Definable Fields in Various Dp-minimal Fields

Model Theory Seminar (The Fields Institute)

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Joint work with Assaf Hasson and Kobi Peterzil

Valued Fields

Definition

A valued field (K, v) is a field together with a group homomorphism $v : K^{\times} \to \Gamma$, where Γ is some ordered abelian group, satisfying that

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3. $\mathbb{R}((t^{\mathbb{Q}}))$ is a real closed valued field.

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The value group of (K, v) is Γ.

History

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Theorem

- *(Sokolovic) An infinite field of finite Morley rank interpretable in DCF_0 is definably isomorphic to the field of constants.*
- *(Sklinos) No infinite field is interpretable in the free group.*

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As a result, this result is also true for any infinite dp-minimal pure field of characteristic 0.

The definition of dp-minimality and 1-h-minimality is immaterial at this moment. It is sufficient to know that it implies that they have nice topological and analytical properties.

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The first has nice 1-dimensional definable sets, the second (in the examples) usually satisfies that acl has exchange.

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1. For every open $V \subseteq G^n$, $a \in V$ and A a small set of parameters, there exists $B \supseteq A$ and a B-definable open subset $U = U_1 \times \cdots \times U_n \subseteq V$ such that $a \in U$ and $\dim(a/B) = \dim(a/A)$.

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2. Let M be small model and $b_1, \ldots, b_n$ some tuples. For any M-definable X, there exists $a \in X$, with $\dim(a/M) = \dim(X)$, such that $\dim(a, b_i/M) = \dim(a/M) + \dim(b_i/M)$ for all $1 \leq i \leq n$.

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The solution: Consider (partial) types of infinitesimals.

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Definition

For any $d \in \mathcal{F}$ with $\dim(d/M) = \dim(\mathcal{F})$, where M is a small model, the infinitesimal neighborhood of d is

$$\nu_{\mathcal{F}}(d) = \{U \cap \mathcal{F} : U \subseteq K^n \text{ definable open and } d \in U\}.$$

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Side note: by applying the above to $\mathcal{F} = K$ one can deduce that the topology on K is necessarily the canonical topology given by Johnson.

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1. $\nu(\widehat{K})$ is a $\mathcal{D}^1$-group
2. for every $c \in \mathcal{F}$, $\lambda_c : \nu(\widehat{K}) \to \nu(\widehat{K})$ mapping $x \mapsto cx$ is a $\mathcal{D}^1$-homomorphism.

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Letting $K_0 = \{aI_n : a \in K\}$, $K_0 \cap \mathcal{F}$ is an infinite definable field (by NIP), but since K_0 is dp-minimal necessarily $K_0 \cap \mathcal{F} = K_0$ (so $K_0 \subseteq \mathcal{F}$).

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Letting $K_0 = \{aI_n : a \in K\}$, $K_0 \cap \mathcal{F}$ is an infinite definable field (by NIP), but since K_0 is dp-minimal necessarily $K_0 \cap \mathcal{F} = K_0$ (so $K_0 \subseteq \mathcal{F}$). Now observe that $\mathcal{F}$ is a finite extension of K_0. $\square$

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Let $(K, v, \dots)$ be a dp-minimal valued field with generic differentiability and let $\mathcal{F}$ be an interpretable field.

Recipe for Interpretable Fields, cont.

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   - V-minimal: Γ is a DOAG (so "linear") and $K/\mathcal{O}$ is linear.

Recipe for Interpretable Fields, cont.

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5. In case of t-convex power bounded, k is also an SW-uniformity.
6. If k is an algebraically closed field then one needs to proceed by proving a local version of Zil'ber's indecomposability theorem.

Generalizations

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2. What about in valued fields of higher dp-rank?

Thank You!