Everett many-worlds theory and social sciences

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Law of Total Probability

$$p(x) + p(\neg x) = 1, \quad p(y) + p(\neg y) = 1$$

the law of total probability is represented by

$$p(y) = p(y, x) + p(y, \neg x) = p(y|x) \cdot p(x) + p(y|\neg x) \cdot p(\neg x)$$

and

$$p(\neg y) = p(\neg y, x) + p(\neg y, \neg x) = p(\neg y|x) \cdot p(x) + p(\neg y|\neg x) \cdot p(\neg x).$$

Quantum Probabilities

- Until *"recently"* quantum physics was the only branch in science that evaluated a probability *p(x)* of a state *x* as the squared magnitude of a probability amplitude *A(x)*, which is represented by a complex number

$$A(x) = \alpha + \beta \cdot i$$

$$A(x)^* \cdot A(x) = (\alpha - \beta \cdot i) \cdot (\alpha + \beta \cdot i) = \alpha^2 + \beta^2 = |A(x)|^2.$$

$$p(x) = |A(x)|^2 = A(x)^* \cdot A(x).$$

- Converting these amplitudes into probabilities leads to an interference term

$$
\begin{aligned}
|A(x) + A(y)|^2 &= |A(x)|^2 + |A(y)|^2 + 2 \cdot \Re(A(x) \cdot A^*(y)) \\
&= p(x) + p(y) + 2 \cdot \Re(A(x) \cdot A^*(y)),
\end{aligned}
$$

  - The summation rule of classical probability theory is violated, resulting in one of the **most fundamental laws** of quantum mechanics

$$|A(x) + A(y)|^2 \neq p(x) + p(y).$$

- A gun that fires electrons and a screen with two narrow slits x and y and a photographic plate
- An emitted electron can pass through slit x or slit y. The electron is spited into a superposition of two states that pass at the same time through both slits.
- Since the electron is unobserved and coherent it is rejoined into a single object at the photographic plate an interference pattern appears.
- The summation rule of classical probability theory is violated

Intensity Waves

- Not knowing through which slit it went through, **the electron is represented as a wave with the amplitudes**

$$a(x,\theta_1) = \sqrt{p(x)} \cdot e^{i \cdot \theta_1} = A(x), \quad a(y,\theta_2) = \sqrt{p(y)} \cdot e^{i \cdot \theta_2} = A(y).$$

$$\begin{aligned} I(z,\theta_1,\theta_2) &= |a(x,\theta_1) + a(y,\theta_2)|^2 = \\ &= (a(x,\theta_1) + a(y,\theta_2)) \cdot (a(x,\theta_1) + a(y,\theta_2))^* \end{aligned}$$

Intensity Waves

It follows

$$I(z,\theta_1,\theta_2) = \left(\sqrt{p(x)} \cdot e^{i \cdot \theta_1} + \sqrt{p(y)} \cdot e^{i \cdot \theta_2}\right) \cdot (\sqrt{p(x)} \cdot e^{-i \cdot \theta_1} + \sqrt{p(y)} \cdot e^{-i \cdot \theta_2}),$$

$$I(z,\theta_1,\theta_2) = p(x) + p(y) + \sqrt{p(y) \cdot p(x)} \cdot e^{i \cdot (\theta_2 - \theta_1)} + \sqrt{p(x) \cdot p(y)} \cdot e^{i \cdot (\theta_1 - \theta_2)}.$$

$$I(z,\theta_1,\theta_2) = p(x) + p(y) + \sqrt{p(x) \cdot p(y)} \cdot \left(e^{i \cdot (\theta_1 - \theta_2)} + e^{-i \cdot (\theta_1 - \theta_2)}\right). \qquad (39)$$

With

$$\cos(\theta_1 - \theta_2) = \frac{\left(e^{i \cdot (\theta_1 - \theta_2)} + e^{-i \cdot (\theta_1 - \theta_2)}\right)}{2}. \qquad (40)$$

we get

$$I(z,\theta_1,\theta_2) = p(x) + p(y) + 2 \cdot \sqrt{p(x) \cdot p(y)} \cdot \cos(\theta_1 - \theta_2) \qquad (41)$$

Intensity Waves

- For binary events,

$$p(x) + p(\neg x) = 1, \quad p(y) + p(\neg y) = 1,$$

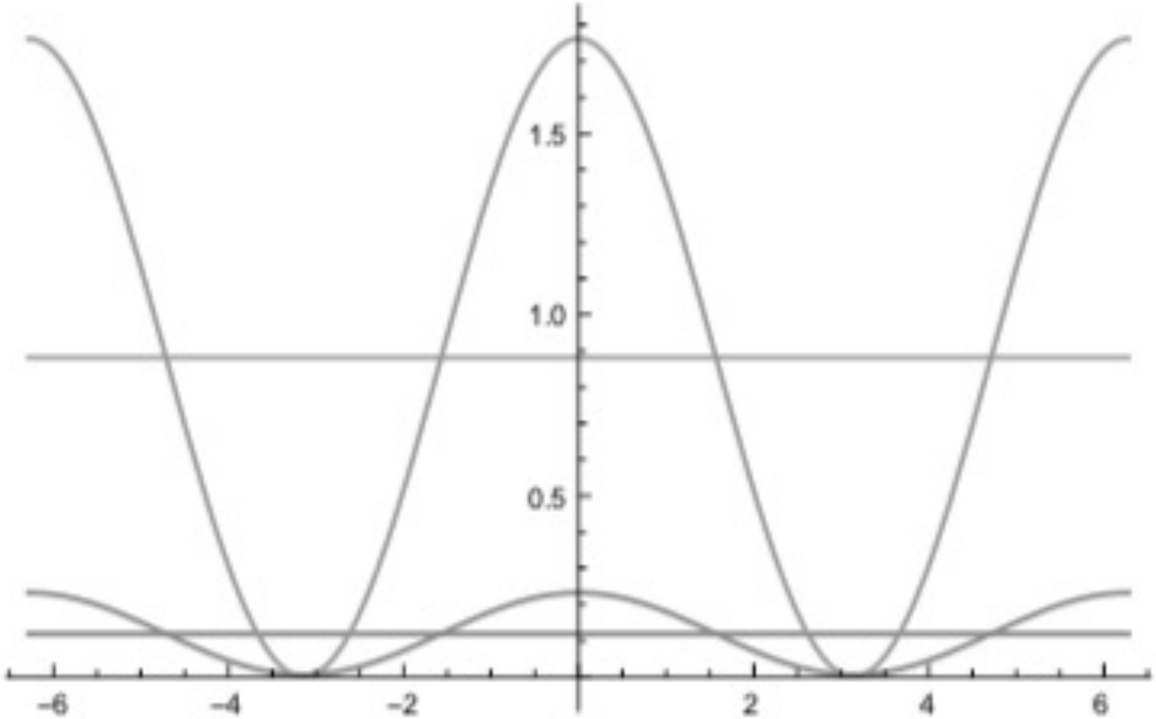
$$\theta = \theta_1 - \theta_2$$

$$I(y, \theta) = p(y) + 2 \cdot \sqrt{p(y, x) \cdot p(y, \neg x)} \cdot \cos(\theta)$$

$$\theta_\neg = \theta_{\neg 1} - \theta_{\neg 2}$$

$$I(\neg y, \theta_\neg) = p(\neg y) + 2 \cdot \sqrt{p(\neg y, x) \cdot p(\neg y, \neg x)} \cdot \cos(\theta_\neg)$$

Intensity Waves



$$I(y, \theta) + I(\neg y, \theta_{\neg}) \neq 1.$$

Probability Waves

- Intensity waves are probability waves if:

1. they are positive

$$0 \leq p(y,\theta), \quad 0 \leq p(\neg y,\theta_\neg);$$

2. they sum to one

$$p(y,\theta) + p(\neg y,\theta_\neg) = p(y) + p(\neg y) = 1;$$

3. they are smaller or equal to one

$$p(y,\theta) \leq 1, \quad p(\neg y,\theta_\neg) \leq 1.$$

1) Positive

- Since the norm is being positive or more precisely non-negative, we can represent a quadratic form by l_2 norm

$$(a(x,\theta_1)+a(y,\theta_2)\cdot a(x,\theta_1)+a(y,\theta_2)) = \|a(x,\theta_1)+a(y,\theta_2)\|^2$$

and it follows

$$0 \leq \|a(x,\theta_1)+a(y,\theta_2)\|^2.$$

2) Law of Balance

- The interference is balanced, which means that the interference each out.

$$\sqrt{p(y,x)\cdot p(y,\neg x)}\cdot\cos(\theta) = -\sqrt{p(\neg y,x)\cdot p(\neg y,\neg x)}\cdot\cos(\theta_{\neg}).$$

2) Law of Balance

- The **smaller probability wave** determines the other probability wave

$$p(y) \leq p(\neg y).$$

$$\theta_{\neg} = \cos^{-1}\left(-\sqrt{\frac{p(y|x) \cdot p(y|\neg x)}{p(\neg y|x) \cdot p(\neg y|\neg x)}} \cdot \cos(\theta)\right)$$

$$p(\neg y) \leq p(y)$$

$$\theta = \cos^{-1}\left(-\sqrt{\frac{p(\neg y|x) \cdot p(\neg y|\neg x)}{p(y|x) \cdot p(y|\neg x)}} \cdot \cos(\theta_{\neg})\right)$$

3) Probability Waves are Smaller Equal One

$$p(y) \leq p(y_\neg)$$

$$p(y) \leq 0.5.$$

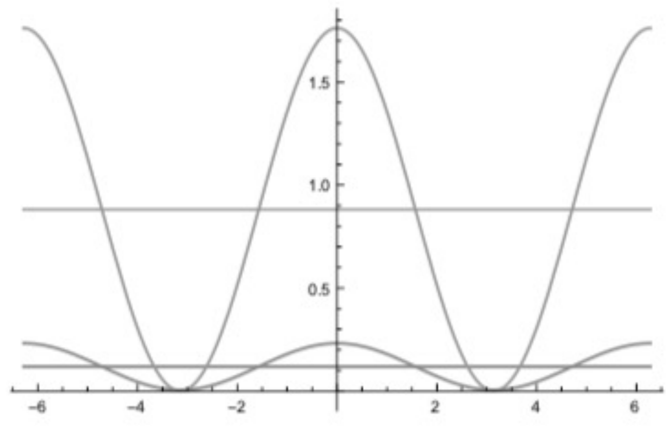
$$\sqrt{p(y,x) \cdot p(y,\neg x)} \leq \frac{p(y,x) + p(y,\neg x)}{2}$$

$$2 \cdot \sqrt{p(y,x) \cdot p(y,\neg x)} \leq p(y) = p(y,x) + p(y,\neg x)$$

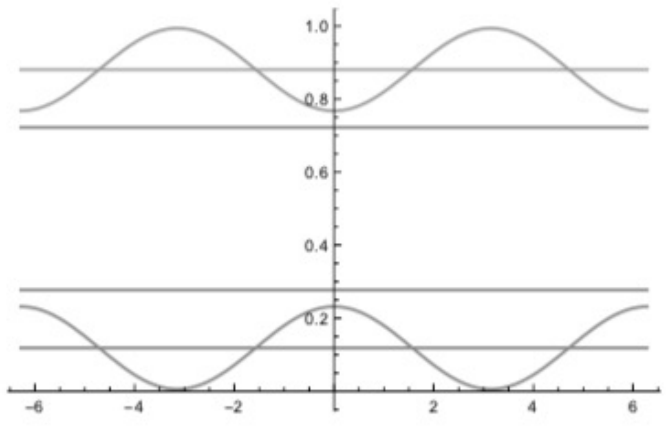
$$p(y,\theta) = p(y) + 2 \cdot \sqrt{p(y|x) \cdot p(x) \cdot p(y|\neg x) \cdot p(\neg x)} \cdot \cos(\theta) \leq 2 \cdot p(y) \leq 1.$$

Probability Waves

- The *bigger wave* is replaced by the negative *smaller one*

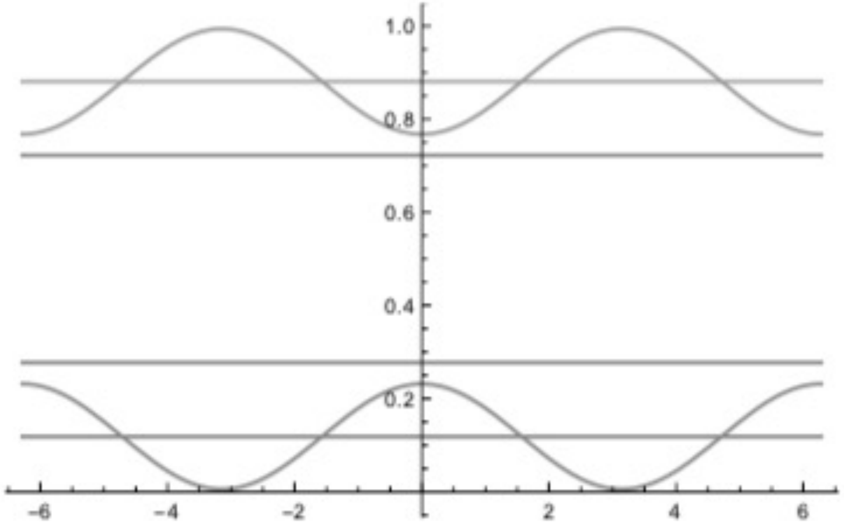


(a)



(b)

Description of Probability Wave

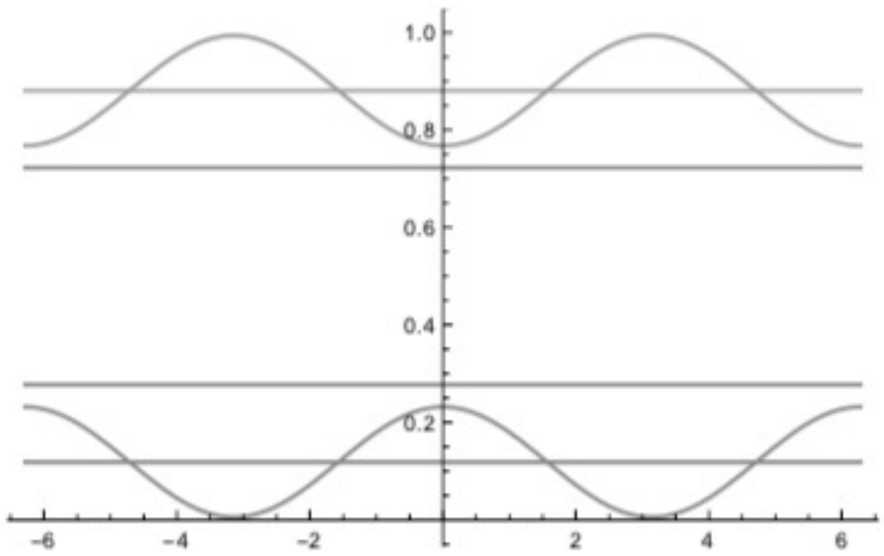


- How can we map a probability wave into ONE probability value?
  - We are interested in the maximal *amplitude size*
  - How to *interpret* the representation?

Principle of Entropy

- The values of the waves that are closest to the equal distribution are chosen
- By doing so the uncertainty is maximized and the information about the probability wave is not lost

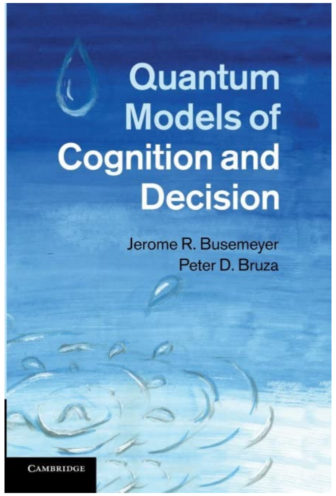
Principle of Entropy

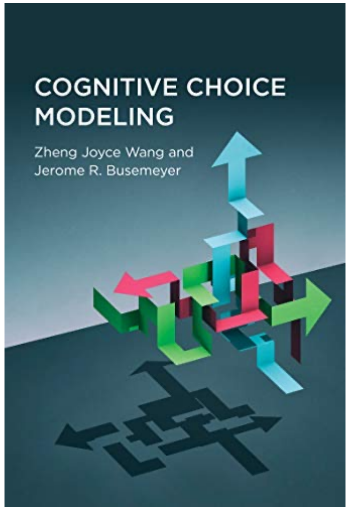


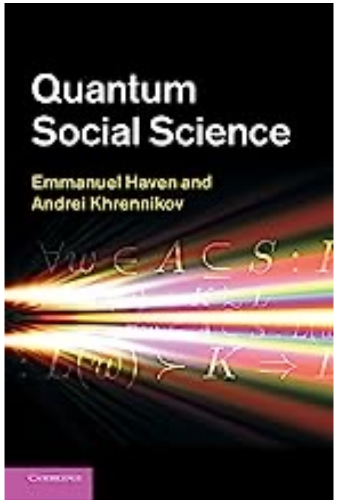
and

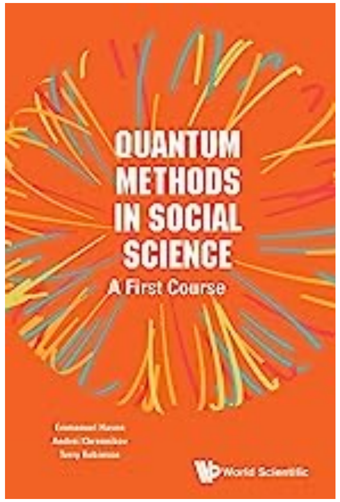
$$p(y) = 0.12 \quad I_y = [0.01, 0.23], \quad p_q(y) = 0.23,$$

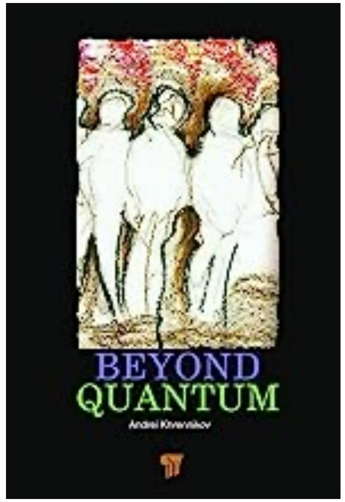
$$p(\neg y) = 0.88 \quad I_{\neg y} = [0.77, 0.99], \quad p_q(\neg y) = 0.77.$$











- Clues from psychology indicate that human cognition is not only based on traditional probability theory as explained by Kolmogorov's axioms but additionally on quantum probability.
- For example, humans when making decisions violate the law of total probability.

- Humans when making decisions violate the law of total probability.

- **The violation can be explained as a quantum interference resulting from the phase represented by the angle θ.**

Prisoner’s Dilemma Game

- Each prisoner is offered by the prosecutors a bargain:
  - By testifying against the other one she can betray the other one (Defect)
  - On the other hand, the prisoner can refuse the deal and cooperate with the other one by remaining silent

Prisoner's Dilemma Game

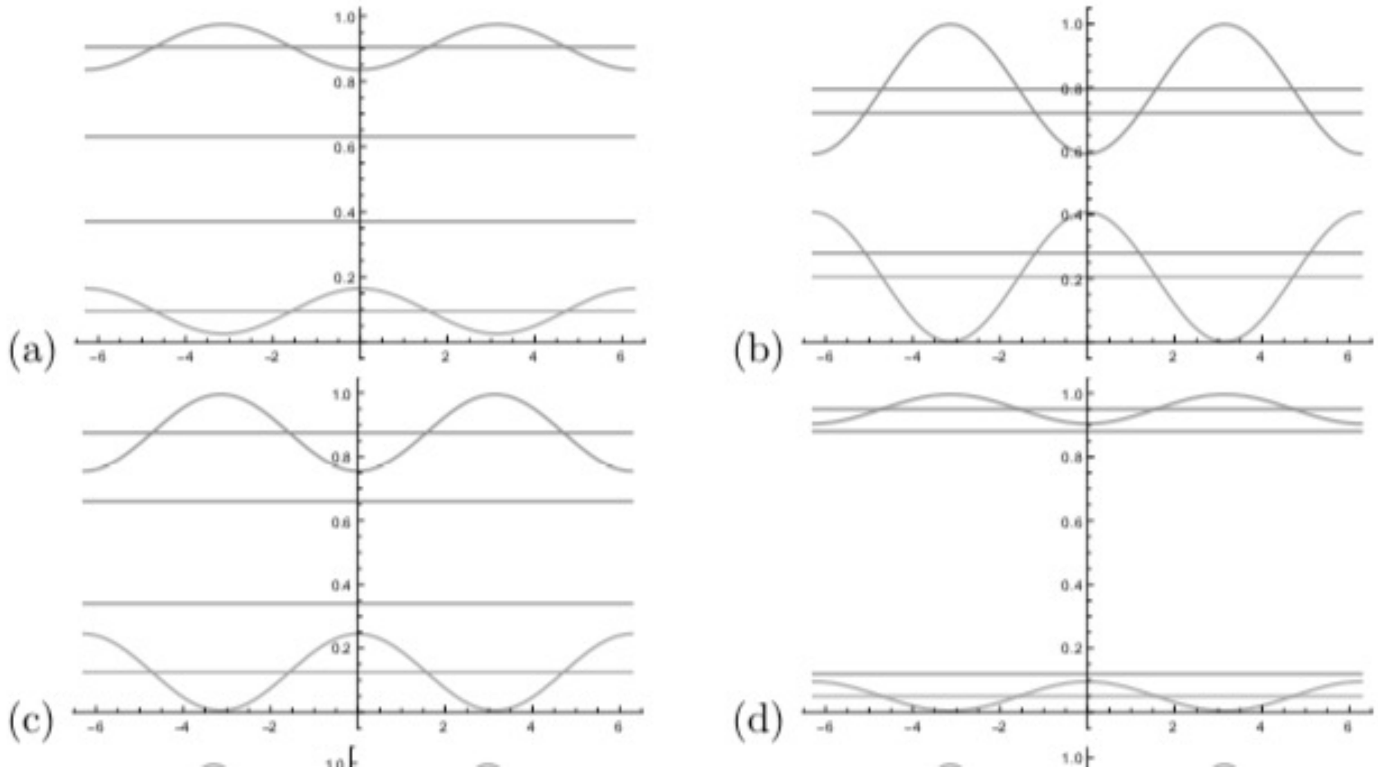
- Probability of prisoner *x* cooperating is $p(x) = 0.5$ and the probability of defecting is $p(\neg x) = 0.5$.

- The participants of the experiment were asked three different questions…

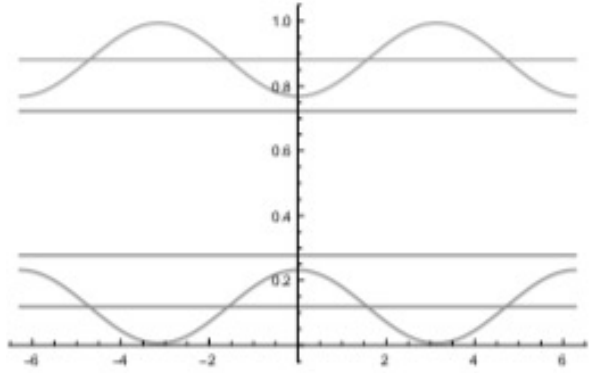
Prisoner’s Dilemma Game

- What is the probability that the prisoner *y* defects given *x* defects, $p(\neg y|\neg x)$.
- What is the probability that the prisoner *y* defects given *x* cooperates, $p(\neg y|x)$.
- What is the probability that the prisoner y defects given there is no information present about knowing if prisoner x cooperates or defects *(Law of Total Probability)*

$$p(\neg y) = p(\neg y, x) + p(\neg y, \neg x) = p(\neg y|x) \cdot p(x) + p(\neg y|\neg x) \cdot p(\neg x).$$



Experiment	$I_{\neg y}$	$p_{sub}(\neg y)$	$p_q(\neg y)$	$p(\neg y)$
(a)	[0.84, 0.97]	0.63	0.84	0.91
(b)	[0.59, 1]	0.72	0.59	0.79
(c)	[0.76,1]	0.66	076	0.88
(d)	[0.90, 1]	0.88	0.90	0.95
(e) Average	[0.77, 0.99]	0.72	0.77	0.88



- Law of **maximum uncertainty** of quantum-like systems. This law indicates how to chose a possible value for the phase

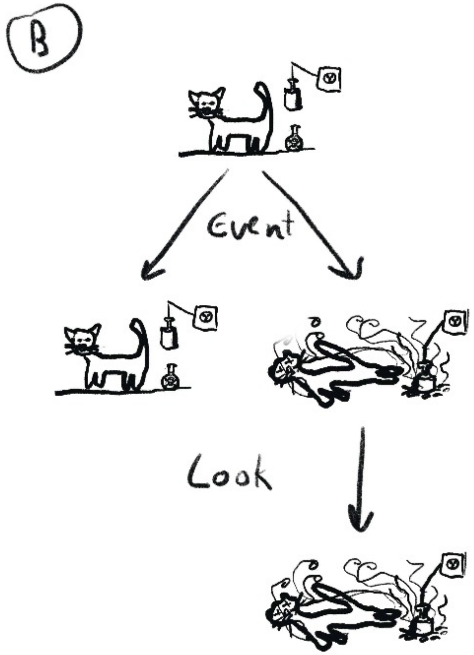
- **Why do humans violate the law of total probability during decision making?**

Quantum Measurment

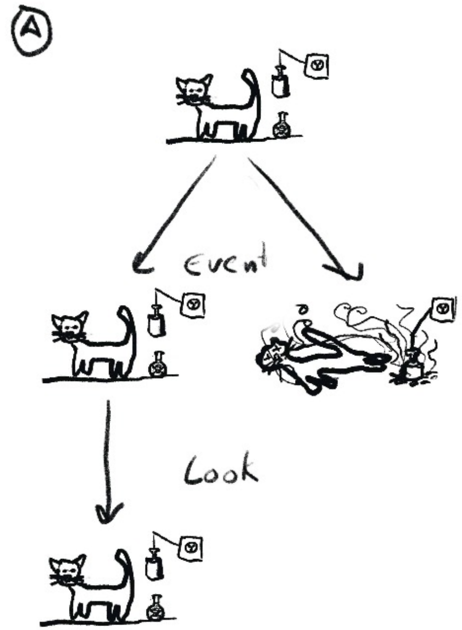
- The wave function in quantum mechanics represents a superposition of states.

- If unobserved it evolves smoothly and continuously; however, during the measurement, it collapses into a definite state

- The state of the cat in superposition is represented as Cat_{dead} +Cat_{alive} after measuring by the observer we get the following interpretation, either



$$(Cat_{dead} + Cat_{alive}) \cdot observer = Cat_{dead} \cdot observer$$



$$(\text{Cat}_{\text{dead}} + \text{Cat}_{\text{alive}}) \cdot \text{observer} = \text{Cat}_{\text{alive}} \cdot \text{observer}$$

- Observer is with the cat that is alive according to the **Copenhagen interpretation**.

- The most popular interpretation, the Copenhagen interpretation, claims that quantum mechanics is a mathematical tool that is used in the calculation of probabilities and has no physical existence; all other questions are metaphysical and should be avoided

- The problem arises by the description of **observer-independent world**, like there is some microscopic quantum world and the macroscopic world is not part of it.

- We have to consider the entire system, including the measuring device (observer), as a single quantum system. It should be instead

$$(\mathrm{Cat}_{dead} + \mathrm{Cat}_{alive}) \cdot \mathrm{observer} = \mathrm{Cat}_{dead} \cdot \mathrm{observer} + \mathrm{Cat}_{alive} \cdot \mathrm{observer}$$

- It leads to the many-worlds theory

Hugh Everett III (1930- 1982), on of the greatest physics of the 20th century, he suggested theory is called the many-worlds theory (multiverse) or many-worlds interpretation (MWI)

- The observer is an integral part of the measured system **but does not play any role** during the measurement itself
- All elements of a superposition are actual, none anymore “real” than another
- After an observation no element of the final superposition is mysteriously selected to be awarded and the others condemned to oblivion.
- **Everett broke with the Copenhagen interpretation** and with the discontinuity of a wave-function collapse.

- The suggested theory is called **the many-worlds theory** (multiverse) or many-worlds interpretation (MWI)
- According to the theory there should be a universe in which every physically possible event has happened

- The MWI received more credibility with the discovery of quantum decoherence in the 1970s
- It has received increased attention in recent decades, becoming one of the **mainstream interpretations** of quantum mechanics.

- The world, which we inhabit is one of a vast number of many worlds, most of which are in practice isolated form one another.
- A splitting of a *history* in *history1* and *history2* can be described as

$$\mathit{history} \rightarrow \mathit{history1} + \mathit{history2}$$

- Histories can branch and can rejoin, rejoin is time reverse of splitting and indicated as

$$\textit{history1} + \textit{history2} \rightarrow \textit{history} \wedge \textit{interference}$$

- *history1* and *history2* merge to one single history (single point) and during the rejoin interference is always present.
- Rejoin is only possible as long as the histories are not entangled with the world in which they appear

- Otherwise they split into two different worlds and it is not possible to rejoin as *history1* and *history2* and they become entangled with the world resulting in a split

$$history \rightarrow (history1 + history2) \cdot world \rightarrow history1 \cdot world + history2 \cdot world$$

- Why do humans violate the law of total probability during decision making?
- We can assume that subconsciously humans model two different histories that a rejoined in the final decision leading to an interference term that violates the law of total probability during decision making.

- It leads to the assumption that two different histories can rejoin into one history on the macro scale of the world.
- Also, that the subconscious decision-making process in the brain adapted to this assumption during the evolution when making predictions during the decision process in the future-

- Classical representations provide a better account of data as individuals gain familiarity with it

- There is a distinction between unknown, which is considered as the truth value, and ignorance, which is considered as the lack of knowledge or as being unaware. An event can be true false or unknown.

- For **unknown** events, the law of total probabilities is applied

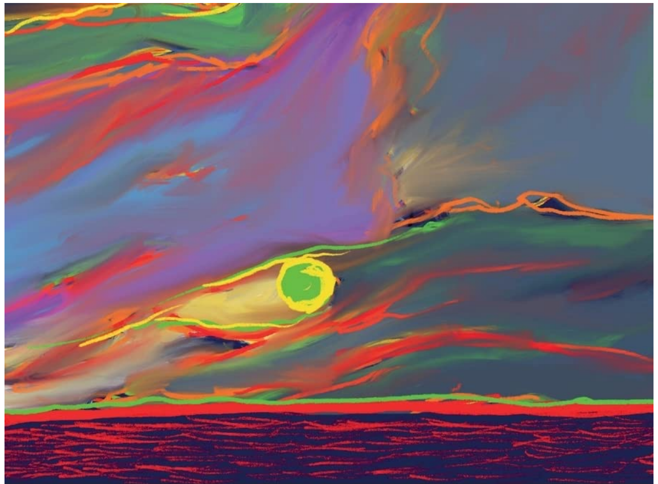
- For the events of which we are **unaware** (**ignorance)**, we apply quantum-like models with the interference resulting from the quantum probabilities (amplitudes).
  - We determine the possible value of the wave that produce the interference by the law of maximum uncertainty of quantum-like system

- An **unknown** event is not known to us because we do not have enough information

- **Ignorance** means that we cannot achieve this information.
- **Ignorance** is not a truth value at all

- Quantum cognition leads to the conclusion that a wave function can be present at the macro scale of our daily life
- Quantum-like models indicate how to describe probabilistic reasoning in artificial intelligence

- Subconsciously humans assume that the two different histories that lead to the same result cannot entangle with their world since they are present in the future, resulting in the interference that violates the law of total probability



Mind, Brain, Quantum AI, and the Multiverse

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