Tameness beyond o-minimality

Philipp Hieronymi

Introductory Workshop on Tame Geometry, Transseries and Applications to Analysis and Geometry, January 2022

UNIVERSITÄT BONN

Universität Bonn

Mathematisches Institut

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!
3. This module will start on March 14th and end on Friday April 8th.

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!
3. This module will start on March 14th and end on Friday April 8th.
4. In this talk I will also say something about tameness phenomena over the ordered real additive group.

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!
3. This module will start on March 14th and end on Friday April 8th.
4. In this talk I will also say something about tameness phenomena over the ordered real additive group.
5. This talk also serves as an invitation to the *tameness research program*.

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!
3. This module will start on March 14th and end on Friday April 8th.
4. In this talk I will also say something about tameness phenomena over the ordered real additive group.
5. This talk also serves as an invitation to the *tameness research program*.
6. Pantelis Eleftheriou and I are planning to organize a mini-workshop around this topic, tentatively scheduled around the end of March.

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!
3. This module will start on March 14th and end on Friday April 8th.
4. In this talk I will also say something about tameness phenomena over the ordered real additive group.
5. This talk also serves as an invitation to the *tameness research program*.
6. Pantelis Eleftheriou and I are planning to organize a mini-workshop around this topic, tentatively scheduled around the end of March.
7. A thank you.

Before we start.

1. This talk is an advertisement for the third module *Tameness beyond o-minimality* of the graduate course on *Tame Phenomena Over the Real Field*.
2. Thus the target audience are graduate students!
3. This module will start on March 14th and end on Friday April 8th.
4. In this talk I will also say something about tameness phenomena over the ordered real additive group.
5. This talk also serves as an invitation to the *tameness research program*.
6. Pantelis Eleftheriou and I are planning to organize a mini-workshop around this topic, tentatively scheduled around the end of March.
7. A thank you.

"A lot of model theory is concerned with discovering and charting the "tame" regions of mathematics, where wild phenomena like space filling curves and Gödel incompleteness are absent, or at least under control. As Hrushovski put it recently:

Model Theory = Geography of Tame Mathematics."

- Lou van den Dries

"What might it mean for a first-order expansion of the field of real numbers to be tame or well behaved? In recent years, much attention has been paid by model theorists and real-analytic geometers to the o-minimal setting[...]. But there are expansions of the real field that define sets with infinitely many connected components, yet are tame in some well-defined sense [...]. The analysis of such structures often requires a mixture of model-theoretic, analytic-geometric and descriptive set-theoretic techniques. An underlying idea is that first-order definability, in combination with the field structure, can be used as a tool for determining how complicated is a given set of real numbers."

- Chris Miller

“A lot of model theory is concerned with discovering and charting the “tame” regions of mathematics, where wild phenomena like space filling curves and Gödel incompleteness are absent, or at least under control. As Hrushovski put it recently:

Model Theory = Geography of Tame Mathematics.”

- Lou van den Dries

“What might it mean for a first-order expansion of the field of real numbers to be tame or well behaved? In recent years, much attention has been paid by model theorists and real-analytic geometers to the o-minimal setting[...]. But there are expansions of the real field that define sets with infinitely many connected components, yet are tame in some well-defined sense [...]. The analysis of such structures often requires a mixture of model-theoretic, analytic-geometric and descriptive set-theoretic techniques. An underlying idea is that first-order definability, in combination with the field structure, can be used as a tool for determining how complicated is a given set of real numbers.”

- Chris Miller

A structure on $\mathbb{R}$ is a sequence $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ such that for each m:

1. $\mathfrak{S}_m$ is a boolean algebra of subsets of $\mathbb{R}^m$.
2. If $X \in \mathfrak{S}_m$ and $Y \in \mathfrak{S}_n$, then $X \times Y \in \mathfrak{S}_{m+n}$.
3. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \; : \; x_i = x_j\} \in \mathfrak{S}_m$.
4. If $X \in \mathfrak{S}_{m+1}$, then the projection of X onto the first m coordinates is in $\mathfrak{S}_m$.
5. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \; : \; x_i < x_j\} \in \mathfrak{S}_m$.
6. The singleton $\{r\} \in \mathfrak{S}_1$ for every $r \in \mathbb{R}$.

Let K be a subfield of $\mathbb{R}$. A K-geometry is a structure satisfying

7. If $X \in \mathfrak{S}_m$ and $T : \mathbb{R}^m \to \mathbb{R}^n$ is K-linear, then $T(X) \in \mathfrak{S}_n$,

A structure on $\mathbb{R}$ is a sequence $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ such that for each m:

1. $\mathfrak{S}_m$ is a boolean algebra of subsets of $\mathbb{R}^m$.
2. If $X \in \mathfrak{S}_m$ and $Y \in \mathfrak{S}_n$, then $X \times Y \in \mathfrak{S}_{m+n}$.
3. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i = x_j\} \in \mathfrak{S}_m$.
4. If $X \in \mathfrak{S}_{m+1}$, then the projection of X onto the first m coordinates is in $\mathfrak{S}_m$.
5. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i < x_j\} \in \mathfrak{S}_m$.
6. The singleton $\{r\} \in \mathfrak{S}_1$ for every $r \in \mathbb{R}$.

Let K be a subfield of $\mathbb{R}$. A K-geometry is a structure satisfying

7. If $X \in \mathfrak{S}_m$ and $T : \mathbb{R}^m \to \mathbb{R}^n$ is K-linear, then $T(X) \in \mathfrak{S}_n$,

A field geometry is a structure satisfying

8. If $X \in \mathfrak{S}_m$ and $p : \mathbb{R}^m \to \mathbb{R}^n$ is a polynomial map, then $p(X) \in \mathfrak{S}_n$,

A structure on $\mathbb{R}$ is a sequence $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ such that for each m:

1. $\mathfrak{S}_m$ is a boolean algebra of subsets of $\mathbb{R}^m$.
2. If $X \in \mathfrak{S}_m$ and $Y \in \mathfrak{S}_n$, then $X \times Y \in \mathfrak{S}_{m+n}$.
3. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i = x_j\} \in \mathfrak{S}_m$.
4. If $X \in \mathfrak{S}_{m+1}$, then the projection of X onto the first m coordinates is in $\mathfrak{S}_m$.
5. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i < x_j\} \in \mathfrak{S}_m$.
6. The singleton $\{r\} \in \mathfrak{S}_1$ for every $r \in \mathbb{R}$.

Let K be a subfield of $\mathbb{R}$. A K-geometry is a structure satisfying

7. If $X \in \mathfrak{S}_m$ and $T : \mathbb{R}^m \to \mathbb{R}^n$ is K-linear, then $T(X) \in \mathfrak{S}_n$,

A field geometry is a structure satisfying

8. If $X \in \mathfrak{S}_m$ and $p : \mathbb{R}^m \to \mathbb{R}^n$ is a polynomial map, then $p(X) \in \mathfrak{S}_n$,

We say that $X \subseteq \mathbb{R}^n$ belongs to $\mathfrak{S}$ (or $\mathfrak{S}$ contains X, X is definable in $\mathfrak{S}$) if $X \in \mathfrak{S}_n$.

A structure on $\mathbb{R}$ is a sequence $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ such that for each m:

1. $\mathfrak{S}_m$ is a boolean algebra of subsets of $\mathbb{R}^m$.
2. If $X \in \mathfrak{S}_m$ and $Y \in \mathfrak{S}_n$, then $X \times Y \in \mathfrak{S}_{m+n}$.
3. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i = x_j\} \in \mathfrak{S}_m$.
4. If $X \in \mathfrak{S}_{m+1}$, then the projection of X onto the first m coordinates is in $\mathfrak{S}_m$.
5. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i < x_j\} \in \mathfrak{S}_m$.
6. The singleton $\{r\} \in \mathfrak{S}_1$ for every $r \in \mathbb{R}$.

Let K be a subfield of $\mathbb{R}$. A K-geometry is a structure satisfying

7. If $X \in \mathfrak{S}_m$ and $T : \mathbb{R}^m \to \mathbb{R}^n$ is K-linear, then $T(X) \in \mathfrak{S}_n$,

A field geometry is a structure satisfying

8. If $X \in \mathfrak{S}_m$ and $p : \mathbb{R}^m \to \mathbb{R}^n$ is a polynomial map, then $p(X) \in \mathfrak{S}_n$,

We say that $X \subseteq \mathbb{R}^n$ belongs to $\mathfrak{S}$ (or $\mathfrak{S}$ contains X, X is definable in $\mathfrak{S}$) if $X \in \mathfrak{S}_n$. We say $\mathfrak{S}$ avoids X if $X \notin \mathfrak{S}_n$.

A structure on $\mathbb{R}$ is a sequence $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ such that for each m:

1. $\mathfrak{S}_m$ is a boolean algebra of subsets of $\mathbb{R}^m$.
2. If $X \in \mathfrak{S}_m$ and $Y \in \mathfrak{S}_n$, then $X \times Y \in \mathfrak{S}_{m+n}$.
3. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i = x_j\} \in \mathfrak{S}_m$.
4. If $X \in \mathfrak{S}_{m+1}$, then the projection of X onto the first m coordinates is in $\mathfrak{S}_m$.
5. If $1 \leq i \leq j \leq m$, then $\{(x_1, \ldots, x_m) \in \mathbb{R}^m \ : \ x_i < x_j\} \in \mathfrak{S}_m$.
6. The singleton $\{r\} \in \mathfrak{S}_1$ for every $r \in \mathbb{R}$.

Let K be a subfield of $\mathbb{R}$. A K-geometry is a structure satisfying

7. If $X \in \mathfrak{S}_m$ and $T : \mathbb{R}^m \to \mathbb{R}^n$ is K-linear, then $T(X) \in \mathfrak{S}_n$,

A field geometry is a structure satisfying

8. If $X \in \mathfrak{S}_m$ and $p : \mathbb{R}^m \to \mathbb{R}^n$ is a polynomial map, then $p(X) \in \mathfrak{S}_n$,

We say that $X \subseteq \mathbb{R}^n$ belongs to $\mathfrak{S}$ (or $\mathfrak{S}$ contains X, X is definable in $\mathfrak{S}$) if $X \in \mathfrak{S}_n$. We say $\mathfrak{S}$ avoids X if $X \notin \mathfrak{S}_n$.

K-Semilinear geometry. Let K be a subfield of $\mathbb{R}$. The collection of all K-semilinear sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f_1(x) \geq 0, \ldots, f_M(x) \geq 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f_1, \ldots f_M, g_1, \ldots, g_N$ are K-affine functions and $k, M, N \in \mathbb{N}$, forms a K-geomertry. This follows from the Fourier-Motzkin elimination method. This class contains all K-polyhedra.

K-Semilinear geometry. Let K be a subfield of $\mathbb{R}$. The collection of all K-semilinear sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f_1(x) \geq 0, \ldots, f_M(x) \geq 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f_1, \ldots f_M, g_1, \ldots, g_N$ are K-affine functions and $k, M, N \in \mathbb{N}$, forms a K-geomertry. This follows from the Fourier-Motzkin elimination method. This class contains all K-polyhedra.

Semialgebraic geometry. The collection of all semialgebraic sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f(x) = 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f, g_1, ..., g_N \in \mathbb{R}[x_1, ..., x_k]$ and $k, N \in \mathbb{N}$, forms a field geometry.

K-Semilinear geometry. Let K be a subfield of $\mathbb{R}$. The collection of all K-semilinear sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f_1(x) \geq 0, \ldots, f_M(x) \geq 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f_1, \ldots f_M, g_1, \ldots, g_N$ are K-affine functions and $k, M, N \in \mathbb{N}$, forms a K-geomertry. This follows from the Fourier-Motzkin elimination method. This class contains all K-polyhedra.

Semialgebraic geometry. The collection of all semialgebraic sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f(x) = 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f, g_1, ..., g_N \in \mathbb{R}[x_1, ..., x_k]$ and $k, N \in \mathbb{N}$, forms a field geometry. This is the Tarski-Seidenberg theorem.

K-Semilinear geometry. Let K be a subfield of $\mathbb{R}$. The collection of all K-semilinear sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f_1(x) \geq 0, \dots, f_M(x) \geq 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f_1, \dots f_M, g_1, \dots, g_N$ are K-affine functions and $k, M, N \in \mathbb{N}$, forms a K-geomertry. This follows from the Fourier-Motzkin elimination method. This class contains all K-polyhedra.

Semialgebraic geometry. The collection of all semialgebraic sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f(x) = 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f, g_1, ..., g_N \in \mathbb{R}[x_1, ..., x_k]$ and $k, N \in \mathbb{N}$, forms a field geometry. This is the Tarski-Seidenberg theorem.

Subexponential geometry. The collection of projections on the first n variables of sets

$$\{(x, y) \in \mathbb{R}^{n+k} : F(x, y) = 0\},$$

where $k \in \mathbb{N}$ and $F : \mathbb{R}^{n+k} \to \mathbb{R}$ is a function from the ring

$$\mathbb{R}[x_1, ..., x_n, y_1, ..., y_k, e^{x_1}, ..., e^{x_n}, e^{y_1}, ..., e^{y_n}],$$

forms a field geometry.

K-Semilinear geometry. Let K be a subfield of $\mathbb{R}$. The collection of all K-semilinear sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f_1(x) \geq 0, \ldots, f_M(x) \geq 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f_1, \ldots f_M, g_1, \ldots, g_N$ are K-affine functions and $k, M, N \in \mathbb{N}$, forms a K-geomertry. This follows from the Fourier-Motzkin elimination method. This class contains all K-polyhedra.

Semialgebraic geometry. The collection of all semialgebraic sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f(x) = 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f, g_1, ..., g_N \in \mathbb{R}[x_1, ..., x_k]$ and $k, N \in \mathbb{N}$, forms a field geometry. This is the Tarski-Seidenberg theorem.

Subexponential geometry. The collection of projections on the first n variables of sets

$$\{(x, y) \in \mathbb{R}^{n+k} : F(x, y) = 0\},$$

where $k \in \mathbb{N}$ and $F : \mathbb{R}^{n+k} \to \mathbb{R}$ is a function from the ring

$$\mathbb{R}[x_1, ..., x_n, y_1, ..., y_k, e^{x_1}, ..., e^{x_n}, e^{y_1}, ..., e^{y_n}],$$

forms a field geometry. This is Wilkie's thorem.

K-Semilinear geometry. Let K be a subfield of $\mathbb{R}$. The collection of all K-semilinear sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f_1(x) \geq 0, \ldots, f_M(x) \geq 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f_1, \ldots f_M, g_1, \ldots, g_N$ are K-affine functions and $k, M, N \in \mathbb{N}$, forms a K-geomertry. This follows from the Fourier-Motzkin elimination method. This class contains all K-polyhedra.
Semialgebraic geometry. The collection of all semialgebraic sets, that is, finite unions of sets of the form

$$\{x \in \mathbb{R}^k : f(x) = 0, g_1(x) > 0, ..., g_N(x) > 0\},$$

where $f, g_1, ..., g_N \in \mathbb{R}[x_1, ..., x_k]$ and $k, N \in \mathbb{N}$, forms a field geometry. This is the Tarski-Seidenberg theorem.
Subexponential geometry. The collection of projections on the first n variables of sets

$$\{(x, y) \in \mathbb{R}^{n+k} : F(x, y) = 0\},$$

where $k \in \mathbb{N}$ and $F : \mathbb{R}^{n+k} \to \mathbb{R}$ is a function from the ring

$$\mathbb{R}[x_1, ..., x_n, y_1, ..., y_k, e^{x_1}, ..., e^{x_n}, e^{y_1}, ..., e^{y_n}],$$

forms a field geometry. This is Wilkie's thorem.

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexpontential geometry, globally subanalytic geometry,...

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexpontential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexpontential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexpontential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. Let $\mathcal{A}$ be a collection of subsets of various $\mathbb{R}^n$'s. Then let $\mathfrak{S}(\mathcal{A})$ be the smallest structure containing $\mathcal{A}$.

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexpontential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. Let $\mathcal{A}$ be a collection of subsets of various $\mathbb{R}^n$'s. Then let $\mathfrak{S}(\mathcal{A})$ be the smallest structure containing $\mathcal{A}$.

The logician observes: Sets in $\mathfrak{S}(A)$ are just sets definable (with parameters) in $(\mathbb{R}, <, (A)_{A \in \mathcal{A}})$.

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexponential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. Let $\mathcal{A}$ be a collection of subsets of various $\mathbb{R}^n$'s. Then let $\mathfrak{S}(\mathcal{A})$ be the smallest structure containing $\mathcal{A}$.

The logician observes: Sets in $\mathfrak{S}(A)$ are just sets definable (with parameters) in $(\mathbb{R}, <, (A)_{A \in \mathcal{A}})$.

Examples.

1. Let $\mathcal{A}_{\text{linear}}$ be the graph of addition. Then $\mathfrak{S}(\mathcal{A}_{\text{linear}})$ is the collection of all $\mathbb{Q}$-semilinear sets (this is the $\mathbb{Q}$-geometry generated by the $\emptyset$).

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexpontential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. Let $\mathcal{A}$ be a collection of subsets of various $\mathbb{R}^n$'s. Then let $\mathfrak{S}(\mathcal{A})$ be the smallest structure containing $\mathcal{A}$.

The logician observes: Sets in $\mathfrak{S}(A)$ are just sets definable (with parameters) in $(\mathbb{R}, <, (A)_{A\in\mathcal{A}})$.

Examples.

1. Let $\mathcal{A}_{\text{linear}}$ be the graph of addition. Then $\mathfrak{S}(\mathcal{A}_{\text{linear}})$ is the collection of all $\mathbb{Q}$-semilinear sets (this is the $\mathbb{Q}$-geometry generated by the $\emptyset$).
2. Let $\mathcal{A}_{\text{alg}}$ be $\mathcal{A}_{\text{linear}}$ plus the graph of multiplication. Then $\mathfrak{S}(\mathcal{A}_{\text{alg}})$ is the collection of all semialgebraic sets (this is the field geometry generated by the $\emptyset$).

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexponential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. Let $\mathcal{A}$ be a collection of subsets of various $\mathbb{R}^n$'s. Then let $\mathfrak{S}(\mathcal{A})$ be the smallest structure containing $\mathcal{A}$.

The logician observes: Sets in $\mathfrak{S}(A)$ are just sets definable (with parameters) in $(\mathbb{R}, <, (A)_{A \in \mathcal{A}})$.

Examples.

1. Let $\mathcal{A}_{\text{linear}}$ be the graph of addition. Then $\mathfrak{S}(\mathcal{A}_{\text{linear}})$ is the collection of all $\mathbb{Q}$-semilinear sets (this is the $\mathbb{Q}$-geometry generated by the $\emptyset$).
2. Let $\mathcal{A}_{\text{alg}}$ be $\mathcal{A}_{\text{linear}}$ plus the graph of multiplication. Then $\mathfrak{S}(\mathcal{A}_{\text{alg}})$ is the collection of all semialgebraic sets (this is the field geometry generated by the $\emptyset$).
3. Let $\mathcal{A}_{\text{exp}}$ to be $\mathcal{A}_{\text{alg}}$ plus the graph of exp. Then $\mathfrak{S}(\mathcal{A}_{\text{alg}})$ is the collection of all subexponential sets.

Definition. We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ **o-minimal** if every element of $\mathfrak{S}_1$ is a finite union of intervals and points.

Examples. Semilinear geometry, semialgebraic geometry, subexponential geometry, globally subanalytic geometry,...(more on o-minimality in Gareth Jones' talk on Friday)

Naive charting goal. *Classify geometries (up to equality).*

Definition. Let $\mathcal{A}$ be a collection of subsets of various $\mathbb{R}^n$'s. Then let $\mathfrak{S}(\mathcal{A})$ be the smallest structure containing $\mathcal{A}$.

The logician observes: Sets in $\mathfrak{S}(A)$ are just sets definable (with parameters) in $(\mathbb{R}, <, (A)_{A\in\mathcal{A}})$.

Examples.

1. Let $\mathcal{A}_{\text{linear}}$ be the graph of addition. Then $\mathfrak{S}(\mathcal{A}_{\text{linear}})$ is the collection of all $\mathbb{Q}$-semilinear sets (this is the $\mathbb{Q}$-geometry generated by the $\emptyset$).
2. Let $\mathcal{A}_{\text{alg}}$ be $\mathcal{A}_{\text{linear}}$ plus the graph of multiplication. Then $\mathfrak{S}(\mathcal{A}_{\text{alg}})$ is the collection of all semialgebraic sets (this is the field geometry generated by the $\emptyset$).
3. Let $\mathcal{A}_{\text{exp}}$ to be $\mathcal{A}_{\text{alg}}$ plus the graph of exp. Then $\mathfrak{S}(\mathcal{A}_{\text{alg}})$ is the collection of all subexponential sets.

Naive charting goal (rephrased). Given $\mathcal{A}$, understand $\mathfrak{S}(A)$.

Wildness. Consider $\mathcal{Z} := \mathcal{A}_{\mathsf{alg}} \cup \{\mathbb{Z}\}$. What is $\mathfrak{S}(\mathcal{Z})$?

Naive charting goal (rephrased). Given $\mathcal{A}$, understand $\mathfrak{S}(A)$.

Wildness. Consider $\mathcal{Z} := \mathcal{A}_{\mathsf{alg}} \cup \{\mathbb{Z}\}$. What is $\mathfrak{S}(\mathcal{Z})$? This contains all closed sets - even all Borel sets. Hence all projections of Borel sets. Then all of the complements. Then again all of the projections... So there is no uniformly finite bound on the number of iterations needed.

Naive charting goal (rephrased). Given $\mathcal{A}$, understand $\mathfrak{S}(A)$.

Wildness. Consider $\mathcal{Z} := \mathcal{A}_{\mathsf{alg}} \cup \{\mathbb{Z}\}$. What is $\mathfrak{S}(\mathcal{Z})$? This contains all closed sets - even all Borel sets. Hence all projections of Borel sets. Then all of the complements. Then again all of the projections... So there is no uniformly finite bound on the number of iterations needed.

All space-filling curve and all nowhere differentiable continuous functions are in $\mathfrak{S}(\mathcal{Z})$.

Naive charting goal (rephrased). Given $\mathcal{A}$, understand $\mathfrak{S}(A)$.

Wildness. Consider $\mathcal{Z} := \mathcal{A}_{\mathsf{alg}} \cup \{\mathbb{Z}\}$. What is $\mathfrak{S}(\mathcal{Z})$? This contains all closed sets - even all Borel sets. Hence all projections of Borel sets. Then all of the complements. Then again all of the projections... So there is no uniformly finite bound on the number of iterations needed.

All space-filling curve and all nowhere differentiable continuous functions are in $\mathfrak{S}(\mathcal{Z})$.

Beyond o-minimality (Miller-Speissegger)

Let $\mathcal{A}_{\mathsf{Spiral}} = \mathcal{A}_{\mathsf{alg}} \cup \{\mathbb{S}_\omega\}$, where $\mathbb{S}_\omega := \{(e^t \cos \omega t, e^t \sin \omega t) : t \in \mathbb{R}\}$. Every set in $\mathfrak{S}(\mathcal{A}_{\mathsf{Spiral}})$ is a union of an open set and finitely many discrete sets (d-minimal). Hence $\mathbb{Z}$ is not in $\mathfrak{S}(\mathcal{A}_{\mathsf{Spiral}})$.

Naive charting goal (rephrased). Given $\mathcal{A}$, understand $\mathfrak{S}(A)$.

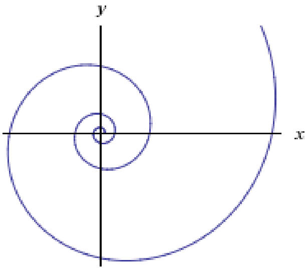
Wildness. Consider $\mathcal{Z} := \mathcal{A}_{\mathrm{alg}} \cup \{\mathbb{Z}\}$. What is $\mathfrak{S}(\mathcal{Z})$? This contains all closed sets - even all Borel sets. Hence all projections of Borel sets. Then all of the complements. Then again all of the projections... So there is no uniformly finite bound on the number of iterations needed.

All space-filling curve and all nowhere differentiable continuous functions are in $\mathfrak{S}(\mathcal{Z})$.

Beyond o-minimality (Miller-Speissegger)

Let $\mathcal{A}_{\mathrm{Spiral}} = \mathcal{A}_{\mathrm{alg}} \cup \{\mathbb{S}_\omega\}$, where $\mathbb{S}_\omega := \{(e^t \cos \omega t, e^t \sin \omega t) : t \in \mathbb{R}\}$. Every set in $\mathfrak{S}(\mathcal{A}_{\mathrm{Spiral}})$ is a union of an open set and finitely many discrete sets (d-minimal). Hence $\mathbb{Z}$ is not in $\mathfrak{S}(\mathcal{A}_{\mathrm{Spiral}})$.

Note that the intersection of $\mathbb{S}_\omega$ onto the x-axis is $e^{2\pi\mathbb{Z}/\omega}$.



Naive charting goal (rephrased). Given $\mathcal{A}$, understand $\mathfrak{S}(A)$.

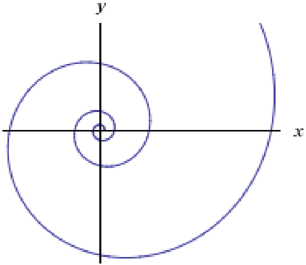
Wildness. Consider $\mathcal{Z} := \mathcal{A}_{\mathrm{alg}} \cup \{\mathbb{Z}\}$. What is $\mathfrak{S}(\mathcal{Z})$? This contains all closed sets - even all Borel sets. Hence all projections of Borel sets. Then all of the complements. Then again all of the projections... So there is no uniformly finite bound on the number of iterations needed.

All space-filling curve and all nowhere differentiable continuous functions are in $\mathfrak{S}(\mathcal{Z})$.

Beyond o-minimality (Miller-Speissegger)

Let $\mathcal{A}_{\mathrm{Spiral}} = \mathcal{A}_{\mathrm{alg}} \cup \{\mathbb{S}_\omega\}$, where $\mathbb{S}_\omega := \{(e^t \cos \omega t, e^t \sin \omega t) : t \in \mathbb{R}\}$. Every set in $\mathfrak{S}(\mathcal{A}_{\mathrm{Spiral}})$ is a union of an open set and finitely many discrete sets (d-minimal). Hence $\mathbb{Z}$ is not in $\mathfrak{S}(\mathcal{A}_{\mathrm{Spiral}})$.

Note that the intersection of $\mathbb{S}_\omega$ onto the x-axis is $e^{2\pi\mathbb{Z}/\omega}$.



A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$.

A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$. We call $(X, \prec)$ a dense ω-order if X is dense in some interval and $(X, \prec)$ is order-isomorphic to $(\mathbb{N}, <)$.

A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$. We call $(X, \prec)$ a dense ω-order if X is dense in some interval and $(X, \prec)$ is order-isomorphic to $(\mathbb{N}, <)$.

A condition for wildness. Let $\mathfrak{S}$ be a field geometry. If $\mathfrak{S}$ contains a dense ω-order, then $\mathbb{Z}$ belongs to $\mathfrak{S}$.

A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$. We call $(X, \prec)$ a dense ω-order if X is dense in some interval and $(X, \prec)$ is order-isomorphic to $(\mathbb{N}, <)$.

A condition for wildness. Let $\mathfrak{S}$ be a field geometry. If $\mathfrak{S}$ contains a dense ω-order, then $\mathbb{Z}$ belongs to $\mathfrak{S}$.

Why bother? The non-existence of a dense ω-order in a geometry has substantial consequences.

A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$. We call $(X, \prec)$ a dense ω-order if X is dense in some interval and $(X, \prec)$ is order-isomorphic to $(\mathbb{N}, <)$.

A condition for wildness. Let $\mathfrak{S}$ be a field geometry. If $\mathfrak{S}$ contains a dense ω-order, then $\mathbb{Z}$ belongs to $\mathfrak{S}$.

Why bother? The non-existence of a dense ω-order in a geometry has substantial consequences.

Notation. (Miller-Speissegger, Dolich-Miller-Steinhorn) Let $\mathfrak{S}$ be a structure. We say $X \subseteq \mathbb{R}^n$ is D_Σ in $\mathfrak{S}$ if there is $Y \subseteq \mathbb{R}^{n+2}$ in $\mathfrak{S}$ such that

1. $X = \bigcup_{r,s>0} Y_{r,s}$,
2. $Y_{r,s} \subseteq Y_{r',s'}$ if $r \leq r'$ and $s' \leq s$.

A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$. We call $(X, \prec)$ a dense ω-order if X is dense in some interval and $(X, \prec)$ is order-isomorphic to $(\mathbb{N}, <)$.

A condition for wildness. Let $\mathfrak{S}$ be a field geometry. If $\mathfrak{S}$ contains a dense ω-order, then $\mathbb{Z}$ belongs to $\mathfrak{S}$.

Why bother? The non-existence of a dense ω-order in a geometry has substantial consequences.

Notation. (Miller-Speissegger, Dolich-Miller-Steinhorn) Let $\mathfrak{S}$ be a structure. We say $X \subseteq \mathbb{R}^n$ is D_Σ in $\mathfrak{S}$ if there is $Y \subseteq \mathbb{R}^{n+2}$ in $\mathfrak{S}$ such that

1. $X = \bigcup_{r,s>0} Y_{r,s}$,
2. $Y_{r,s} \subseteq Y_{r',s'}$ if $r \leq r'$ and $s' \leq s$.

Strong Baire Category Theorem. (Fornasiero-H.-Walsberg) Let $\mathfrak{S}$ be a geometry that does not contain a dense ω-order. Then every D_Σ-set either has interior or is nowhere dense.

A more reasonable charting goal. *Classify geometries up to tameness conditions.*

Notation. Let $(X, \prec)$ be such that $X \subseteq \mathbb{R}$ and $\prec$ is a linear order on X. We say $(X, \prec)$ belongs to a structure $\mathfrak{S}$ if X and $\prec$ (as a subset of $\mathbb{R}^2$) belong to $\mathfrak{S}$. We call $(X, \prec)$ a dense ω-order if X is dense in some interval and $(X, \prec)$ is order-isomorphic to $(\mathbb{N}, <)$.

A condition for wildness. Let $\mathfrak{S}$ be a field geometry. If $\mathfrak{S}$ contains a dense ω-order, then $\mathbb{Z}$ belongs to $\mathfrak{S}$.

Why bother? The non-existence of a dense ω-order in a geometry has substantial consequences.

Notation. (Miller-Speissegger, Dolich-Miller-Steinhorn) Let $\mathfrak{S}$ be a structure. We say $X \subseteq \mathbb{R}^n$ is D_Σ in $\mathfrak{S}$ if there is $Y \subseteq \mathbb{R}^{n+2}$ in $\mathfrak{S}$ such that

1. $X = \bigcup_{r,s>0} Y_{r,s}$,
2. $Y_{r,s} \subseteq Y_{r',s'}$ if $r \leq r'$ and $s' \leq s$.

Strong Baire Category Theorem. (Fornasiero-H.-Walsberg) Let $\mathfrak{S}$ be a geometry that does not contain a dense ω-order. Then every D_Σ-set either has interior or is nowhere dense.

Definition. (Walsberg) We say an geometry $\mathfrak{G}$ is type A if it does not contain a dense ω-order. It is called type C if it contains every compact set, and type B if it is neither type A or type C.

Definition. We say a geometry $\mathfrak{G}$ is field-type if there is a field $(I, \oplus, \otimes)$, where $I \subseteq \mathbb{R}$ is an interval, such that both $\oplus$ and $\otimes$ are in $\mathfrak{G}$ (as subsets of $\mathbb{R}^3$) and $(I, <, \oplus, \otimes)$ is isomorphic to $(\mathbb{R}, <, +, \cdot)$.

Definition. (Walsberg) We say an geometry $\mathfrak{G}$ is type A if it does not contain a dense ω-order. It is called type C if it contains every compact set, and type B if it is neither type A or type C.

Definition. We say a geometry $\mathfrak{G}$ is field-type if there is a field $(I, \oplus, \otimes)$, where $I \subseteq \mathbb{R}$ is an interval, such that both $\oplus$ and $\otimes$ are in $\mathfrak{G}$ (as subsets of $\mathbb{R}^3$) and $(I, <, \oplus, \otimes)$ is isomorphic to $(\mathbb{R}, <, +, \cdot)$.

field-type \ dense ω-order	no	yes
yes	**type A, field-type** For any k, every continuous function in $\mathfrak{G}$ is C^k almost everywhere.	**type C** Every continuous function on compact domain is in $\mathfrak{G}$.
no	**type A, affine** Every continuous function in $\mathfrak{G}$ is affine almost everywhere.	**type B** Every C^2 function in $\mathfrak{G}$ is affine.

Definition. (Walsberg) We say an geometry $\mathfrak{S}$ is type A if it does not contain a dense ω-order. It is called type C if it contains every compact set, and type B if it is neither type A or type C.

Definition. We say a geometry $\mathfrak{S}$ is field-type if there is a field $(I, \oplus, \otimes)$, where $I \subseteq \mathbb{R}$ is an interval, such that both $\oplus$ and $\otimes$ are in $\mathfrak{S}$ (as subsets of $\mathbb{R}^3$) and $(I, <, \oplus, \otimes)$ is isomorphic to $(\mathbb{R}, <, +, \cdot)$.

field-type \ dense ω-order	no	yes
yes	**type A, field-type** For any k, every continuous function in $\mathfrak{S}$ is C^k almost everywhere.	**type C** Every continuous function on compact domain is in $\mathfrak{S}$.
no	**type A, affine** Every continuous function in $\mathfrak{S}$ is affine almost everywhere.	**type B** Every C^2 function in $\mathfrak{S}$ is affine.

Remarks.

1. If $\mathfrak{S}$ is a field geometry (ie. $\mathcal{A}_{\mathsf{alg}}$ is contained in $\mathfrak{S}$), then $\mathfrak{S}$ is type A or type C.
2. Actually: if $\mathfrak{S}$ is an $\mathbb{R}$-geometry, then $\mathfrak{S}$ is type A or type C.

Remarks.

1. If $\mathfrak{S}$ is a field geometry (ie. $\mathcal{A}_{\mathsf{alg}}$ is contained in $\mathfrak{S}$), then $\mathfrak{S}$ is type A or type C.
2. Actually: if $\mathfrak{S}$ is an $\mathbb{R}$-geometry, then $\mathfrak{S}$ is type A or type C.
3. If $\mathfrak{S}$ is o-minimal or d-minimal (or satisfies any other notion of geometric tameness), then $\mathfrak{S}$ is type A.

Remarks.

1. If $\mathfrak{S}$ is a field geometry (ie. $\mathcal{A}_{\mathsf{alg}}$ is contained in $\mathfrak{S}$), then $\mathfrak{S}$ is type A or type C.
2. Actually: if $\mathfrak{S}$ is an $\mathbb{R}$-geometry, then $\mathfrak{S}$ is type A or type C.
3. If $\mathfrak{S}$ is o-minimal or d-minimal (or satisfies any other notion of geometric tameness), then $\mathfrak{S}$ is type A.
4. For model-theorists: If (the expansion of $(\mathbb{R}, <, +)$ corresponding to) $\mathfrak{S}$ satisfies any Shelah-style combinatorical tameness notion (such as distality, NIP or NTP2), then $\mathfrak{S}$ is type A.

Remarks.

1. If $\mathfrak{S}$ is a field geometry (ie. $\mathcal{A}_{\mathsf{alg}}$ is contained in $\mathfrak{S}$), then $\mathfrak{S}$ is type A or type C.
2. Actually: if $\mathfrak{S}$ is an $\mathbb{R}$-geometry, then $\mathfrak{S}$ is type A or type C.
3. If $\mathfrak{S}$ is o-minimal or d-minimal (or satisfies any other notion of geometric tameness), then $\mathfrak{S}$ is type A.
4. For model-theorists: If (the expansion of $(\mathbb{R}, <, +)$ corresponding to) $\mathfrak{S}$ satisfies any Shelah-style combinatorical tameness notion (such as distality, NIP or NTP2), then $\mathfrak{S}$ is type A.

Motto. *Type A is the ultimate generalization of o-minimal.*

Remarks.

1. If $\mathfrak{S}$ is a field geometry (ie. $\mathcal{A}_{\mathsf{alg}}$ is contained in $\mathfrak{S}$), then $\mathfrak{S}$ is type A or type C.
2. Actually: if $\mathfrak{S}$ is an $\mathbb{R}$-geometry, then $\mathfrak{S}$ is type A or type C.
3. If $\mathfrak{S}$ is o-minimal or d-minimal (or satisfies any other notion of geometric tameness), then $\mathfrak{S}$ is type A.
4. For model-theorists: If (the expansion of $(\mathbb{R}, <, +)$ corresponding to) $\mathfrak{S}$ satisfies any Shelah-style combinatorical tameness notion (such as distality, NIP or NTP2), then $\mathfrak{S}$ is type A.

Motto. *Type A is the ultimate generalization of o-minimal.*

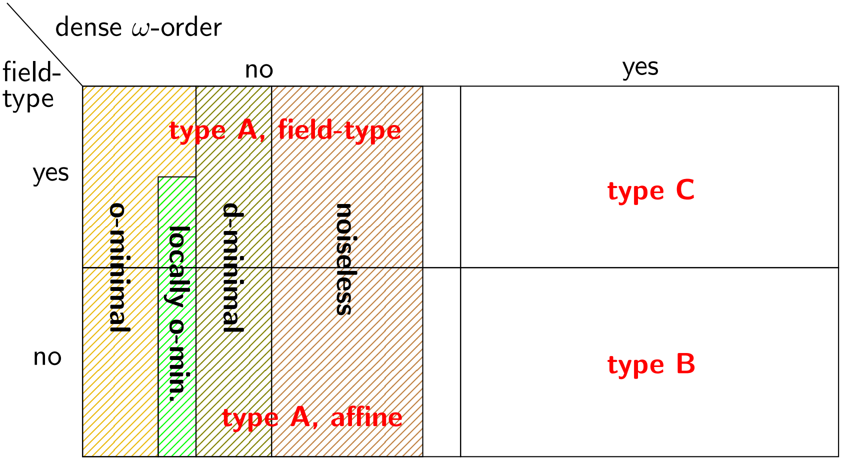
In the module of the graduate course, I will develop the theory of type A field geometries.

Remarks.

1. If $\mathfrak{S}$ is a field geometry (ie. $\mathcal{A}_{\mathsf{alg}}$ is contained in $\mathfrak{S}$), then $\mathfrak{S}$ is type A or type C.
2. Actually: if $\mathfrak{S}$ is an $\mathbb{R}$-geometry, then $\mathfrak{S}$ is type A or type C.
3. If $\mathfrak{S}$ is o-minimal or d-minimal (or satisfies any other notion of geometric tameness), then $\mathfrak{S}$ is type A.
4. For model-theorists: If (the expansion of $(\mathbb{R}, <, +)$ corresponding to) $\mathfrak{S}$ satisfies any Shelah-style combinatorical tameness notion (such as distality, NIP or NTP2), then $\mathfrak{S}$ is type A.

Motto. *Type A is the ultimate generalization of o-minimal.*

In the module of the graduate course, I will develop the theory of type A field geometries.



Definition We call a structure $\mathfrak{S} := (\mathfrak{S}_m)_{m=1}^{\infty}$ noiseless (or i-minimal) if every element of $\mathfrak{S}_1$ either has interior or is nowhere dense.

Definition (Miller-Speissegger) Let $\mathfrak{S}$ be a structure, and let $\mathcal{O}$ be the collection of all open sets in $\mathfrak{S}$. The open core of $\mathfrak{S}$ (written $\mathfrak{S}^\circ$) is the structure generated by $\mathcal{O}$.

This gives rise to new tameness conditions: o-minimal open core, d-minimal open core, noiseless open core...

Definition (Miller-Speissegger) Let $\mathfrak{S}$ be a structure, and let $\mathcal{O}$ be the collection of all open sets in $\mathfrak{S}$. The open core of $\mathfrak{S}$ (written $\mathfrak{S}^\circ$) is the structure generated by $\mathcal{O}$.

This gives rise to new tameness conditions: o-minimal open core, d-minimal open core, noiseless open core...

Example. The open core of $\mathfrak{S}(\mathcal{A}_{\mathrm{alg}} \cup \mathbb{Q}^{\mathrm{rc}})$ is semialgebraic geometry, and hence o-minimal.

Definition (Miller-Speissegger) Let $\mathfrak{S}$ be a structure, and let $\mathcal{O}$ be the collection of all open sets in $\mathfrak{S}$. The open core of $\mathfrak{S}$ (written $\mathfrak{S}^\circ$) is the structure generated by $\mathcal{O}$.

This gives rise to new tameness conditions: o-minimal open core, d-minimal open core, noiseless open core...

Example. The open core of $\mathfrak{S}(\mathcal{A}_{\mathrm{alg}} \cup \mathbb{Q}^{\mathrm{rc}})$ is semialgebraic geometry, and hence o-minimal.

Miller's conjecture. Let $\mathfrak{S}$ be a field geometry that does not contain $\mathbb{Z}$. Then the open core of $\mathfrak{S}$ is noiseless.

Definition (Miller-Speissegger) Let $\mathfrak{S}$ be a structure, and let $\mathcal{O}$ be the collection of all open sets in $\mathfrak{S}$. The open core of $\mathfrak{S}$ (written $\mathfrak{S}^\circ$) is the structure generated by $\mathcal{O}$.

This gives rise to new tameness conditions: o-minimal open core, d-minimal open core, noiseless open core...

Example. The open core of $\mathfrak{S}(\mathcal{A}_{\mathsf{alg}} \cup \mathbb{Q}^{\mathsf{rc}})$ is semialgebraic geometry, and hence o-minimal.

Miller's conjecture. Let $\mathfrak{S}$ be a field geometry that does not contain $\mathbb{Z}$. Then the open core of $\mathfrak{S}$ is noiseless.

A stronger form. Let $\mathfrak{S}$ be a type A geometry. Then the open core of $\mathfrak{S}$ is noiseless.

Definition (Miller-Speissegger) Let $\mathfrak{S}$ be a structure, and let $\mathcal{O}$ be the collection of all open sets in $\mathfrak{S}$. The open core of $\mathfrak{S}$ (written $\mathfrak{S}^\circ$) is the structure generated by $\mathcal{O}$.

This gives rise to new tameness conditions: o-minimal open core, d-minimal open core, noiseless open core...

Example. The open core of $\mathfrak{S}(\mathcal{A}_{\mathsf{alg}} \cup \mathbb{Q}^{\mathsf{rc}})$ is semialgebraic geometry, and hence o-minimal.

Miller's conjecture. Let $\mathfrak{S}$ be a field geometry that does not contain $\mathbb{Z}$. Then the open core of $\mathfrak{S}$ is noiseless.

A stronger form. Let $\mathfrak{S}$ be a type A geometry. Then the open core of $\mathfrak{S}$ is noiseless.

Restated stronger form. Let $\mathfrak{S}$ be a type A geometry generated by open sets. Then $\mathfrak{S}$ is noiseless.

Definition (Miller-Speissegger) Let $\mathfrak{S}$ be a structure, and let $\mathcal{O}$ be the collection of all open sets in $\mathfrak{S}$. The open core of $\mathfrak{S}$ (written $\mathfrak{S}^\circ$) is the structure generated by $\mathcal{O}$.

This gives rise to new tameness conditions: o-minimal open core, d-minimal open core, noiseless open core...

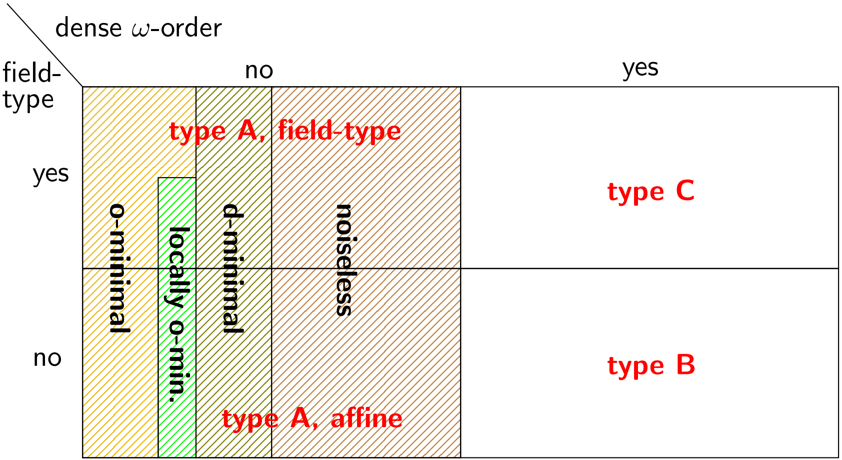
Example. The open core of $\mathfrak{S}(\mathcal{A}_{\mathrm{alg}} \cup \mathbb{Q}^{\mathrm{rc}})$ is semialgebraic geometry, and hence o-minimal.

Miller's conjecture. Let $\mathfrak{S}$ be a field geometry that does not contain $\mathbb{Z}$. Then the open core of $\mathfrak{S}$ is noiseless.

A stronger form. Let $\mathfrak{S}$ be a type A geometry. Then the open core of $\mathfrak{S}$ is noiseless.

Restated stronger form. Let $\mathfrak{S}$ be a type A geometry generated by open sets. Then $\mathfrak{S}$ is noiseless.

Assuming the strong form of Miller's conjecture and $\mathfrak{S}$ being generated by open sets.



A few comments about type B geometries.

1. Let C be the usual ternary Cantor set. Then the $\mathbb{Q}$-geometry generated by $\{C\}$ is type B.
2. Let K be a quadratic subfield of $\mathbb{R}$ with $K \neq \mathbb{Q}$. Then the K-geometry generated by $\{\mathbb{Z}\}$ is type B.
3. While both geometries do not satisfy any model-theoretic/geometric tameness, the theories of the corresponding expansions of $(\mathbb{R}, <, +)$ are decidable.

Job description of a model theorist/tame geometer.

discovering: determine which collections generate geometries that can be considered tame.

charting: understand the tameness of the various geometries and how they relate to each other.

A few comments about type B geometries.

1. Let C be the usual ternary Cantor set. Then the $\mathbb{Q}$-geometry generated by $\{C\}$ is type B.
2. Let K be a quadratic subfield of $\mathbb{R}$ with $K \neq \mathbb{Q}$. Then the K-geometry generated by $\{\mathbb{Z}\}$ is type B.
3. While both geometries do not satisfy any model-theoretic/geometric tameness, the theories of the corresponding expansions of $(\mathbb{R}, <, +)$ are decidable.

Job description of a model theorist/tame geometer.

discovering: determine which collections generate geometries that can be considered tame.

charting: understand the tameness of the various geometries and how they relate to each other.

> *'Contemporary model theory has tended to concentrate on the tame region of mathematics, although exploration of the borderline or middle ground between tame and foundational is a fascinating topic.'*

- Anand Pillay

A few comments about type B geometries.

1. Let C be the usual ternary Cantor set. Then the $\mathbb{Q}$-geometry generated by $\{C\}$ is type B.
2. Let K be a quadratic subfield of $\mathbb{R}$ with $K \neq \mathbb{Q}$. Then the K-geometry generated by $\{\mathbb{Z}\}$ is type B.
3. While both geometries do not satisfy any model-theoretic/geometric tameness, the theories of the corresponding expansions of $(\mathbb{R}, <, +)$ are decidable.

Job description of a model theorist/tame geometer.

discovering: determine which collections generate geometries that can be considered tame.

charting: understand the tameness of the various geometries and how they relate to each other.

> *'Contemporary model theory has tended to concentrate on the tame region of mathematics, although exploration of the borderline or middle ground between tame and foundational is a fascinating topic.'*

- Anand Pillay

Thank you!