Complementarity and topology in quantum and quantum-like theories

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1. **Plotnitsky A 2021 *Reality without realism: Matter, thought, and technology of quantum physics.* Heidelberg: Springer**
2. **Plotnitsky A 2022 "Most tantumising state of affairs": Mathematical and non-mathematical in qPlouantum-like understanding of thinking. *Front. Psychol*. 13:934776. doi: 10.3389/fpsyg.2022.934776**
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Singularity is observation.
--Stephen Smale

an artificial word like "complementarity" which does not belong to our daily concepts serves … to remind us of the epistemological situation here encountered.
--Niels Bohr

Topology is too important to be left to mathematicians.
--John Archibald Wheeler

The argument of this paper is grounded in the irreducible interference of observational instruments in our interactions with nature in quantum physics and, thus, in the constitution of quantum phenomena vs. classical physics or relativity, where this interference can be disregarded, at least in principle. The irreducible character of this interference, seen by Niels Bohr as the principal distinction between quantum physics and classical physics or relativity, was the basis his interpretation of quantum phenomena and quantum theory, specifically quantum mechanics (QM) and, by implication, quantum field theory (QFT), developed by him in conjunction with the concepts of complementarity.

Classical physics and relativity, special and general

All these theories are based in the assumption that we can observe the phenomena considered without appreciably disturbing them, and as a result, identify them with the corresponding objects and their independent behavior. It is this assumption that makes these theories realist because it allows then ideally to represent this behavior and to predict it, (ideally) exactly or, as in classical statistical physics or chaos theory, probabilistically, by using this representation.

"Our … description of physical phenomena [is] based of the idea that the phenomena concerned may be observed *without disturbing them appreciably.*"
--N. Bohr

Bohr is careful to refer to the idea and hence the assumption rather than saying that such is in fact the case. This assumption is, however, workable in these theories for all practical purposes.

Quantum physics

In quantum By contrast,

"Any observation of atomic phenomena will involve an *interaction [of the object under investigation] with the agency of observation* not to be neglected."
--N. Bohr

It is the irreducible nature of this interaction and thus the interference of technology into physical reality in quantum physics that is responsible for changing the nature of probability, from classical to quantum. Importantly, this interaction *gives rise* to a quantum phenomenon rather than *disturbs* the object observed.

Interpretation

There is no such a thing as THE (single) Copenhagen interpretation, as even Bohr changed his interpretation several times.

In the interpretation adopted here (following Bohr's ultimate interpretation) QM does not represent the physical emergence of quantum phenomena. Nothing can be said, by means of QM or otherwise, or even thought, concerning what happens between quantum experiments, which define quantum events or phenomena.

"There is no description of what happens to the system between the initial observation and the next measurement. …The demand to "describe what happens" in the quantum-theoretical process between two successive observations is a contradiction in adjecto, since the word "describe" refers to the use of classical concepts, while these concepts cannot be applied in the space between the observations; they can only be applied at the points of observation."
–W. Heisenberg

Nor, does QM represent quantum phenomena themselves, which are represented by classical physics, which, however, cannot predict them.

QM only predicts, in general probabilistically, the outcomes of quantum experiments, registered classically. No other predictions are possible on experimental grounds, because the repetition of the identically prepared quantum experiments in general leads to different outcomes.

However, the nature of the probabilities used is different from those of classical physics, even in realist interpretations of QM. These probabilities are nonadditive: the joint probability of two or more mutually exclusive alternatives in which an event might occur is not equal to the sum of the probabilities for each alternative, as in classical probability theory. Quantum predictions, moreover, must respond to the fact that quantum phenomena contain correlations, which are not found in classical physics, and in fact are expressly inconsistent with classical phenomena.

How, then, does QM calculate these probabilities? Although routine now, the mathematics of QM was a radical change from all mathematics previously used in physics, in particular in the following aspects—the use of complex numbers, ℂ, noncommutativity, and Born's (or an analogous) rule:

1.While all previous physics used, *fundamentally*, mathematics over real numbers, ℝ, and was finite-dimensional, QM uses Hilbert spaces over complex numbers, ℂ, which are abstract vector spaces of both finite and infinite dimensions. I speak of "fundamentally" because classical physics or relativity may use complex numbers, but (as when using Fourier analysis) only practically for calculation, and not essentially. They do not figure in the final solutions of the equations used and related to what is observed; and everything that we can, in principle, observe is always represented by real (technically, rational) numbers. QM had to find a different way to use its formalism in dealing with observed quantum phenomena.

2.The second key feature is the noncommutativity of Hilbert space-vectors and especially operators, known as "observables," which are *mathematical* entities over ℂ, as opposed to classical physics and relativity, where all observable quantities are represented by commuting functions of real variables. These complex quantities are only related to *physically* observable real quantities by using (3).

3.Born's rule or an analogous rule (such as von Neumann's projection postulate or Lüder's postulate), establishes the relation between "quantum amplitudes," associated with complex Hilbert-space vectors, and probabilities as real numbers, by using square moduli or, equivalently, the multiplication of these quantities and their complex conjugates, which are real quantities. Technically, these amplitudes are first linked to probability densities.

The probabilities involved are nonadditive: they obey the law of the addition of the so-called "amplitudes," which are complex quantity, associated with possible alternatives events, to the sum of which Born's rule is then applied, given real numbers corresponding to probabilities of these events. In the simplest case, when ψ is a wave function for a particle in the (position) Hilbert space, Born's rule says that the probability density function *p (x, y, z)* for predicting a measurement of the position at time t_1 is equal to $|\psi(x, y, z, t_1|^2$. Integrating over this density gives the probability or (if one repeats the experiment many times) statistics of finding the particle in a given area. Although Born's or similar rules are connected naturally to the formalism, they are added to rather than contained in it. We do not know why these rules work, but they do.

On a wave function and time dependent Schrödinger equation

$$(4'') \quad \nabla^2\psi - \frac{8\pi^2}{h^2}V\psi \mp \frac{4\pi i}{h}\frac{\partial\psi}{\partial t} = 0$$

***We will require the complex wave function ψ to satisfy one of these two equations*. Since the conjugate complex function $\bar{\psi}$ will then satisfy the *other* equation, we may take the real part of ψ as the real wave function (if we require it).**
--E. Schrödinger "Quantisierung als Eigenwertproblem III," Annalen der Physik, Vol. 80, 1926, 437-490. 1926

Note the first orders derivative in time.

Dirac's equation:

$$\left(\boldsymbol{\beta} \boldsymbol{m} \boldsymbol{c}^{2}+\sum_{k=1}^{3} \boldsymbol{\alpha}_{k} \boldsymbol{p}_{k} \boldsymbol{c}\right) \boldsymbol{\psi}(\boldsymbol{x}, \boldsymbol{t})=\boldsymbol{i} \hbar \frac{\partial \boldsymbol{\psi}(\boldsymbol{x}, \boldsymbol{t})}{\partial \boldsymbol{t}}$$

The new mathematical elements here are the 4×4 matrices α_k and β and the four-component wave function ψ. The Dirac matrices are all Hermitian,

$$\alpha_i^2 = \beta^2 = I_4$$

(I_4 is the identity matrix), and they mutually anticommute:

$$\alpha_i \beta + \beta \alpha_i = 0$$
$$\alpha_i \alpha_j + \alpha_j \alpha_i = 0$$

The equation unfolds into four coupled linear first-order partial differential equations for the four quantities that make up the wave function. The matrices form a basis of the corresponding Clifford algebra. One can think of Clifford algebras as quantizations of Grassmann's exterior algebras, in the same way that the Weyl algebra is a quantization of symmetric algebra. *p* is the momentum operator, but in a more complicated Hilbert space than in quantum mechanics. The wave function $\psi(t, x)$ takes value in $X = C^4$ (Dirac's spinors are elements of *X*). For each *t*, *y (t, x)* is an element of $H = L^2 (R^3) \otimes C^4$. This mathematical architecture allows one to predict the probabilities of (high-energy) QED events.

This structure, ultimately defined by the role of noncommutative self-adjoint operators, $\langle Ax, y\rangle = \langle xA^*, y \rangle$, A= A^*, a highly nontrivial concept, leads to the topological structure difference from that of the mathematics predicting the (Kolmogorovian) probability in classical physics. The mathematical structure of predictions is fundamentally different in classical and quantum physics, including topologically.

The algebra of quantum theory vs. geometry of classical mechanics, leads to a different (nontrivial) topology of the mathematics of predictions.

That does not mean that topology explain quantum physics, but it reflects a new type of physics.

What is the physics that correspond to this structure, specifically the absence of the single probability space, and led to its invention, beginning with Heisenberg?

Complementarity (=topological singularity)

"In the last resort an artificial word like "complementarity" which does not belong to our daily concepts serves … to remind us of the epistemological situation here encountered."
--Niels Bohr

The same is true about such terms as "entanglement," "superposition," "amplitude," etc. indeed nearly all terms we use in quantum physics.

"In this connection I warned especially against phrases, often found in the physical literature, such as "disturbing of phenomena by observation" or "creating physical attributes to atomic objects by measurements." Such phrases, which may serve to remind of the apparent paradoxes in quantum theory, are at the same time apt to cause confusion, since words like "phenomena" and "observations," just as "attributes" and "measurements," are used in a way hardly compatible with common language and practical definition."
--Niels Bohr

"Complementarity" was never used as a noun before Bohr.

As a general concept, complementarity is defined by:

(A) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet

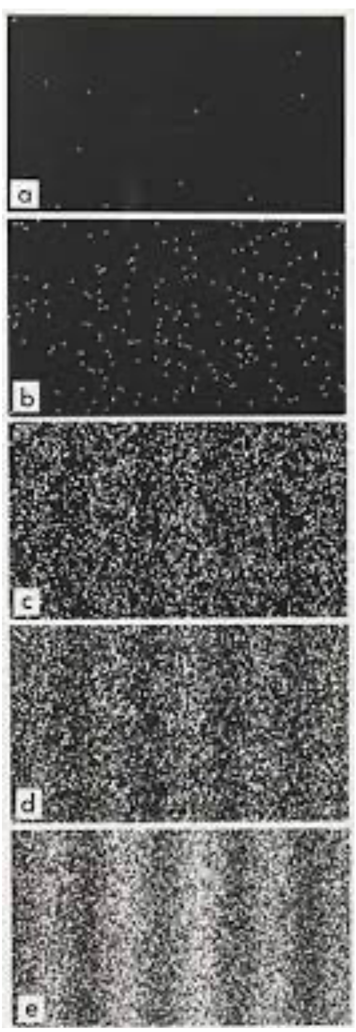
(B) the possibility of considering each one of them separately at any given point; and

(C) the necessity of considering all of them at different moments of time for a comprehensive account of the totality of phenomena that one must consider in quantum physics.

A paradigmatic example is the mutually exclusivity of the *exact* simultaneous position and momentum measurements in view of the uncertainty relations, $\Delta q \Delta p \cong h$ (where q is the coordinate, p is the momentum in the corresponding direction), which are experimentally confirmed laws independent of any theory, but with which QM is fully in accord. Both variables can be measured simultaneously inexactly. When is comes to exact measurement, at any moment in time, one can perform either one measurement or the other, and, in Bohr's ultimate interpretation, even define one or the other corresponding phenomenon, but never both together, thus in accordance with (A). On the other hand, one can always decide and thus has a freedom (at least a sufficient freedom) of choice to perform either measurement, as reflected in (B) and (C).

Wave-particle complementarity, with which the concept of complementarity is often associated, had not played a significant role in Bohr's understanding of quantum phenomena. His solution to the dilemma of whether quantum objects are particles or waves was that they were neither, any more than anything else. When either "picture" is used, it refers not to quantum objects but only to either one or the other of two *sets of discrete individual effects*, described classically, of the interactions between quantum objects and measuring instruments—particle-like, which may be individual or collective, or wave-like, which are always collective, composed of a large number of discrete individual effects. One needs on the order of 100,000 to observe the corresponding pattern of such effects.

A famous example of the second is a manifold of "interference" effects, composed of discrete traces of the collisions between the quantum objects considered and the screen in the double-slit experiment in the corresponding setup, when both slits are open and there are no means to know through which slit each object has passed. Alternatively, one observes a discrete set of random, rather than interference-like, effects. While these *two sets of effects* are complementary (with the statistics for each correspondingly predicted by QM), the properties observed pertain to two mutually exclusive *sets* of discrete phenomena observed in instruments and not to any continuous phenomena or continuous reality responsible for phenomena. However, the reality ultimately responsible for quantum phenomena cannot be assumed to be discrete either.



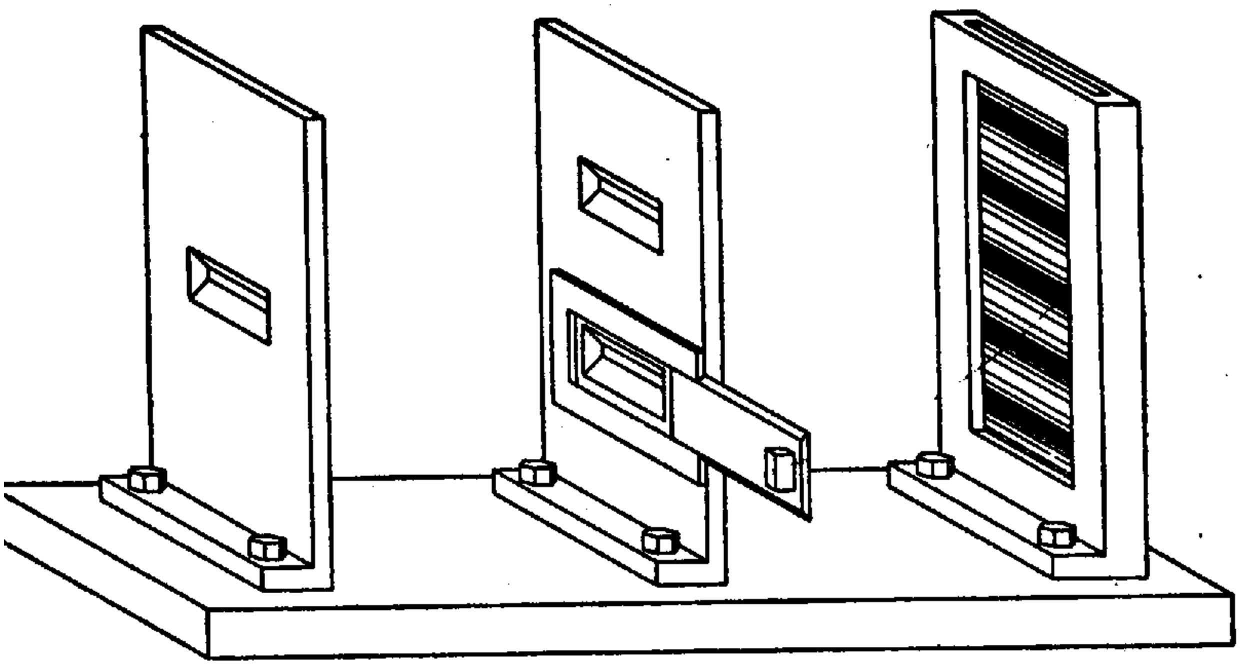


FIG. 4

According to Bohr:

"Evidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as complementary in the sense that only the totality of the phenomena [some of which are mutually exclusive] exhaust the *possible* information about the objects."

In classical mechanics, it is possible to comprehend all the information about each object at each moment in time within a single picture because the interference of measurement can be neglected: this assumption allows one to identify the phenomenon and the object under investigation and to establish determinately the quantities defining this information, such as the position and the momentum of each object, in the same experiment.

In quantum physics, this interference cannot be neglected. This leads to different experimental conditions for each measurement on and their complementarity, in correspondence with the uncertainty relations, which preclude the simultaneous exact measurement of both variables, always possible, in principle, in classical physics. The situation implies two incompatible pictures of what is observed in measuring instruments. Hence, the *possible* information about a quantum object, the information *to be possibly found* in measuring instruments, could only be exhausted by the mutually incompatible evidence obtainable under different experimental conditions.

On the other hand—and this is crucial—once made, either measurement, say, that of the position, will provide the *complete actual* information about the system's state, as complete as possible, at this moment in time. One could never obtain the complementary information, provided by the momentum measurement, at this moment in time, because to do so one would need simultaneously to perform a complementarity experiment on it, which is impossible. By (B), however, one can always decide to perform either one or the other experiment at any given moment in time. Each measurement establishes the only reality there is, and the alternative decision would establish a different reality. Instead of reflecting an arbitrarily selection of one or the other parts of a preexisting physical reality, our decisions concerning which experiment to perform establish the *single* reality that defines what *type* of quantity can be observed or predicted and precludes the complementary alternative.

Hence, parts (B) and (C) of the above definition are as important as part (A) and disregarding them can lead to misunderstandings of Bohr's concept, often misleadingly identified with just (A). Bohr's complementarity is not only about a mutual exclusivity of certain entities but also about performing quantum experiments and making predictions by human agents, in some of which a mutual exclusivity becomes necessary. That we have a free choice as concerns what kind of experiment to perform is in accordance with the very idea of experiment, including in classical physics. In classical physics or relativity, however, this freedom does not matter in fundamental terms because it only defines which part of the already established reality one decides to consider. In principle, all variables necessary for defining the future course of reality, in accord with classical causality, can always be determined at any moment in time, as there is no complementarity or the uncertainty relations. By contrast, quantum physics and complementarity give this freedom a fundamental role. By implementing our decision, we define the character of physical reality and its future course, which allows us to make only certain types of predictions and exclude certain other, *complementary*, types of predictions.

Moreover, each new measurement, M_2 at a later moment in time t_2, creates, in Schrödinger's phrase, a new "expectation-catalog," enabled by QM (cum Born's rule) for possible future measurements. This new measurement, as a new unique event, even if one measures the same variable, makes the previous expectation-catalog, defined by a previous measurement, M_1, at time t_1 meaningless as concerns any prediction after M_2. In each such sequence, one deals with a *quantum* Markov chain: the probability of a future event is defined only by the state of things at present and not the preceding history, but replacing the standard (additive) probabilities law of classical Markov chains with nonadditive quantum probability laws. It also follows that one can change one's decision and perform an alternative measurement at any point, which is an important fact that plays a key role in Bell's theorems and related findings.

Two concepts essential in classical physics and relativity, "measurement" and "causality," become no longer applicable in quantum theory in RWR interpretations. In Bohr's or the present interpretation, a quantum measurement *does not measure* or even *observe* any property of the reality ultimately responsible for quantum phenomena, which this reality is not assumed to possess before or even during the act of observation. By using an instrument (capable of registering quantum phenomena) an observation *creates* a quantum phenomenon by an interaction between this instrument and the quantum object, with the latter concepts only applicable at the time of observation in the present view. Each quantum phenomenon is the product of a unique act, event, of creation by an interaction by means of experimental technology.

The category of event: it is only what has been observed (classically in measuring instruments). One can make predictions concerning possible future events. But, whatever mathematics is used to predict an event is not an "event," for one think because in quantum physics no predicted event is guaranteed to happen. It may not, in which case there is no event.

"[In quantum physics] no phenomenon is a phenomenon unless it is a registered phenomenon."
--John A. Wheeler

Then what is so observed can be measured classically just as one measures what is observed in classical physics. In quantum physics, observations technologically construct quantum phenomena, while measurements then classically measure physical properties observed in instruments and not those of quantum objects.

This concept of quantum measurement, in conjunction with complementarity, lead to the corresponding understanding of the key feature of QM, the noncommutativity of certain quantum variables, such as those associated with the measurements of a momentum, P, and a coordinate Q, $PQ - QP = i\hbar$ ($PQ \neq QP$).

This formula is connected to the uncertainty relations, $\Delta q \Delta p \cong h$ (*q* is the coordinate, *p* the momentum in the same direction), which are part of the experimental confirmation of QM. The uncertainty relations are an experimental law, independent of any theory. In Bohr and the present view, as correlative to complementarity, the uncertainty relations are understood not only as the impossibility of *exactly measuring* both variables simultaneously, but the impossibility of simultaneously *defining* both variables.

Commonly, noncommutativity is seen as relating to the fact that, if one measures two physical properties involved in one order and then in the other, the outcome would in general be different, which is not the case in classical physics. However, the present understanding of quantum measurement, as a creation of quantum phenomenon, unique each time, offers a deeper view of this noncommutativity and the difference in the outcomes in reversing the order of measuring complementary variables.

It is true that if, in the experiment with the initial preparation of measuring instruments at time t_{01}, one makes first the position measurement, M_{1Q}, at time t_{11} and then the momentum measurement, M_{2P} at time t_{21} and then, with the same initial preparation of measuring instruments at time t_{02} reverse the order of the quantities we measure, by first measuring the momentum, M_{1P}, at time t_{12} and then the position at time t_{22}, M_{2Q}, the outcome will be different. As is, however, reflected in my double indexing, each set of measurements happens at a different set of time intervals and in fact requires a different quantum object. One can never reverse the order of measurements for the same quantum object.

This situation, rarely properly realized, has important implications in Q-L theories of psychological phenomena.

This situation reflects and confirms the fact that each quantum event is unique and is the product of a unique act of creation defined by our decision which experiment to perform and which technological set up to use. QM, however, or QFT enable us to predict, PROBALISTICALLY, the future course of reality, which also define, correlatively to complementarity, a new concept of causality, QUANTUM CAUSALITY, vs. classical, deterministic causality. It is, the mathematics—algebra, geometry, and topological—of quantum theory that enable us to do so.

Thank you!