Analytic cyclic homology (Ralf Meyer) 30.9.2021

How do we compute HP?
Why do we need to go beyond HP?

Local cyclic homology by Puschnigg gives good results for C^*-algebras like $C(X)$:

$$HA_*(C(X)) \cong HA_*(C^\infty(X))$$

Entire cyclic cohomology is very close to analytic cyclic cohomology. Why pick this growth condition?

Cuntz-Quillen machinery gives various kinds of cyclic homology theories.

$$A \longmapsto \underset{\text{pro-algebra}}{\mathcal{T}A} \longmapsto X(\mathcal{T}A)$$

$HH_n(\mathcal{T}A) = 0$ for $n \geq 2$

$$HP_*(A) = HP_*(\mathcal{T}A) = H_*(X(\mathcal{T}A))$$

$$\underbrace{JA}_{\text{pro-nilpotent}} \rightarrowtail \mathcal{T}A \twoheadrightarrow A \qquad \exists n : J^n = 0$$

tensor algebra

$HH_0(A) = A/[A,A]$

$$\begin{tikzcd} HP_0(A) \arrow[r] & HH_0(A) \\ F(A) \arrow[u, dashed, "\exists!"] \arrow[ur] \end{tikzcd}$$

F is invariant under pro-nilpotent extensions

To get analytic cyclic homology, look at complete bornological algebras over $\mathbb{C}$.

bounded subsets

$S \subseteq A$ $\quad \bigcup_{n=1}^{\infty} (r \cdot S)^n$ bounded

A analytically nilpotent $\Leftrightarrow$ $\forall S \subseteq A$ bounded: $\bigcup S^n$ is bounded

$C^\infty(X) \hookrightarrow C(X)$ $\qquad$ $HA_*(A, \text{precompact}) \to HA_*(A, \text{bdd})$

φ_t "linear "asymptotic homomorphism" $\varphi_t \xrightarrow[T]{} id$

$\| \varphi_t(a)\, \varphi_t(b) - \varphi_t(ab) \|_\infty \to 0$ $\qquad$ uniform on compacts

$g(\dots) \to 0$

$\mathcal{T} A \to B$ $\quad \leftrightarrow$ lin. maps $A \to B$ with analytically nilpotent curvature $A \otimes A \xrightarrow{\omega} B$

$\cup\!\!\!|$ bounded

S

$\mathcal{T}(C(X)) \hookrightarrow \mathcal{T}(C^\infty(X))$

locally defined inverses $\qquad$ $\cup\!\!\!|$

S

How to define analytic cyclic homology for algebras over $\mathbb{Z}_p$?

Want ideally: $HA_*(A)$ depends only on A/pA

Want to get Monsky-Washnitzer's cohomology / rigid cohomology

pro-(complete bornological algebras over $\mathbb{Z}_p$)

bornologically torsion-free $\quad V \hookrightarrow V \otimes_{\mathbb{Z}_p} \mathbb{Q}_p$

$\cup$

analytically nilpotent algebras $\Big\{$ A/pA pro-nilpotent

pro-dagger algebra

$S \subseteq A_n$ bdd $\Rightarrow \bigcup_{n=0}^{\infty} p^n S^{n+1}$ bdd