Mini-symposium on Sensor Network Localization and Dynamical Distance Geometry

FIELDS

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Optically solving the **Distance Geometry Problem***

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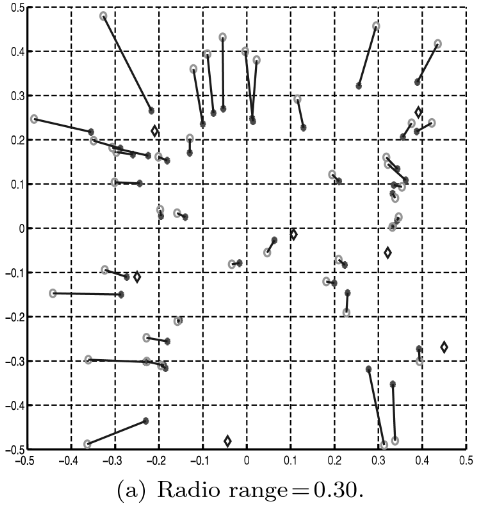
*arXiv:2105.12118

Motivation

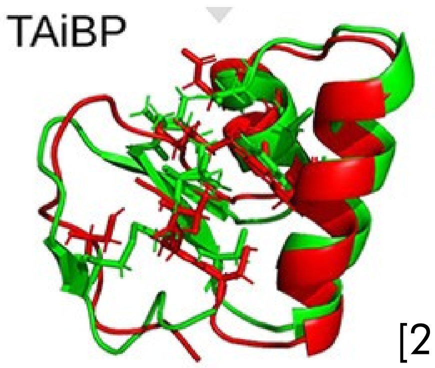
Sensor network localization



(a) Radio range = 0.30.

[1]

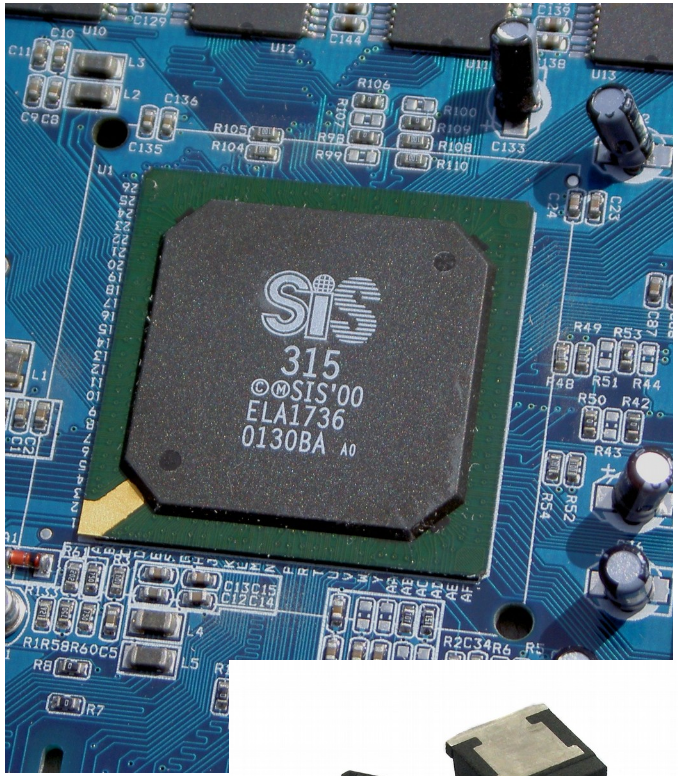
Molecular conformation

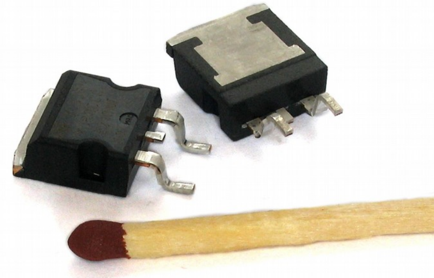


[2]

Distance Geometry Problem

New computing platforms





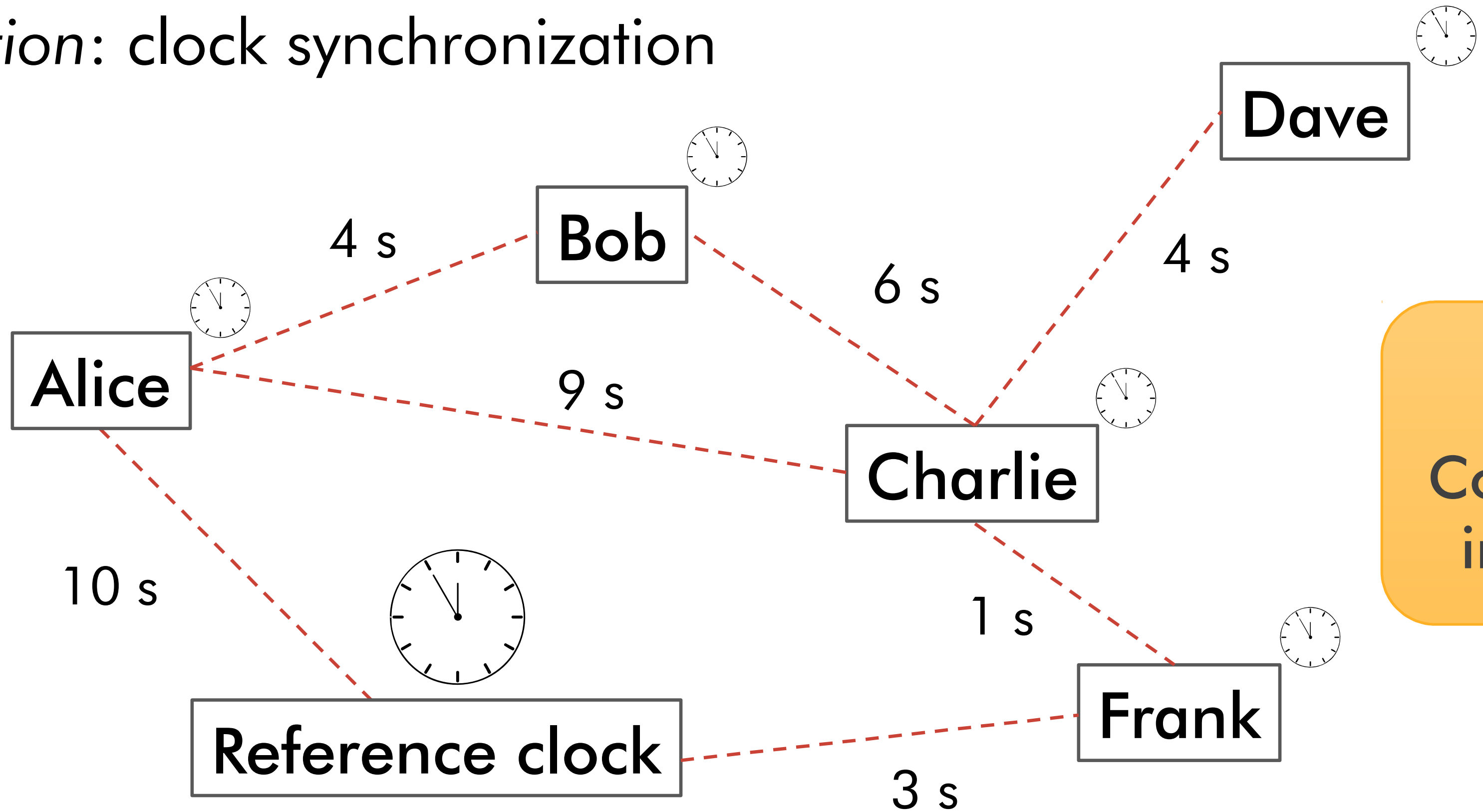
[1] Biswas *et al.*, ACM Trans. Sens. Netw. **2** (2006)

[2] Malliavin *et al.*, J. Chem. Inf. Model. **59** (2019)

Which problem?

Distance Geometry Problem in dimension 1

Application: clock synchronization



NP-hard!

Complexity of algorithms increases exponentially

A. Singer, *Appl. Comput. Harmon. A* **30** (2011)

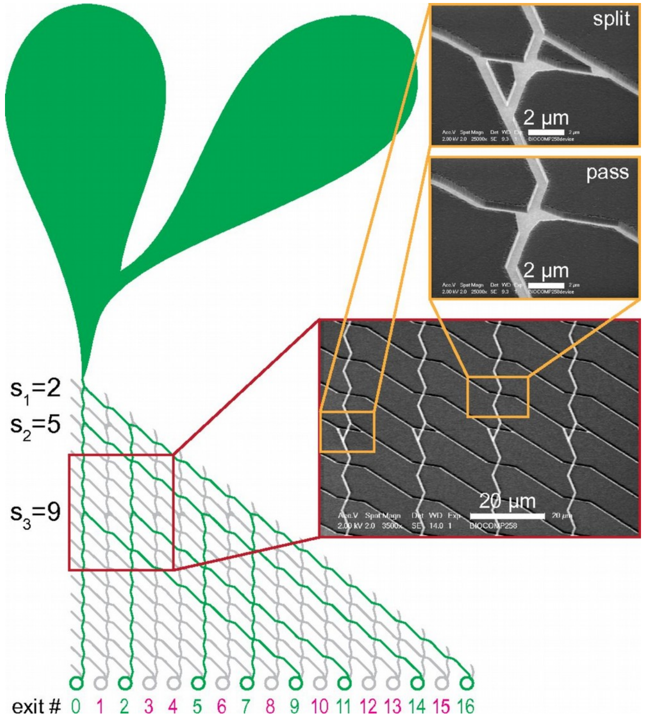
Which platform?

Solving Subset-Sum Problem in different platforms

Parallel computing

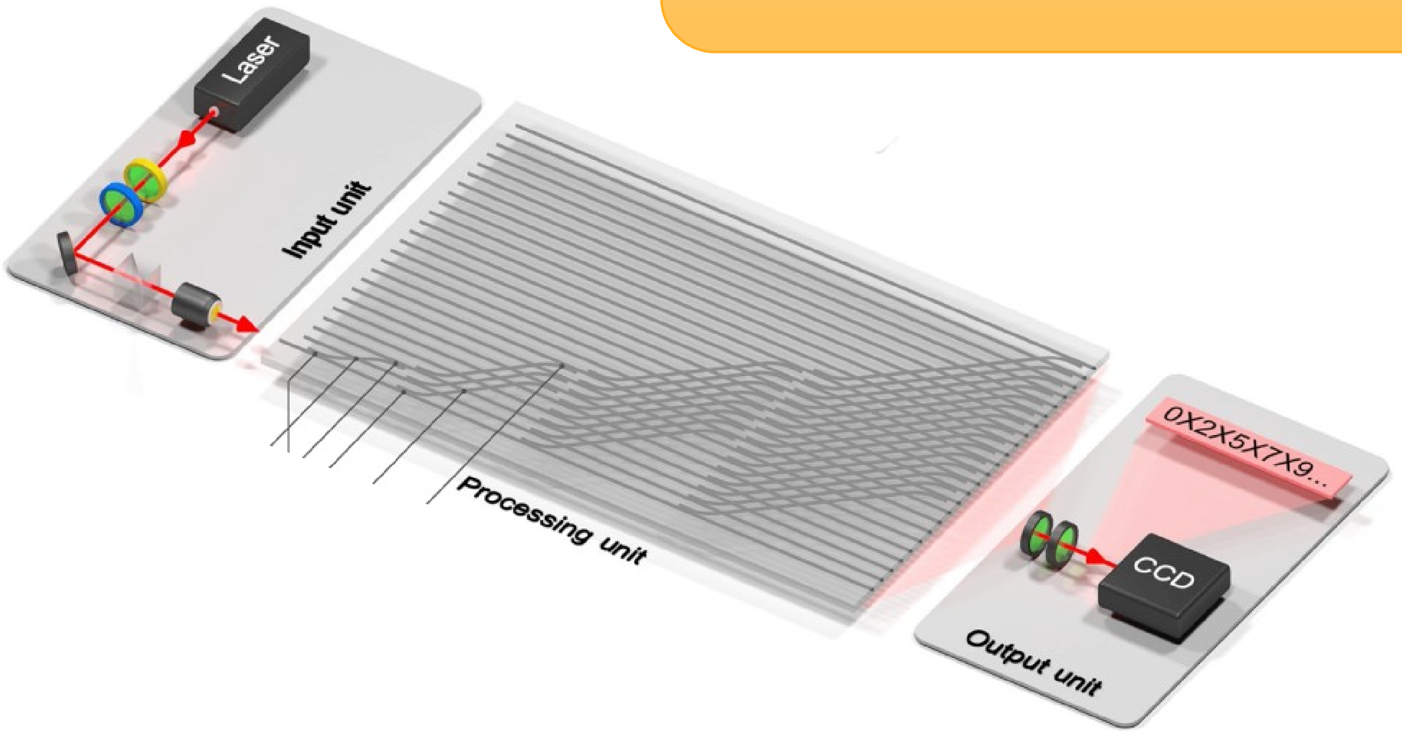
Reduce algorithm complexity

Molecular network



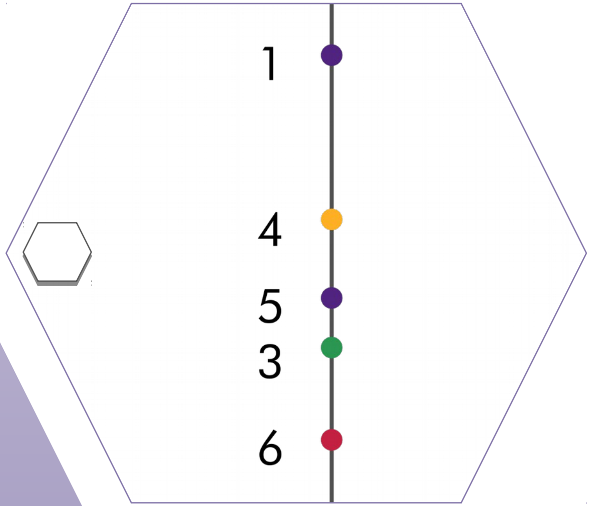
Nicolau Jr. *et al.*, *PNAS* **113** (2016)

Optical network



Xu *et al.*, *Science Adv.* **6** (2020)

Outline

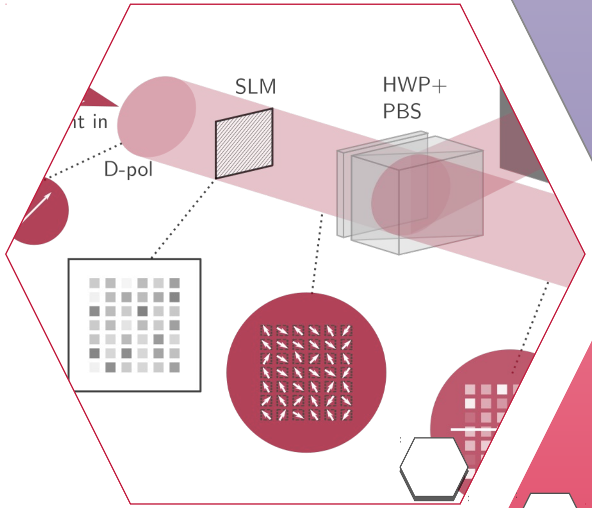


Paradoxical Distance Geometry Problem

Matrix-by-vector reformulation

$$= \begin{pmatrix} -1 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Optical scheme



Paradoxical DGP

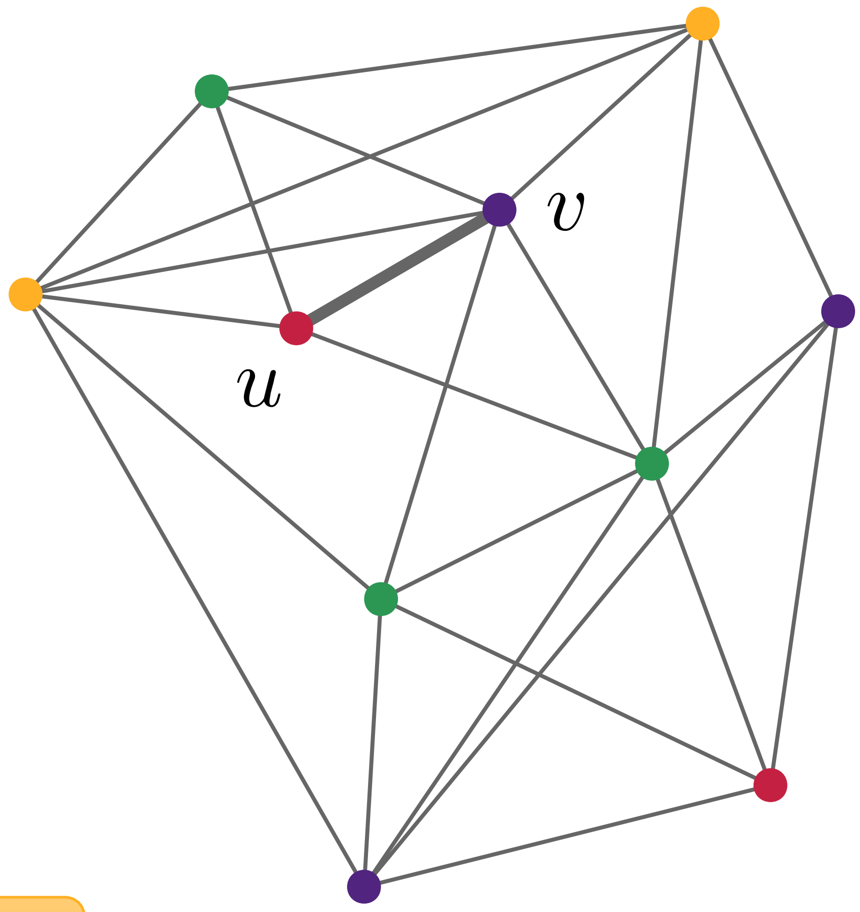
Distance Geometry Problem

Graph $G = (V, E, d)$

Vertices V

Edges E

Distances d *



* Weight $d(u, v) \in \mathbb{R}_+$ when $\{u, v\} \in E$

NP-hard!

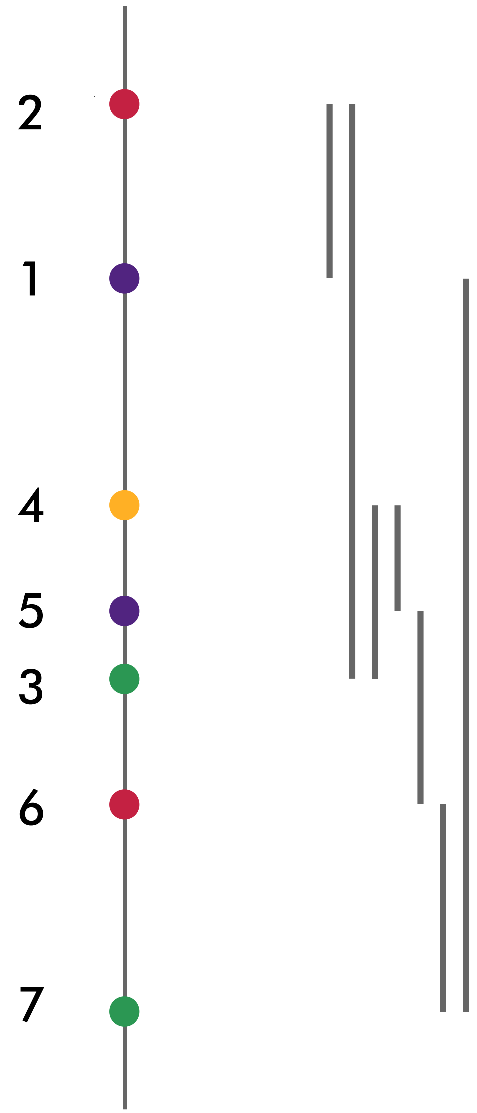
Paradoxical DGP_1

DGP_1 : distance geometry problem in dimension 1

Definition of paradoxical DGP_1

i. Vertex order exists: for every vertex (different from the first), the edge k $\{k-1, k\}$ belongs to E;

ii. No other edge is in E except $\{1, n\}$ (distance from first to last vertex in the order, $n = |V|$).



Paradoxical DGP_1

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ii. No other edge is in E except $\{1, n\}$ (distance from first to last vertex in the order, $n = |V|$).

- Presence of $d(1, n)$ → **only two solutions** are possible;
- $d(1, n)$ involves the very first and the very last vertex → information can be exploited **only in the last moment.**

BuildUp algorithms

Construct possible DGP solutions:

i. One vertex position at a time;

ii. Predefined vertex order.

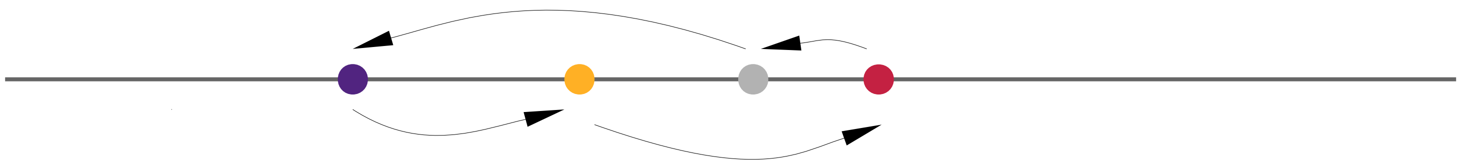
→ can be used to generate a search tree (multiple realizations)

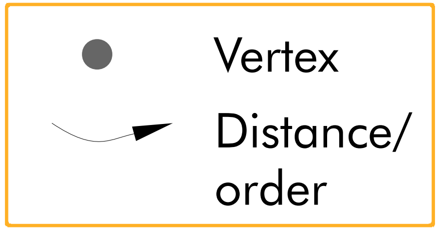
For paradoxical DGP_1

- **# of potential realizations keeps exponentially growing** until reaching the last vertex;
- at most two realizations are feasible (correspond to all distances)

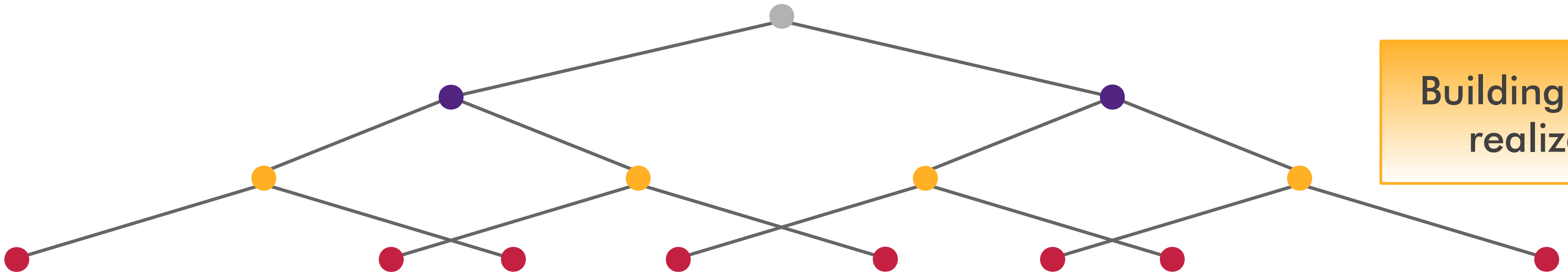
Branch-and-Prune I

Expected solution



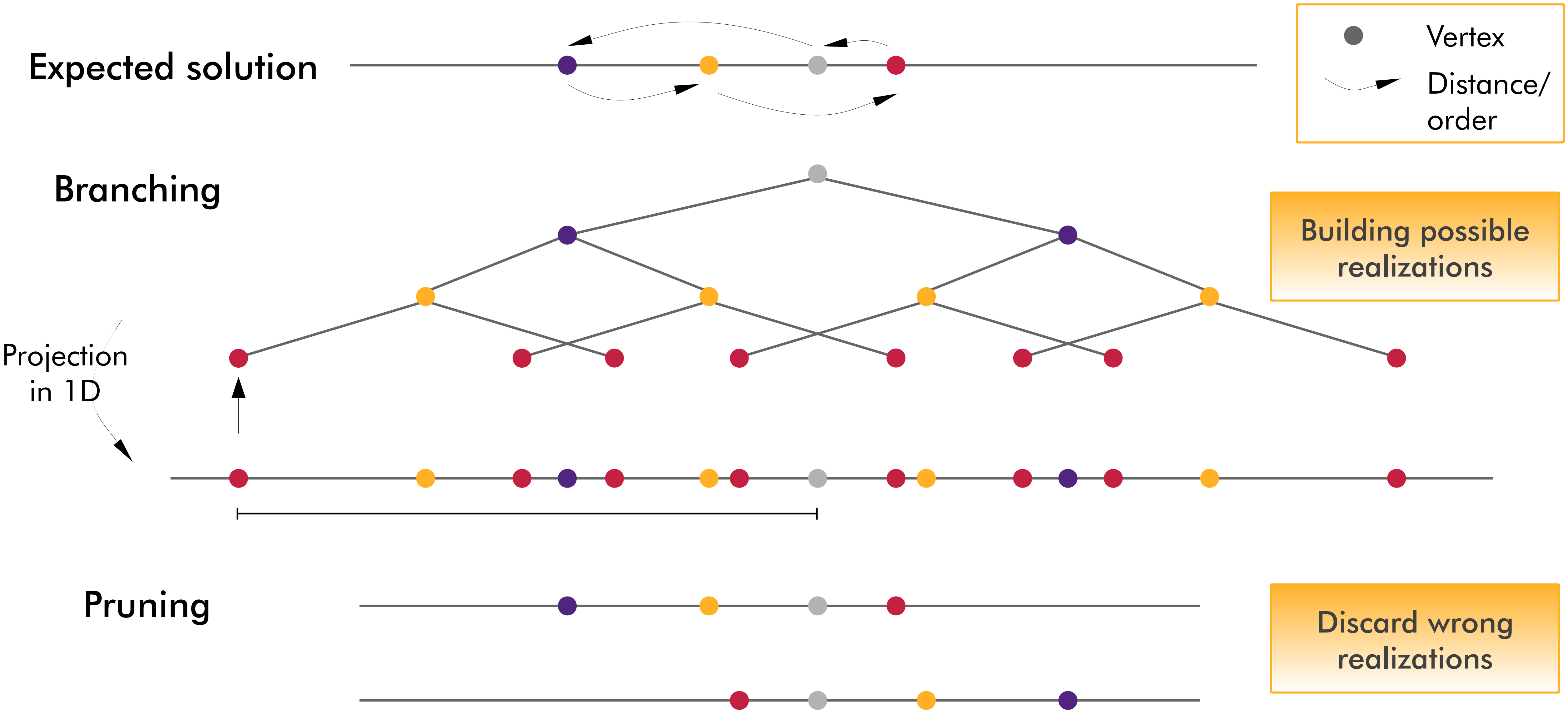


Branching



Building possible realizations

Branch-and-Prune I



Branch-and-Prune II

For a paradoxical DGP$_1$ instance:

i. Initialize position of vertex 1: $x_1 = 0$

ii. Possible positions for all subsequent vertices:

$$x_k = x_{k-1} + s_k d_{k-1,k}, \quad \forall k = 2, \ldots, n,$$
$$d_{k-1,k} = d(k-1, d); \qquad s_k = -1 \ \text{ or } \ s_k = +1$$

iii. Branching: computing $2p$ positions for the vertex k (p is the number of available positions for the preceding vertex).

iv. Pruning: at the last layer n, $d_{1,n}$ is used to select only two solutions of the tree.

HERE: uniformize branching/pruning by including a "virtual" vertex $n{+}1$, for which $d_{n,n+1} = d_{1,n}$

→ feasibility of a solution: $x_1 = x_{n+1}$

Matrix-by-vector reformulation

Basic idea

$$x_k = x_{k-1} + s_k d_{k-1,k}, \quad \forall k = 2, \ldots, n,$$
$$s_k = -1 \;\; \text{or} \quad s_k = +1, \quad \text{and} \quad x_1 = 0$$

- A solution is represented by a boolean n-vector (s_k): $(-1, +1, +1, \ldots, -1, -1)$
- … when multiplied by a vector containing the distances: $\mathbf{y} = (d_{12}, d_{23}, \ldots, d_{n-1,n}, d_{1n})^T$

  → position of vertex $n+1$ (check feasibility)

$$x_{k+1} = (-1, +1, \ldots, -1, -1) \begin{bmatrix} d_{12} \\ d_{23} \\ \vdots \\ d_{n-1,n} \\ d_{1n} \end{bmatrix} = -d_{12} + d_{23} + \ldots - d_{n-1,n} - d_{1n}$$

Binary matrix

- Take all possible realizations at once → binary matrix $\mathbf{M}$:
  - 2^n rows – potential realizations with fictive vertex
  - n columns – n vertices

$$x_1 = 0$$

- Only one vertex in G: $\mathbf{M} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad \mathbf{y} = (d_{11}) \quad \Rightarrow \mathbf{r} = \begin{pmatrix} -d_{11} \\ d_{11} \end{pmatrix}.$

- Two vertices in G: $\mathbf{M} = \begin{pmatrix} -1 & -1 \\ 1 & -1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} d_{12} \\ d_{12} \end{pmatrix} \quad \Rightarrow \mathbf{r} = \begin{pmatrix} -d_{12} - d_{12} \\ d_{12} - d_{12} \\ -d_{12} + d_{12} \\ +d_{12} + d_{12} \end{pmatrix}.$

Pattern length: 2^j

Binary matrix

- Take all possible realizations at once → binary matrix $\mathbf{M}$:
  - 2^n rows – potential realizations with fictive vertex
  - n columns – n vertices

$x_1 = 0$

- Only one vertex in G: $\mathbf{M} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad \mathbf{y} = (d_{11}) \quad \Rightarrow \mathbf{r} = \begin{pmatrix} -d_{11} \\ d_{11} \end{pmatrix}.$
- Three vertices:

$$\mathbf{M} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} d_{12} \\ d_{23} \\ d_{13} \end{pmatrix} \quad \Rightarrow \mathbf{r} = \begin{pmatrix} -d_{12} - d_{23} - d_{13} \\ d_{12} - d_{23} - d_{13} \\ -d_{12} + d_{23} - d_{13} \\ d_{12} + d_{23} - d_{13} \\ -d_{12} - d_{23} + d_{13} \\ d_{12} - d_{23} + d_{13} \\ -d_{12} + d_{23} + d_{13} \\ d_{12} + d_{23} + d_{13} \end{pmatrix}$$

Pattern length: 2^j

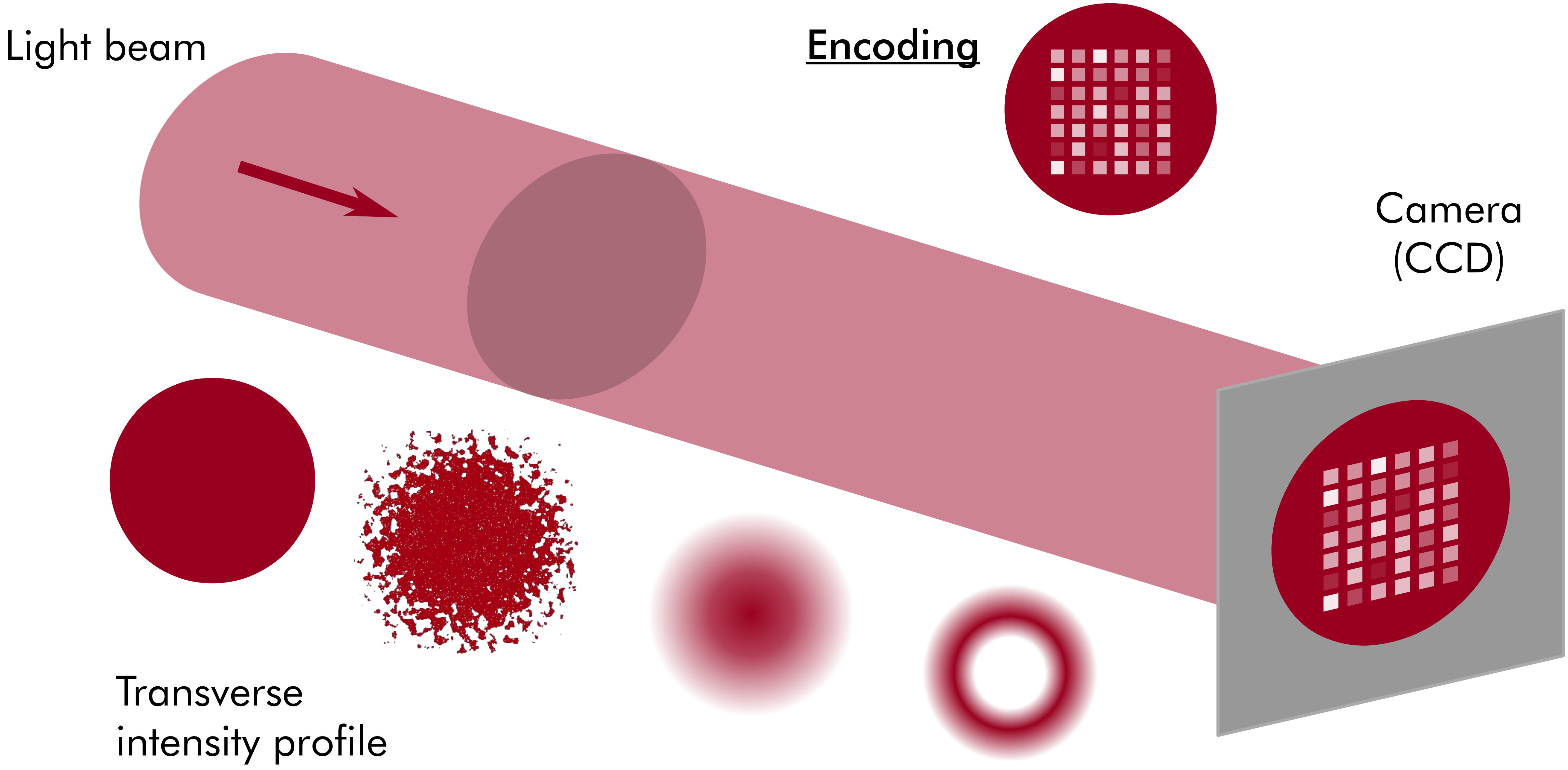
Matrix-by-vector multiplication reformulation

- Realizations vector: $\mathbf{r} = \mathbf{M} \times \mathbf{y}$
- Distance vector: $\mathbf{y} = \begin{bmatrix} d_{12} \\ d_{23} \\ \vdots \\ d_{n-1,n} \\ d_{1n} \end{bmatrix}$
- Binary matrix: $\mathbf{M}_{ij} = \begin{cases} -1 & \text{if } (i-1)2^{1-j} \bmod 2 = 0, \\ 1 & \text{otherwise.} \end{cases}$

Feasible solutions: null components of $\mathbf{r}$

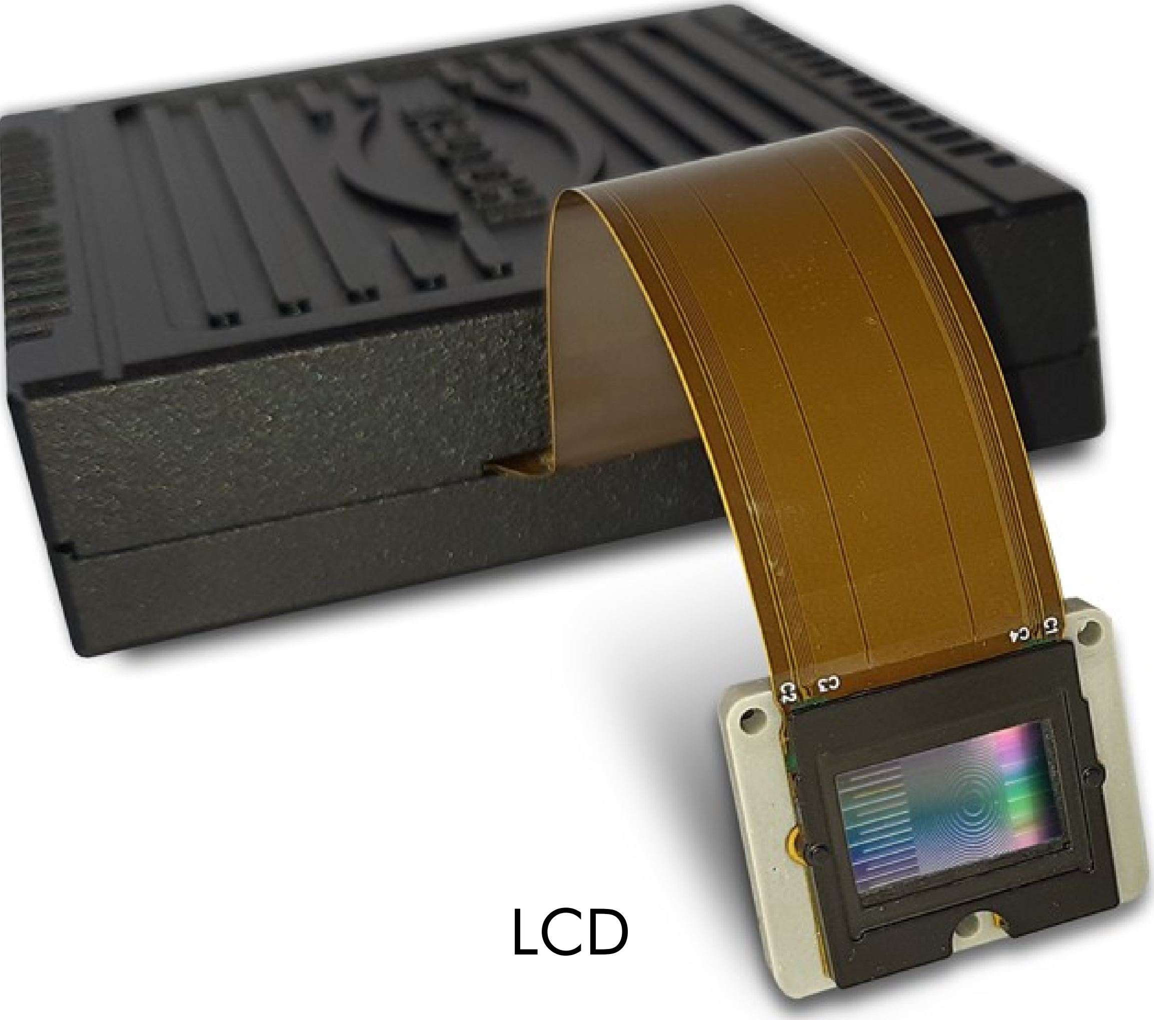
Optical scheme

Optical matrices?

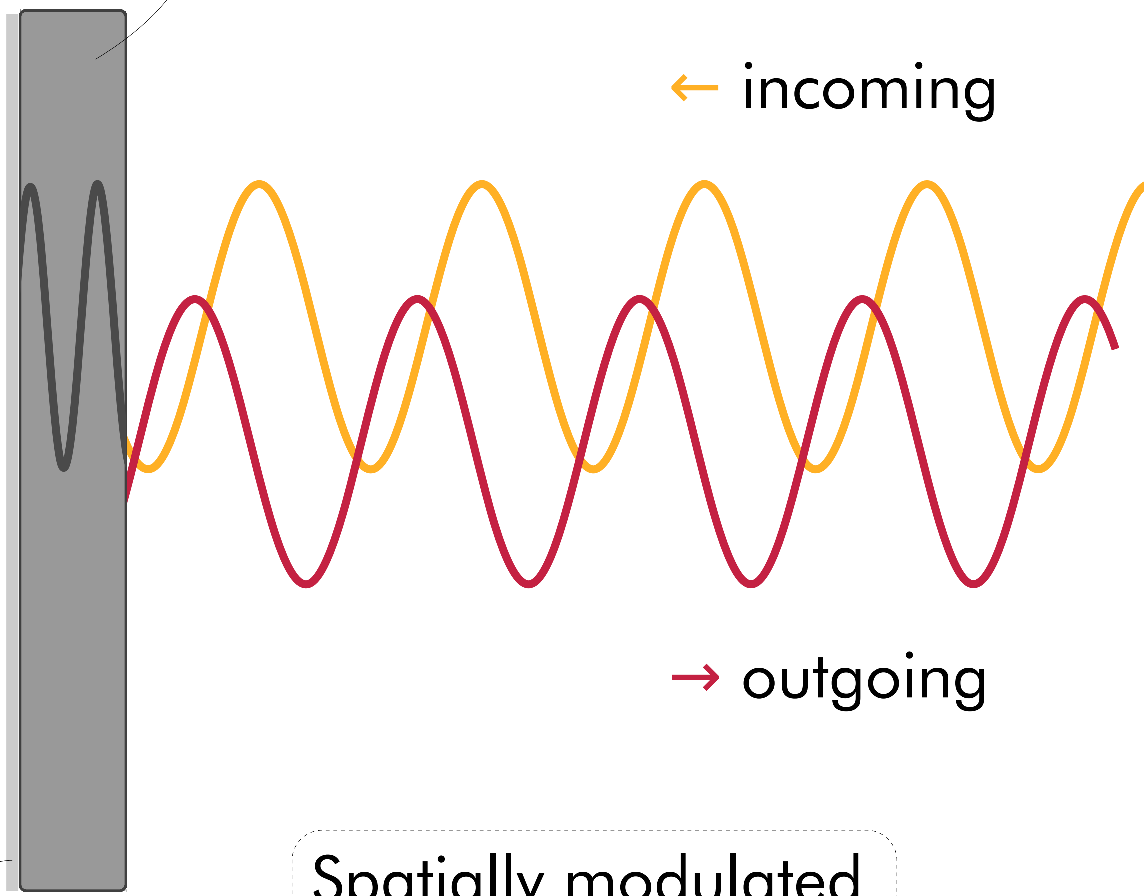


Encoding scheme I

SLM = spatial light modulator



LCD



Spatially modulated
DEPHASING

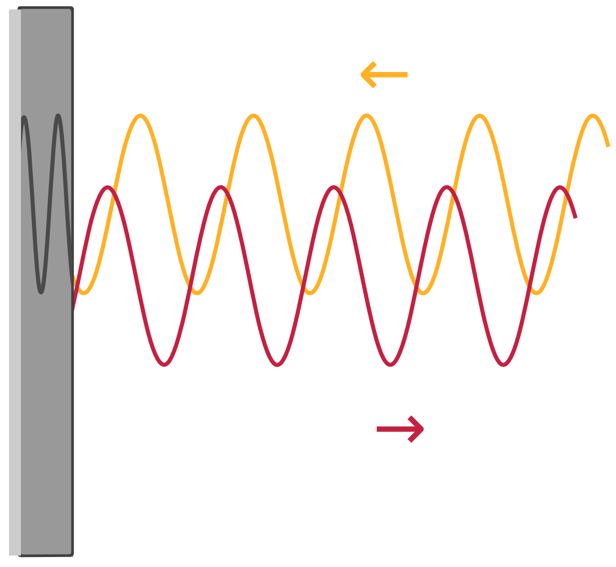
https://holoeye.com/gaea-4k-phase-only-spatial-light-modulator/

Encoding scheme II

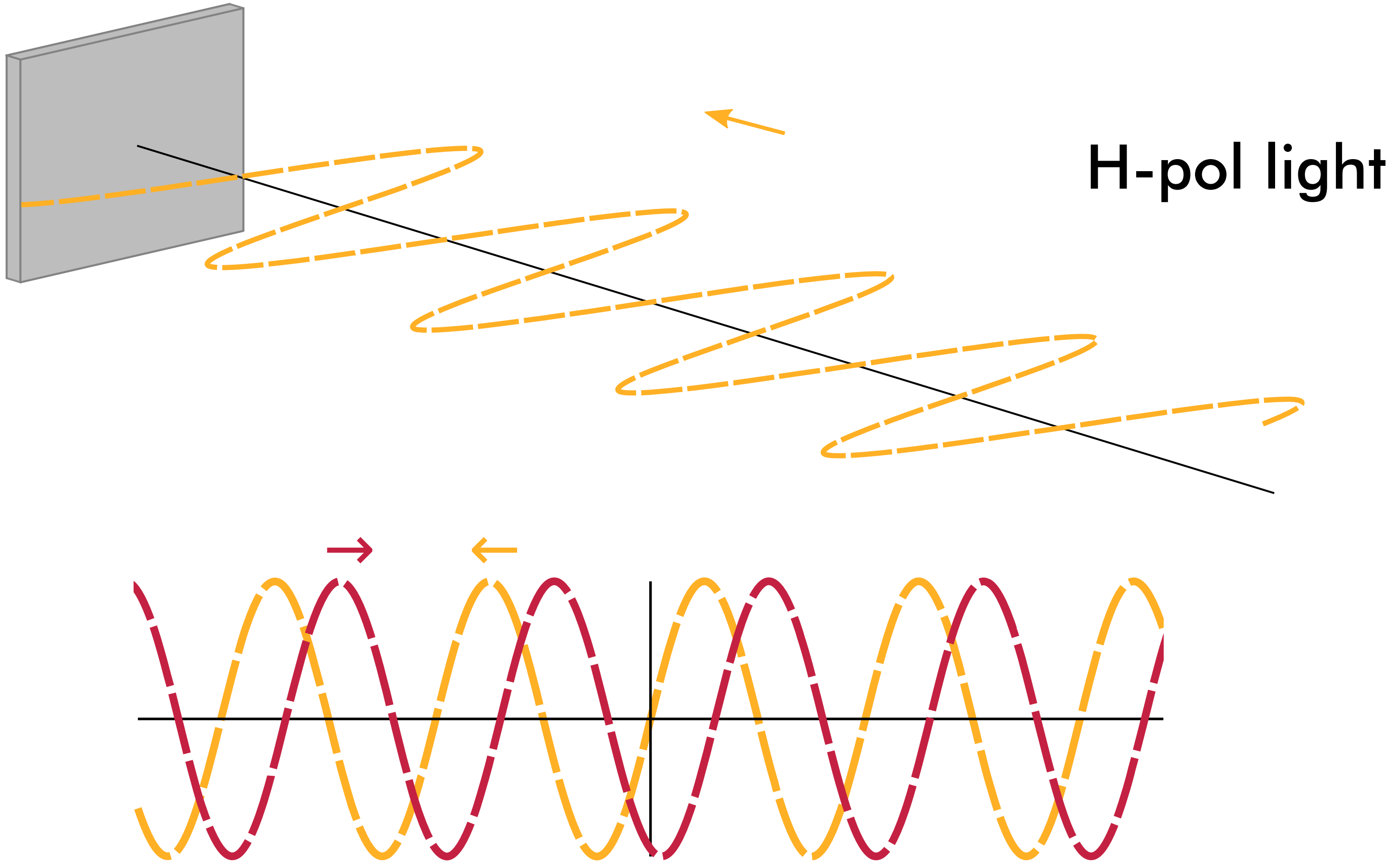
Spatially modulated
DEPHASING

?

Spatially modulated
INTENSITY

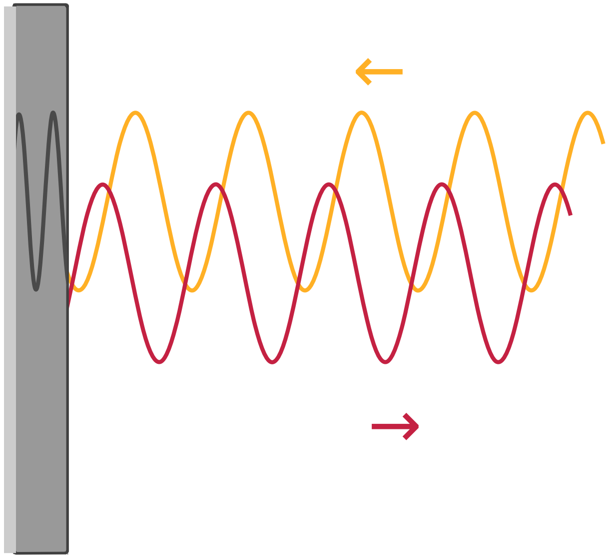


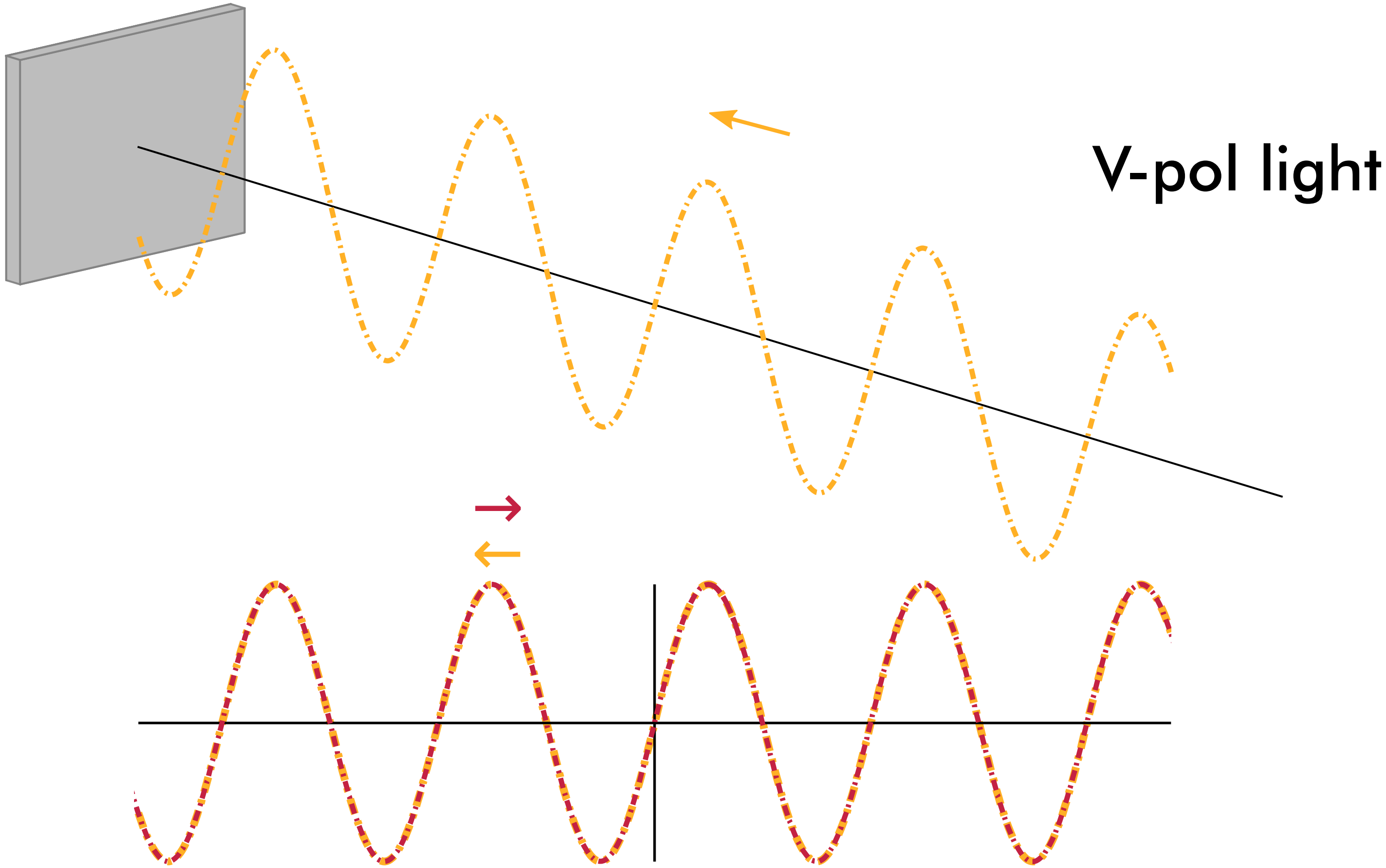
Polarization-dependent



Encoding scheme II

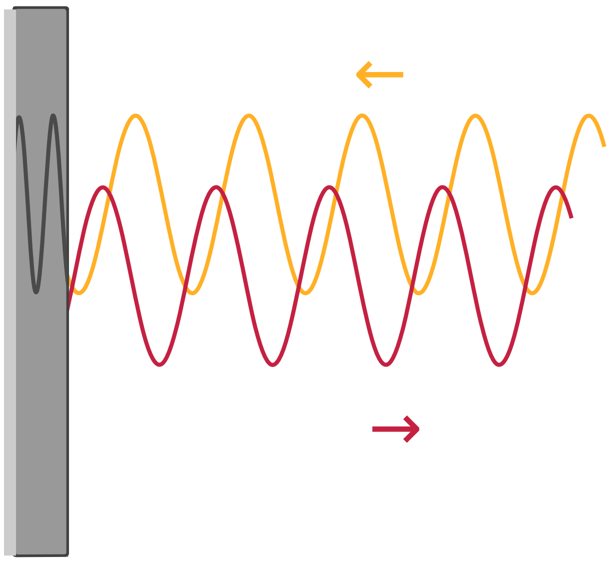
Spatially modulated **DEPHASING** → ? → Spatially modulated **INTENSITY**


Polarization-dependent

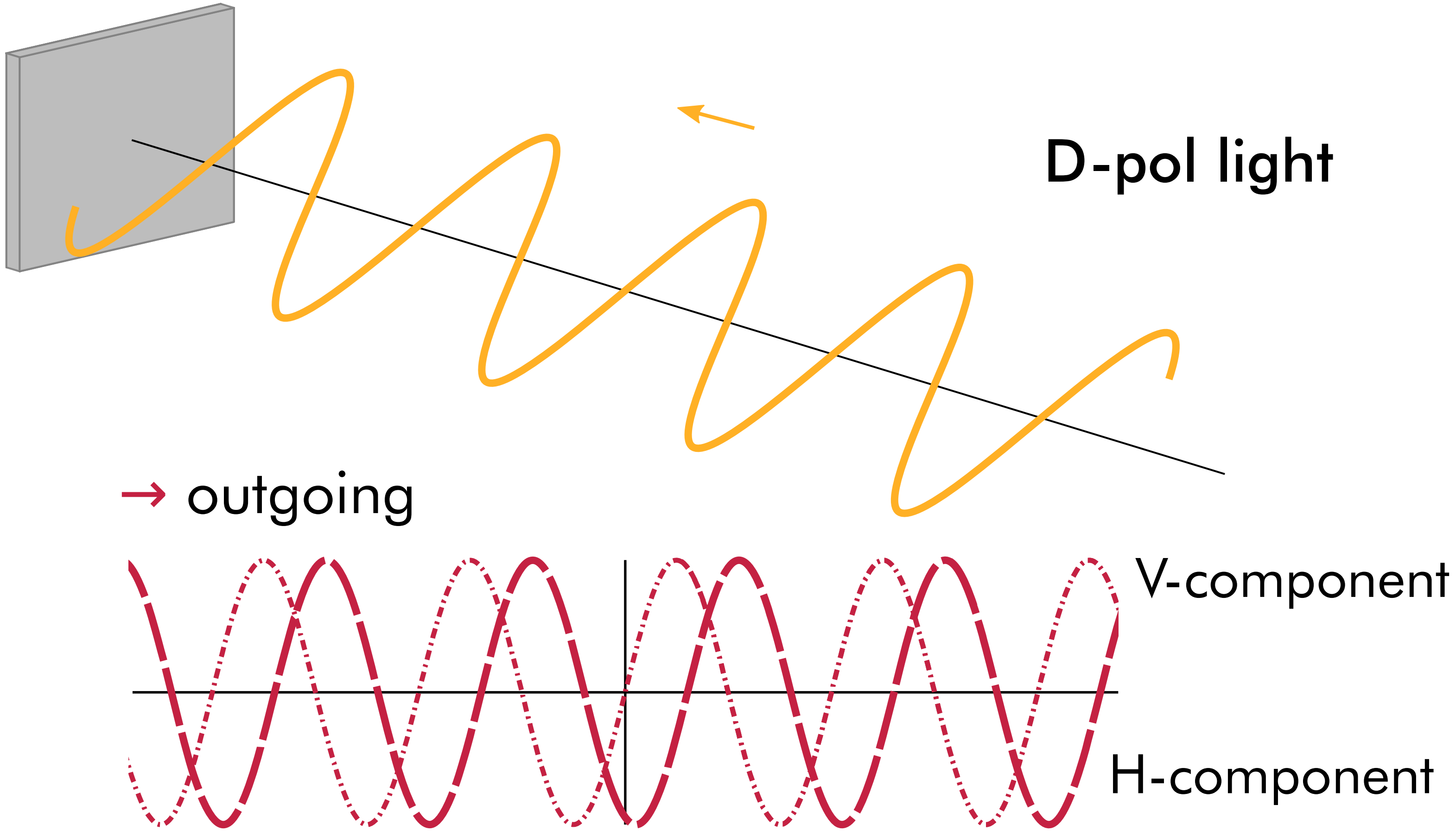


Encoding scheme II

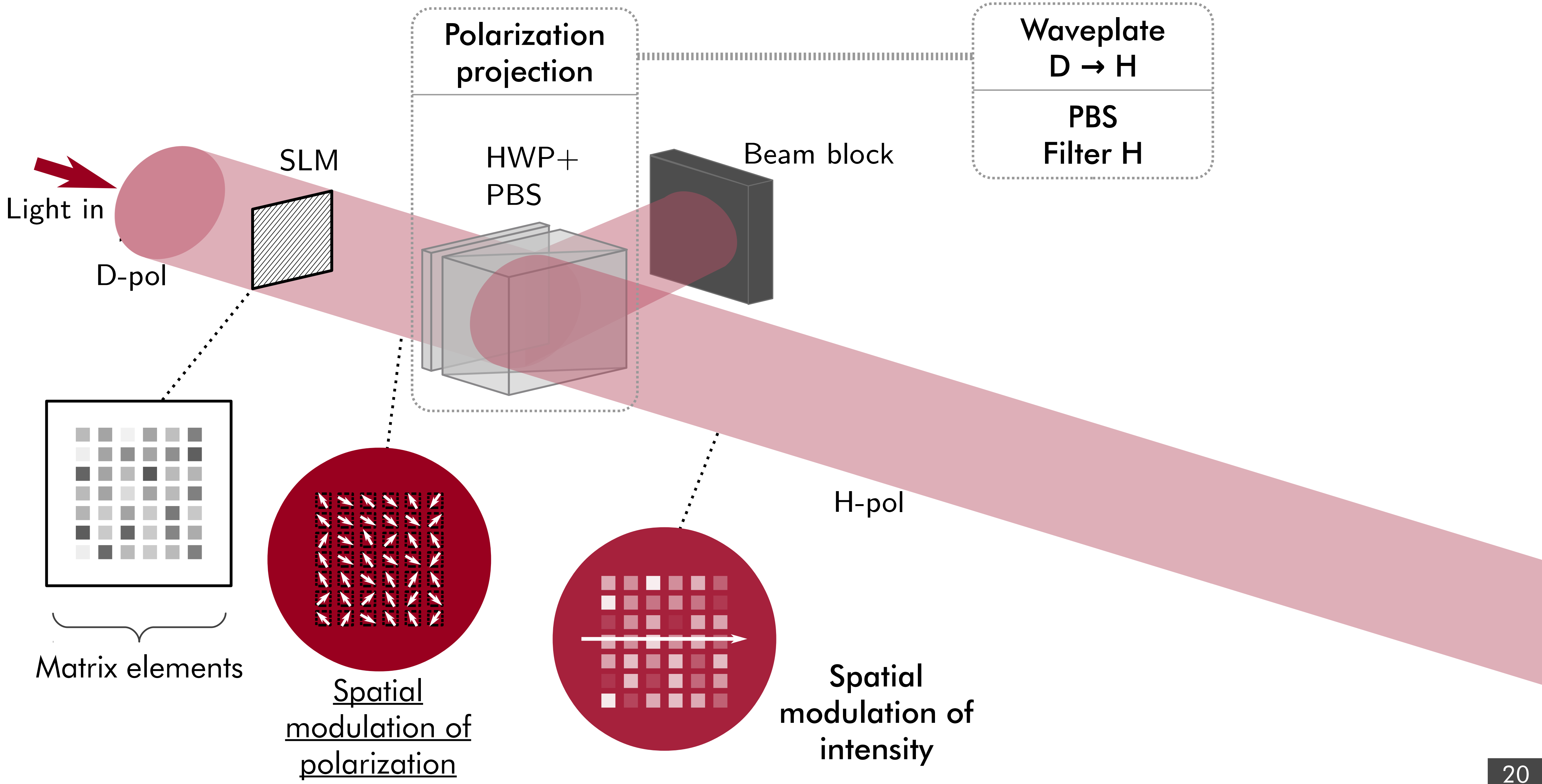
Spatially modulated **DEPHASING** → Use D-pol light (in) Filter D-pol (out) → Spatially modulated **INTENSITY**



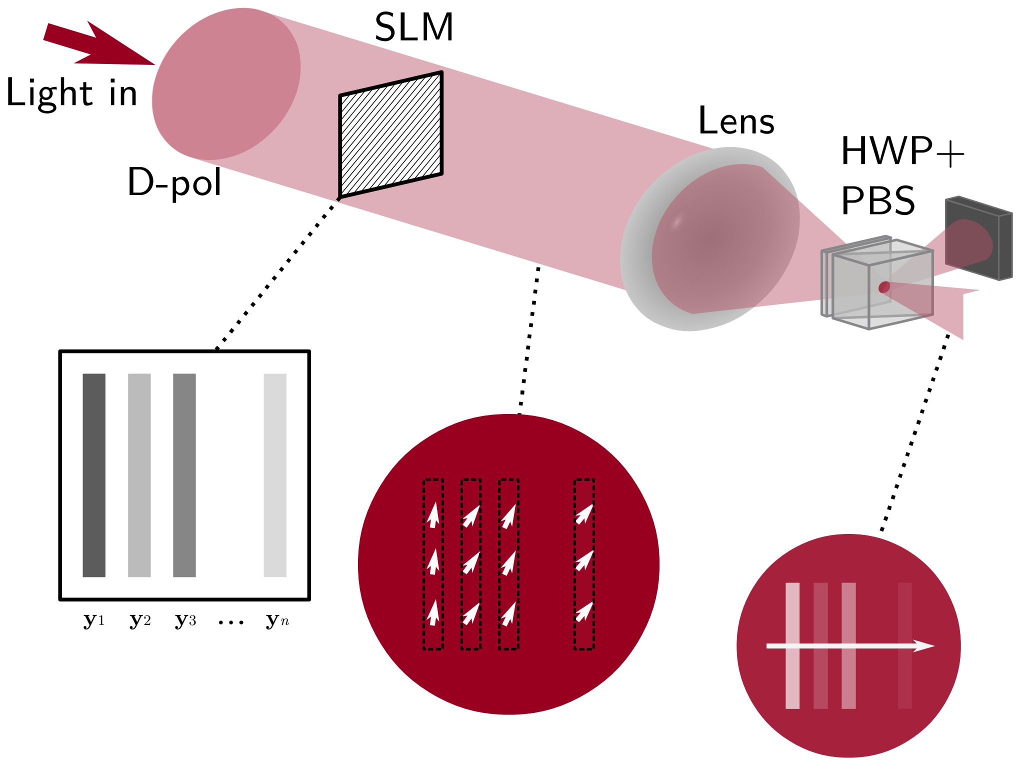
Polarization-dependent

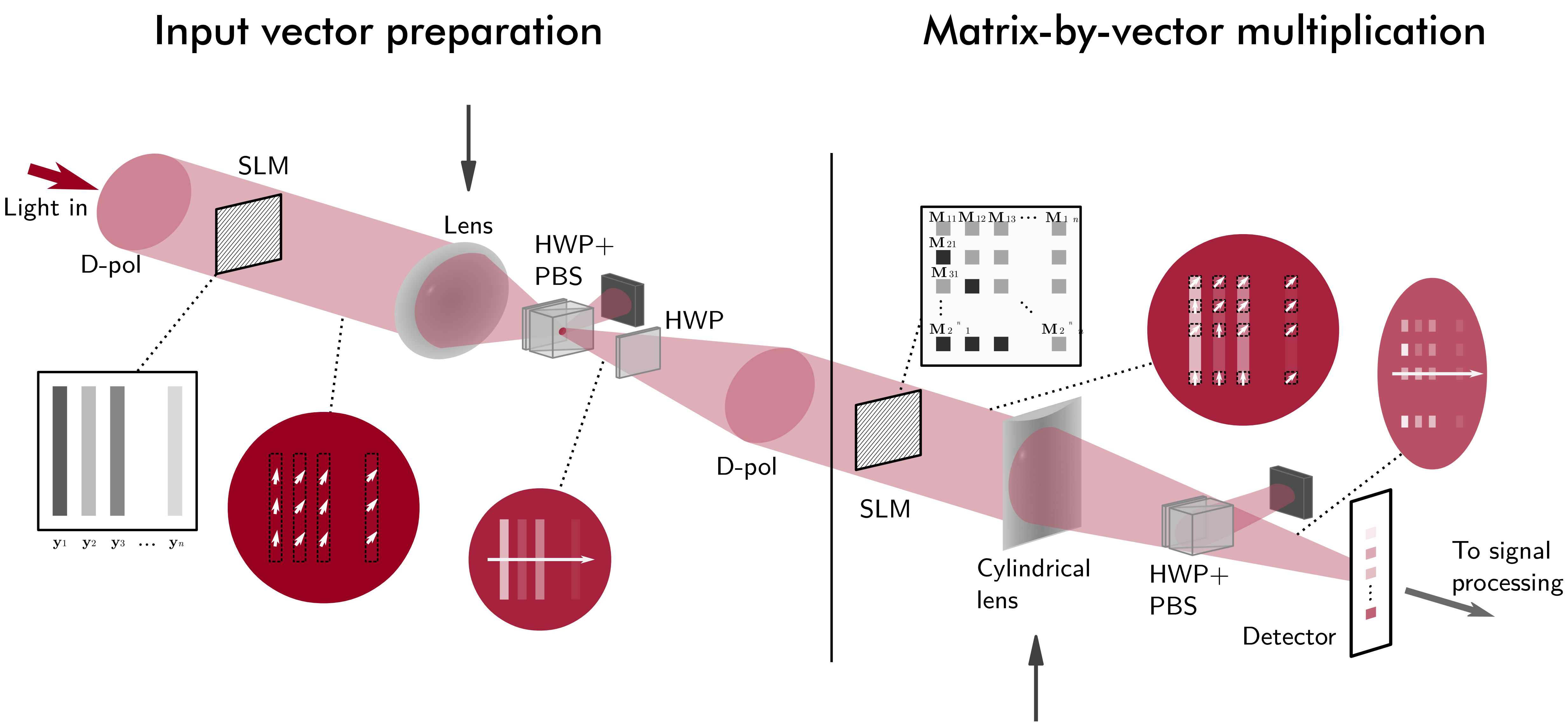


Preparing vectors and matrices

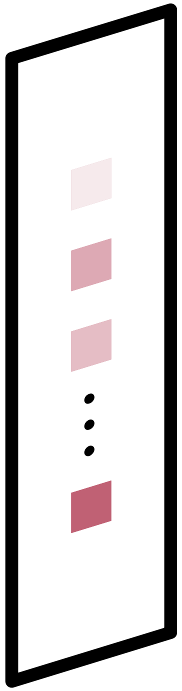


Input vector preparation




Input vector preparation
Matrix-by-vector multiplication
Light in
D-pol
SLM
Lens
HWP+
PBS
HWP
D-pol
SLM
Cylindrical
lens
HWP+
PBS
Detector
To signal
processing

Output signal



Detected signal

Realizations vector:

$$\mathbf{r} = \mathbf{M} \times \mathbf{y}$$

Feasible solutions

- presence of pixel with a reference intensity value
- corresponding to $x_1 = 0$

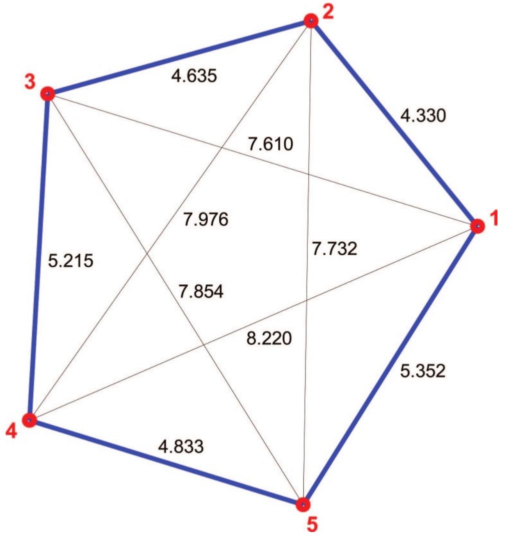
Discussion and Outlook

Performance

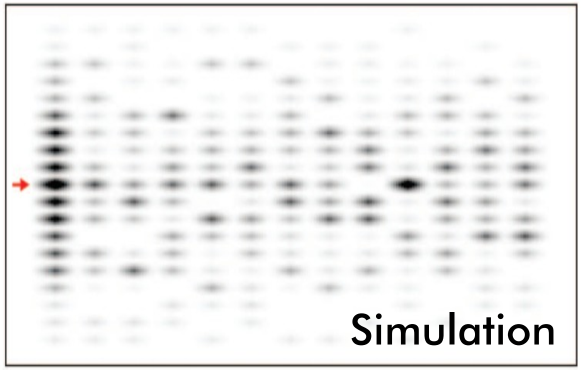
- A figure of merit – MAC-operations/s
  - **Here**: 0.1 TOPS
  - **Photonic chip**: 2 TOPS [1]
  - **CPU**: 2.6 TOPS [2]
- Possible limitations
  - Noise level
  - Refreshing rate of SLM
  - SLM resolution
- Speeding up
  - Larger / faster SLM
  - Many SLMs in parallel
  - Alternative modulation scheme

NP-hard!

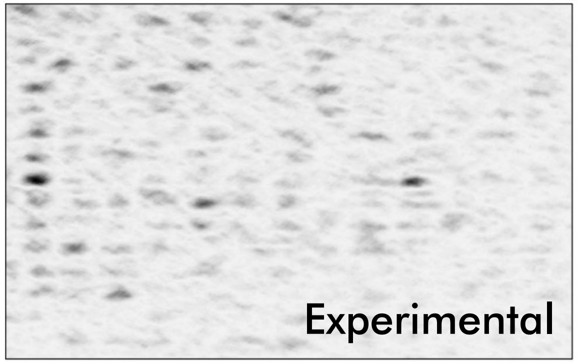
Other optical schemes



Traveling Salesman Problem [3]



Simulation



Experimental

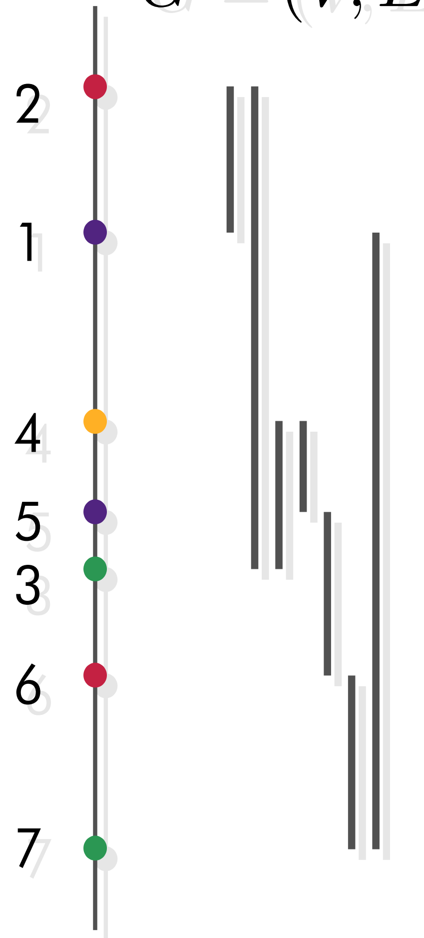
[1] Feldmann *et al.*, Nature **589** (2021)

[2] Jouppi *et al.*, Proc. ISCA '17 (2017)

[3] Shaked *et al.*, Appl. Optics **46** (2007)

Summary

$$G = (V, E, d)$$



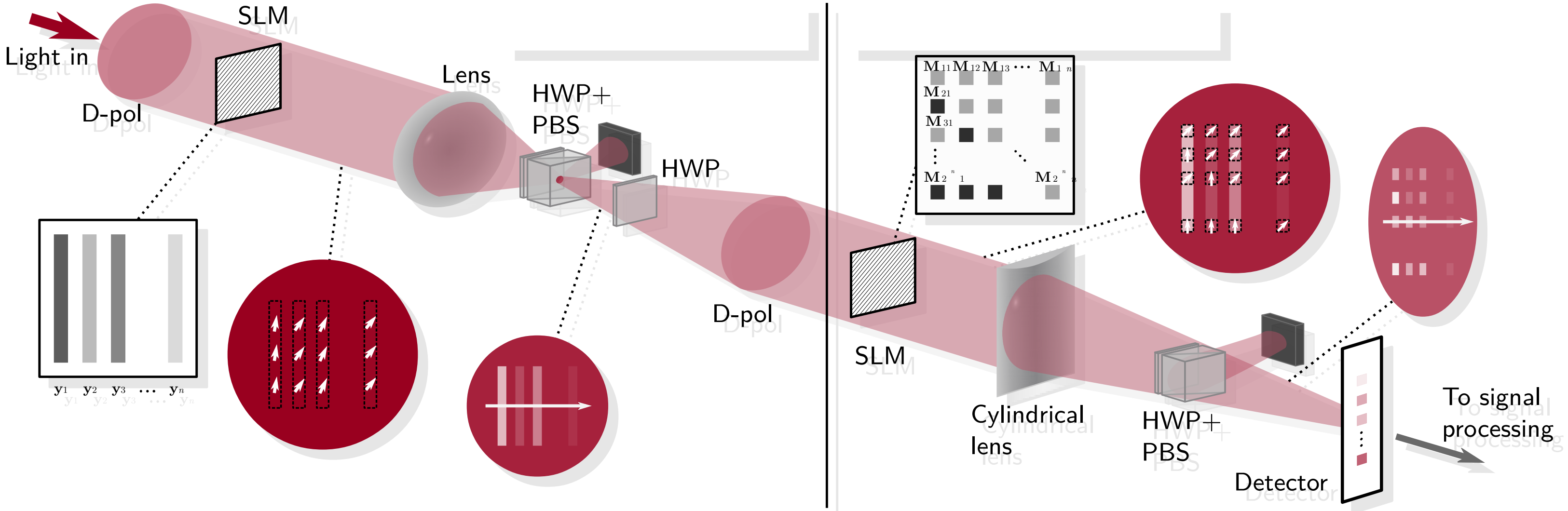
Paradoxical DGP_1

$$\mathbf{r} = \mathbf{M} \times \mathbf{y}$$

$$d \rightarrow \mathbf{y}$$

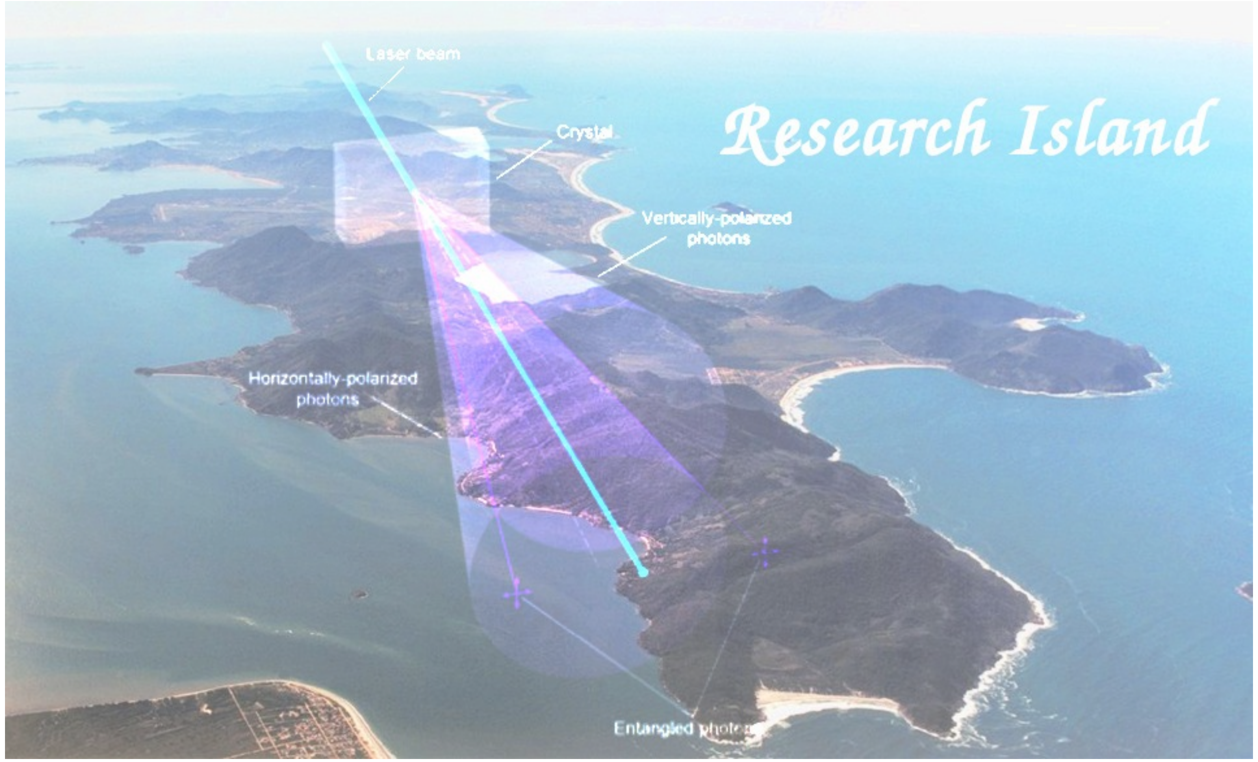
$$\mathbf{M} = \begin{pmatrix} -1 & -1 & -1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Matrix-by-vector reformulation allows for...



...Optical computing of matrix-by-vector multiplication

Acknowledgements



Thank you for your attention!

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