Talk Plan

1. The farpoint map
2. Rhombus maps
3. Main Theorem
4. Ideas of the proof
5. cocircularity and positivity

My paper: The Farthest Point Map on the Regular Dodecahedron
arXiv 2104:02567 (preprint)

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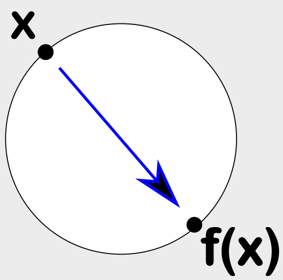
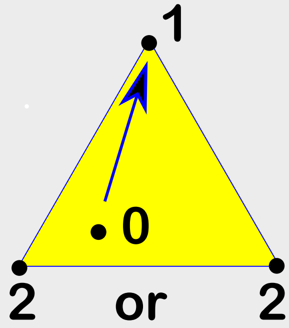
Farpoint Map:

X = compact metric space
f(x)=point of x farthest from x.





Interesting cases:

Surface of a polyhedron equipped
with its INTRINSIC metric

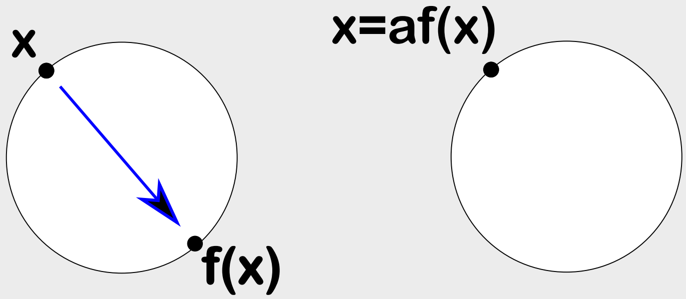
regular tetrahedron (J. Rouyer)
all tetrahedra (Y. Tumarkin, unpublished)
some general results (J. Rouyer)

octahedron (R.E.S.)
cube (similar to octahedron)
centrally symmetric octahedra (Zili Wang)

icosahedron (similar to octahedron)
dodecahedron

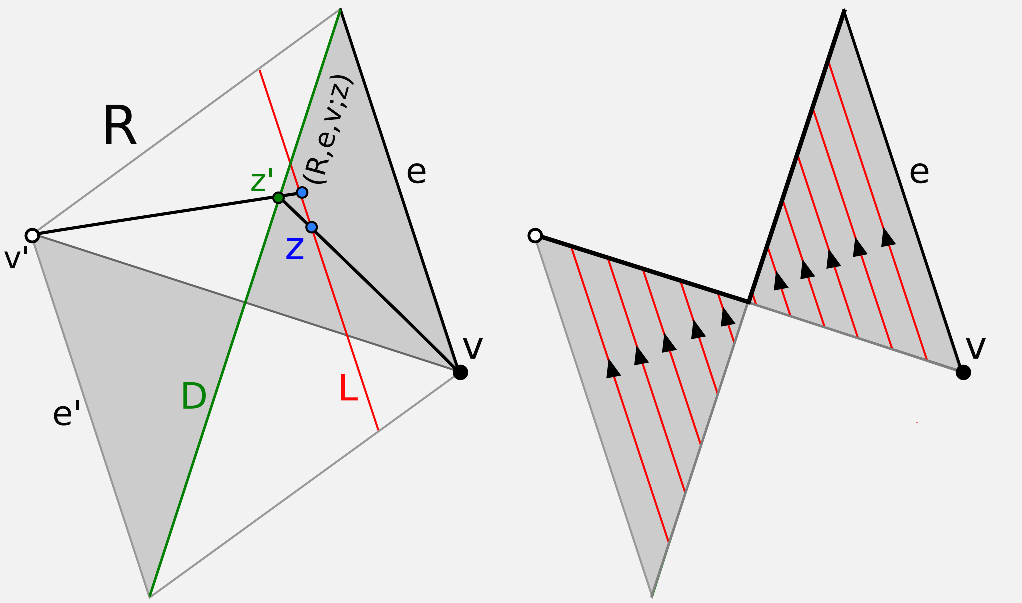
Anti-Farpoint Map:

When the space is centrally symmetric it is better to compose the farpoint map (f) with the antipodal map (a).

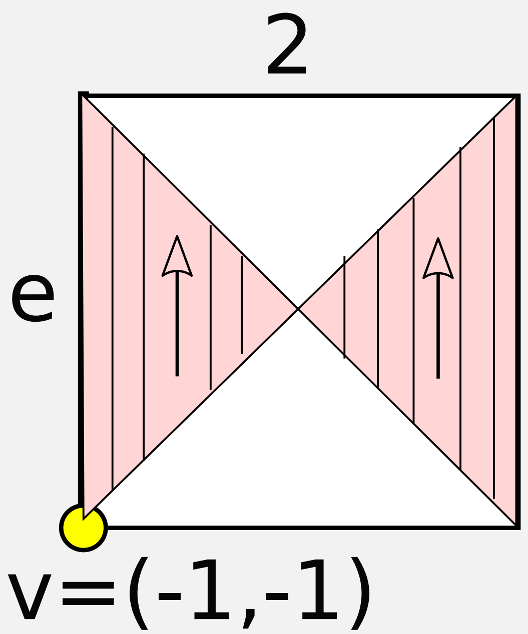


Rhombus maps

Input: (R,e,v)



Example

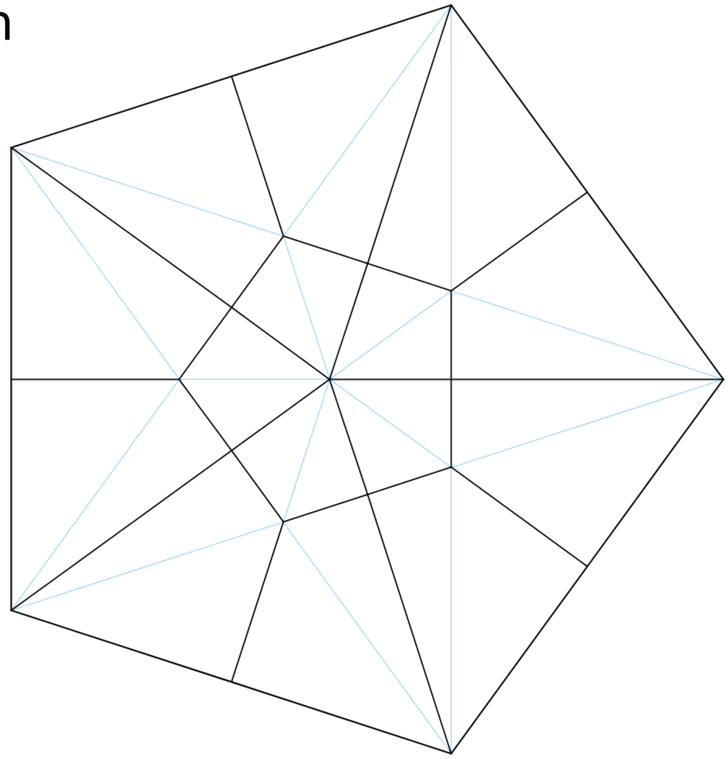


$$f(x,y) = \left(x, \frac{x^2+y}{1+y}\right)$$

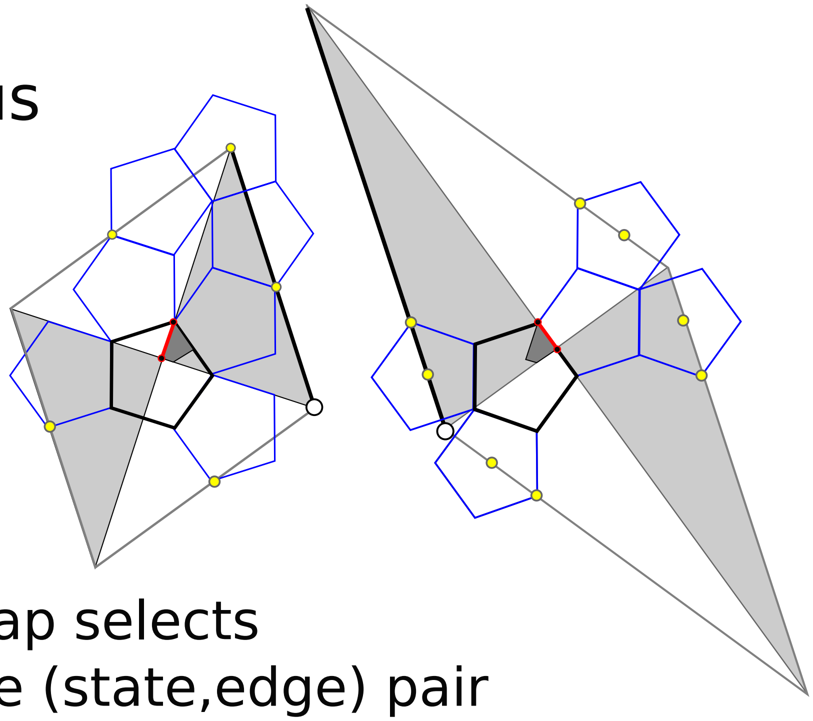
All others
are conjugate
to this one
by similarities.

Decomposition
of a pentagon
face into
15 states.

Each
state
has
4 sides

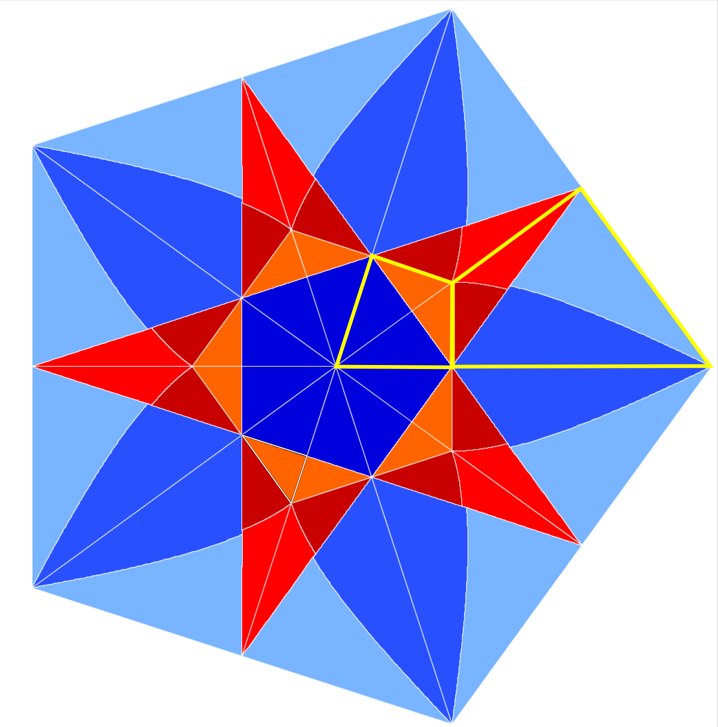


2 of 40
rhombus
maps



Each map selects
a unique (state,edge) pair

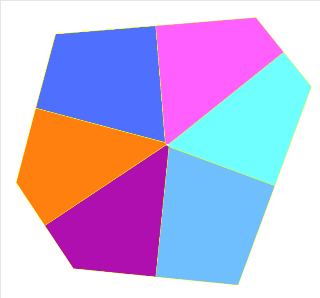
Theorem:
each state
is divided
into 4
cities.
In each city
the af map
is the special
associated
rhombus map.



Steps in the proof.

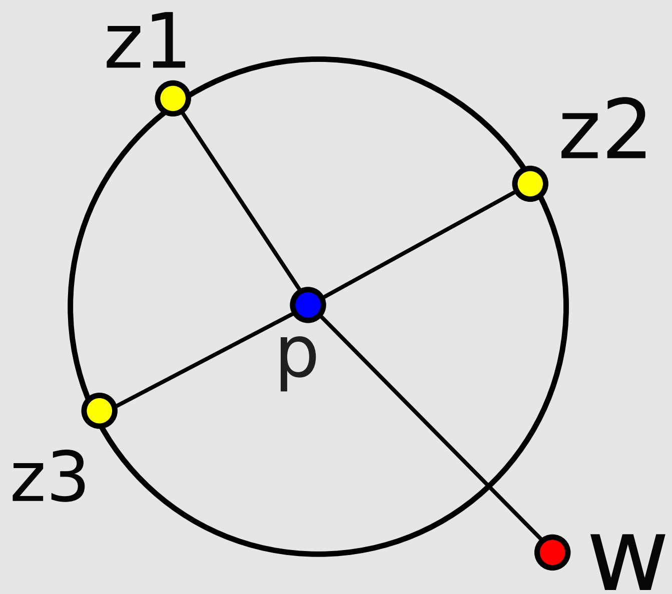
1. F(p) is on the opposite face

2. F(p) is a subset of the vertices of the Voronoi decomposition of a certain convex hexagon H(p).



3. Figure out algebraically which vertices win. (co-circularity/positivity).

cocircularity



|P-w|>|P-zi|

when Im(CR(w,z1,z2,z3))>0

Positive Dominance

Let $P(x)=a_0+a_1 x + \ldots + a_n x^n$

Let $A_0 = a_0$

$A_1 = a_0 + a_1$

$A_2 = a_0 + a_1 + a_2$

...

Lemma: P>0 on [0,1] if $A_0, A_1, \ldots > 0$.

Proof: Let x be in (0,1]

$P(x) \geq (a_0+a_1)x + a_2 x^2 + \ldots$

$= x\,Q(x) > 0$ by induction on the degree. ∎

Multivariable generalization

$$
\begin{aligned}
P(x,y) = {} & a00 + a01\,x + a20\,x^2 + \ldots \\
& + a10\,y + a11\,xy + a21\,x^2 y + \ldots \\
& + a20\,y + a21\,xy^2 + a22\,x^2 y^2 \ldots
\end{aligned}
$$

Lemma: If all rooted rectangular sums are pos. then P>0 on [0,1]^2.

Proof idea: induction on the number of variables. (A polynomial ring is a ring over a polynomial ring.)

Enhancing the positivity criterion using sudvisision.

consider

$P(x)=3 - 4x + 2x^2$.

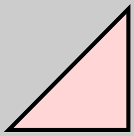
$P(x/2)=3-2x+x^2/2$ POSDOM

$P(1-x/2)=1+x^2/2$ POSDOM.

Hence $P>0$ on $[0,1]$.

We are interested in positivity on
triangles. Idea: precompose with
a polynomial map from a [0,1]^2 to a triangle.

$$f(x,y)=(x,xy)$$

P>0 on  iff pf>0 on