280	3.41
COCA test	0	0.8	0.2	0	0.217	2.8
Google Images test	0.0205	0.2042	0.7651	0.0103	0.202	2.4107
2021 cognitive test	0.0494	0.1235	0.7778	0.0494	0.114	2.79

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- This entanglement is due to the fact that the combination *The Animal Acts* carries meaning in such a way that, depending on which animals and which acts are considered in the human conceptual realm, the majority of (micro)-states are attributed intrinsically to the combination and not to the individual animals or the individual acts, and hence are not product states but entangled states.

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- We have proved that a **quantum representation in Hilbert space** faithfully reproduces all empirical data.
- In an empirical Bell-type test on human participants, a questionnaire is submitted to all participants which contains an introductory text explaining the conceptual entities and precise tasks involved in the test.

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- We focus on experiment AB , which can be realised by considering two examples of *Animal*, namely, *Horse* and *Bear*, and two examples of *Acts*, namely, *Growls* and *Whinnies*.
- Then, the four possible outcomes of AB are obtained by juxtaposing words, so that we get the four options *The Horse Growls*, *The Horse Whinnies*, *The Bear Growls*, *The Bear Whinnies*.

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- Next, each participant has to choose which one among these four options the participant considers as a good example of *The Animal Acts*.
- We denote by $p(HG)$, $p(HW)$, $p(BG)$ and $p(BW)$ the probability that *The Horse Growls*, *The Horse Whinnies*, *The Bear Growls* and *The Bear Whinnies*, respectively, are chosen in a given test.
- These probabilities can be computed from empirical data.

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- On the other side, a representation of *The Animal Acts* as a combination of the entities *Animal* and *Acts* requires the composed entity to be represented in the tensor product Hilbert space $\mathbb{C}^2 \otimes \mathbb{C}^2$, where an isomorphism I_{AB} , which we choose to be the identity operator, maps an ON basis of \mathbb{C}^4 into the canonical ON basis $\{(0, 1) \otimes (0, 1), (0, 1) \otimes (1, 0), (1, 0) \otimes (0, 1), (1, 0) \otimes (1, 0)\}$.

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- Suppose that the composite bipartite entity *The Animal Acts* is initially in the pure state represented by the unit vector

$$|\Psi_{AB}\rangle = \sqrt{p(HG)}|HG\rangle + \sqrt{p(HW)}|HW\rangle + \sqrt{p(BG)}|BG\rangle + \sqrt{p(BW)}|BW\rangle \quad (1)$$

where $\{|HG\rangle, |HW\rangle, |BG\rangle, |BW\rangle\}$ is an ON basis of eigenvectors of the product self-adjoint operator which represents AB in $\mathbb{C}^2 \otimes \mathbb{C}^2$.

3.6 The von Neumann entropy in in *The Animal Acts*

- To calculate the von Neumann entropies associated with the states of the component concepts *Animal* and *Acts*, we firstly write $\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$ and get $S(\rho_{AB}) = 0$, as ρ_{AB} represents a pure state.

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- Then, we take the partial traces of ρ_{AB} with respect to *Acts* and *Animal*, i.e.

$$\rho_{\text{Animal}} = \text{Tr}_{\text{Acts}}\rho_{AB} \quad \rho_{\text{Acts}} = \text{Tr}_{\text{Animal}}\rho_{AB}$$

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- Next, we diagonalise the self-adjoint operators ρ_{Animal} and ρ_{Acts} and use the formula for the von Neumann entropy.
- In both cases, the von Neumann entropy is greater than zero in all studies, which is consistent with the fact that the concepts *Animal* and *Acts* are in non-pure, or density, states.
- In other words, the process of concept combination, or composition, **reduces the entropy** of the component entities.

3.7 A general result in human language

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Result

If we introduce and quantify entropy in language, then this behaves 'macroscopically' in a **non-classical way**.

- 1 Introduction
- 2 Quantization of energy and Bose–Einstein statistics in large texts
- 3 Entanglement and entropy in the combination of concepts
- 4 Towards a thermodynamic theory of human cognition

4.1 The general characteristics of a thermodynamics of language

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- 3 a thermodynamics of language has to be a **non-classical theory**, as energy and entropy exhibit **genuine quantum-type aspects**.

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- On the one side, classical thermodynamics rests on the intuition that a gas consists, at the micro-level, of entities that are random, and **maximisation of entropy** expresses this intuition.

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- Then, it should not be a surprise that a decrease in entropy occurs, as a consequence of a meaning-due entanglement.

4.2 Broadening the range of validity of our findings

A new hypothesis on the nature of entanglement:

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- Indeed, we believe that the genuine quantum aspects identified in our investigation, i.e. quantization of energy and entanglement-induced reduction of entropy, are not peculiar of language, but they generally hold in **human cognition** and also in **human culture**, e.g., for what concerns cultural artefacts.
- E.g., every **collaboration** between humans exploits the effect we have identified here, namely, the creation of entanglement correlations in any collaboration of different individuals which lowers the entropy, hence the uncertainty, of the collaboration as compared to the entropies, hence uncertainties, carried by every individual who participates in the collaboration.

4.3 Warping effects in categorical perception

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- Our **perception of colours** is an example of this warping effect: two stimuli that both fall within the category of 'green' are perceived as more similar than two stimuli of which one falls within the category of 'green' and the other within the category of 'blue' even if, from a physics perspective, both pairs of stimuli have the same difference in frequency.

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- The simultaneous presence of these two effects, a contraction within a given category, and a dilation between different categories, causes us to perceive a **discrete set of colours**.
- This discretization in categorical perception can be naturally explained as a phenomenon of **quantization of energy**.

4.4 Zipf's law

- The attribution of defined energies to words in large texts also provides a **theoretical foundation** to **Zipf's law**, originally identified in a **purely empirical way** in linguistic areas, but systematically present in a variety of human-created systems, e.g.,
 - ▶ rankings of the size of cities
 - ▶ rankings of the size of income
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Conclusion

These examples indicate that the quantum-type thermodynamic behaviour we have identified is not limited to language/cognition, but concerns relevant areas of human culture.

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**Many Thanks for
Your Attention!**

... Any Questions?