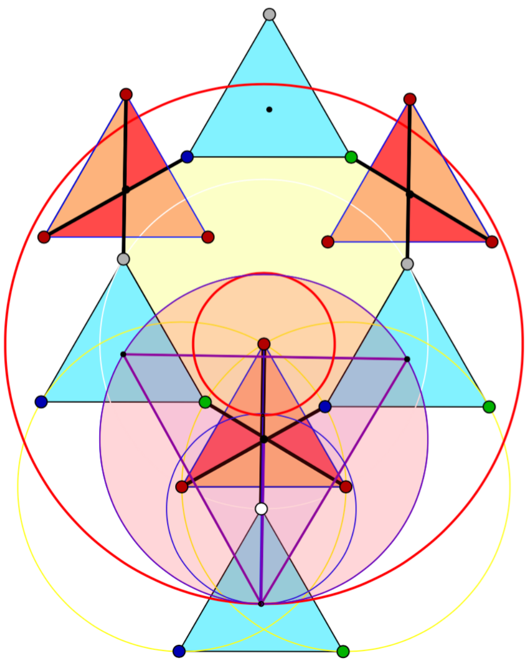
AUXETIC DEPLOYMENTS



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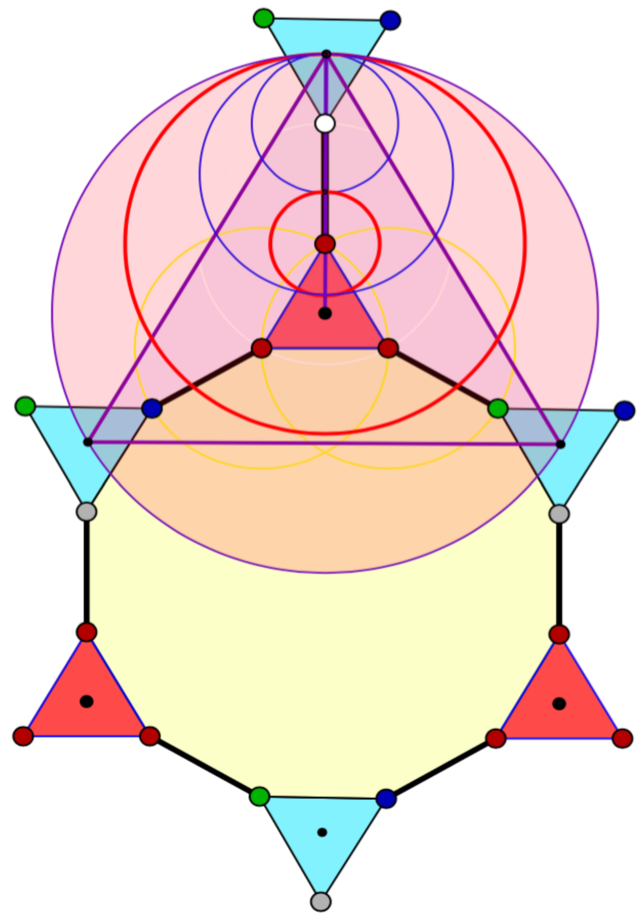
joint work with

Ileana Streinu Smith College

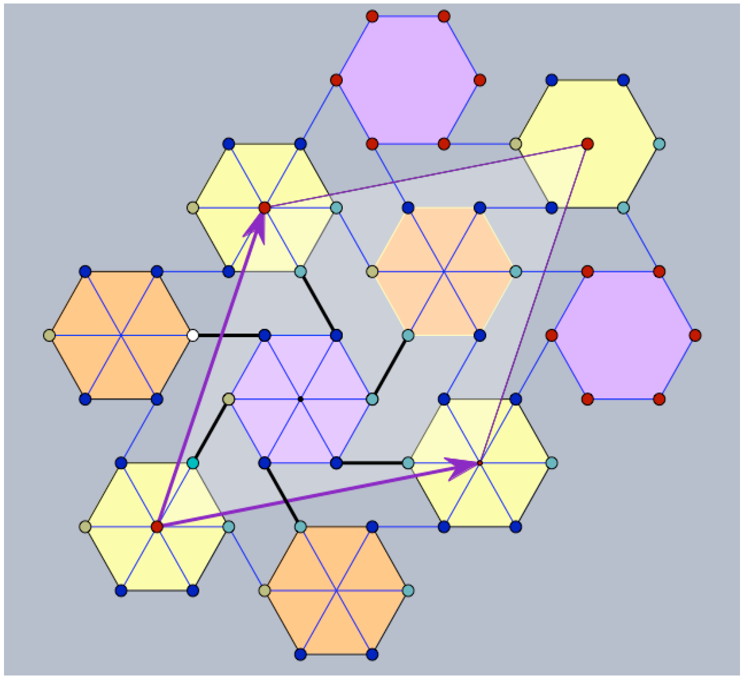
Fields Institute virtual workshop on "Materials and Periodicity"
March 2, 2021

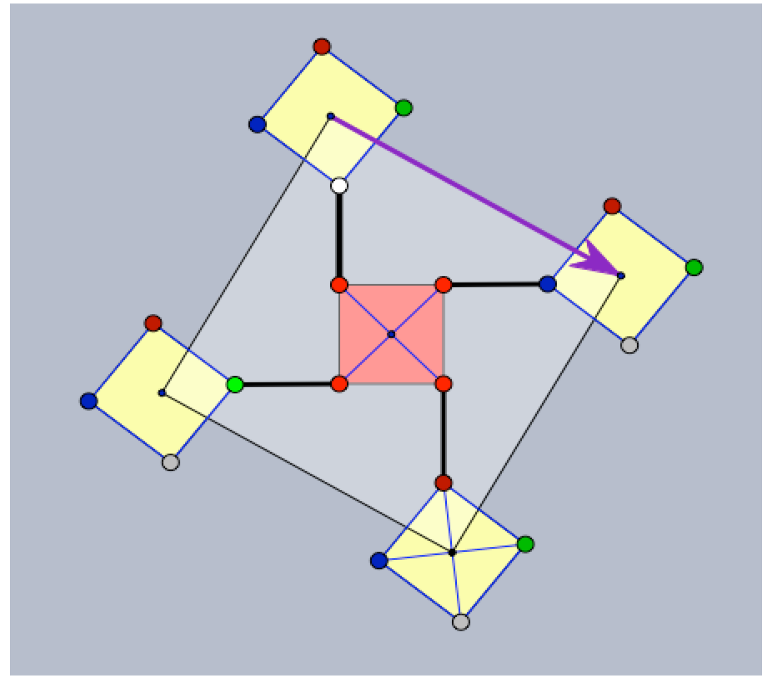
Overview:

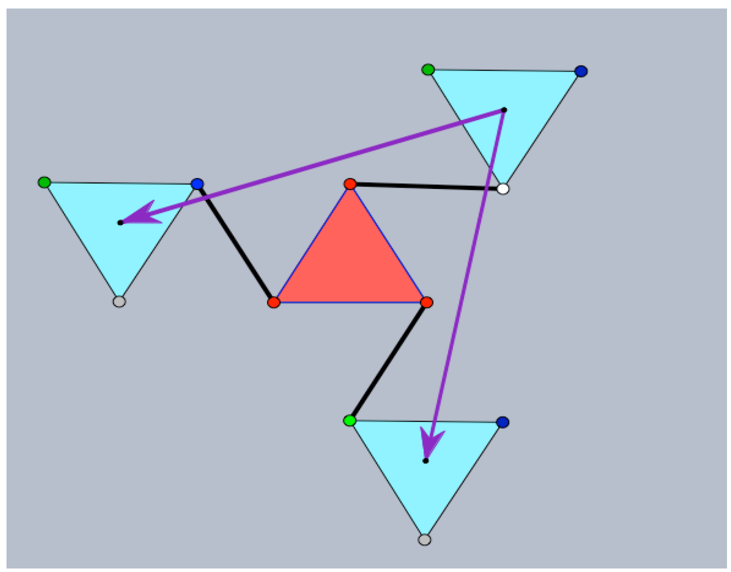
1. Three planar periodic frameworks
2. From tiling to full deployment
3. Auxetic deformations
4. Geometric underpinnings
5. The unit cell area function
6. Deployment trajectories



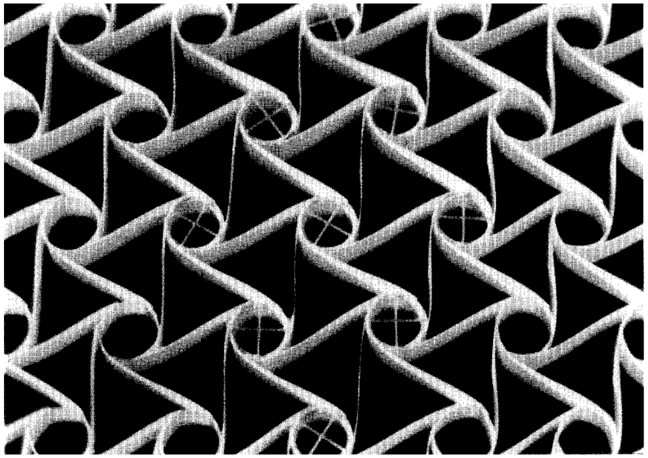
Three planar periodic frameworks







Siblings in materials literature



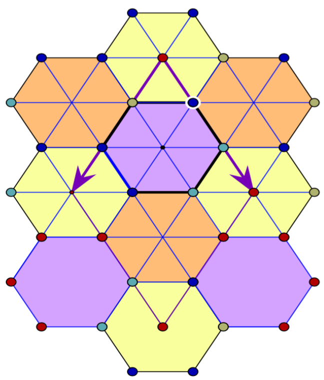
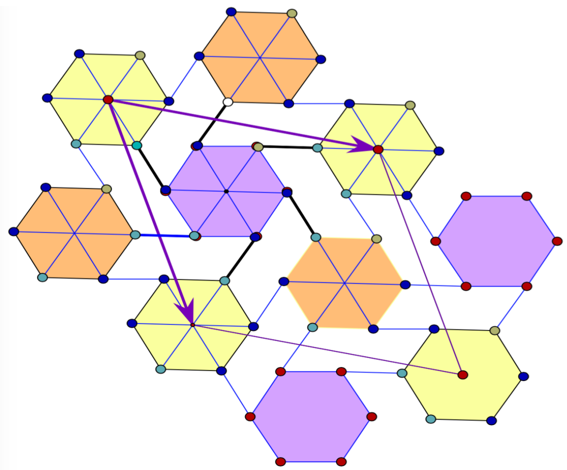
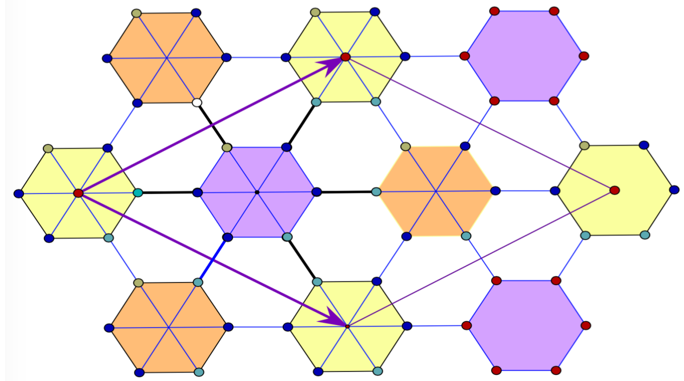
From D. Prall and R.S. Lakes:
PROPERTIES OF A CHIRAL HONEYCOMB WITH A POISSON'S RATIO OF - 1
Int. J. Mech. Sci. Vol. 39, No. 3, pp. 305 314. 1997

Hexachiral version from F. Scarpa et al. **Shape memory polymer hexachiral auxetic structures with tunable stiffness,** Smart Mater. Struct. **23** (2014) 045007

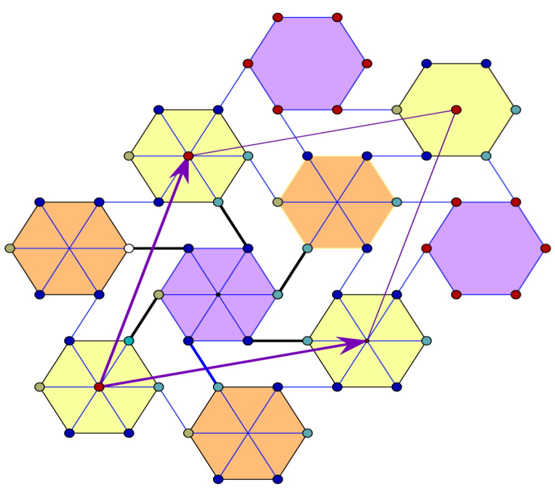
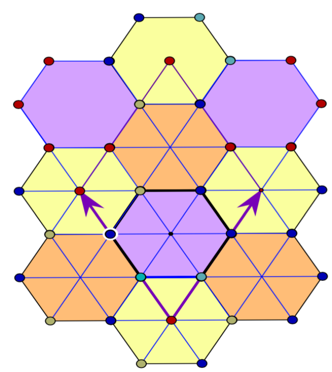
Ornamental motifs

Tiles, Ottoman period, Iznik 1560.
Harvard Art Museums.

From tiling to full deployment

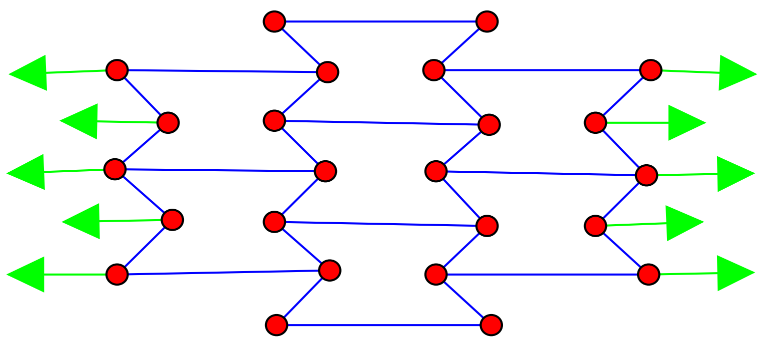
The framework with hexagons has a single degree of freedom
and a unique deformation trajectory.

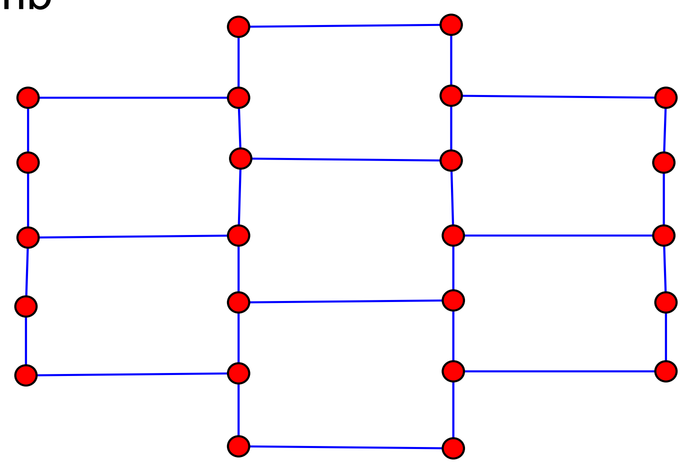



Auxetic behavior

In elasticity theory, auxetic behavior is an expression of negative Poisson's ratios. Given two orthogonal directions, a stretch in the first direction leads to a widening in the second (orthogonal) direction.

Typical illustration: the "reentrant honeycomb"



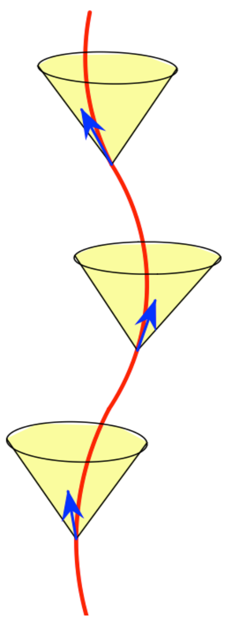


Reference:
G.N. Greaves, A.L. Greer, R.S. Lakes and T. Rouxel: Poisson's ratio and modern materials, Nature materials 10 (2011), 823-837.

Geometric auxetics

For periodic frameworks, there is a purely geometric approach to auxetic behavior, based on the evolution of the periodicity lattice.

Definition. A one-parameter deformation of a periodic framework is an auxetic path when for any $t_1 < t_2$, the linear operator taking the period lattice Λ_{t2} to Λ_{t1} is a contraction i.e. has operator norm at most 1.



Theorem: A one-parameter deformation of a periodic framework is an auxetic path when the curve given by the Gram matrices of a basis of periods has all velocity vectors (tangents) in the positive semidefinite cone.

This is analogous to `causal-lines' in special relativity i.e. curves with all their tangents in the `light cone'.

Reference:
Borcea and Streinu: *Geometric auxetics*, Proc. Roy. Soc. A 471 (2015), 20150033.

Infinitesimal auxetic deformations and spectrahedral cones

For a d-periodic framework F, the connection between auxetics and spectrahedra is provided by the Gram map

Deformation space (F) → Sym(dxd)

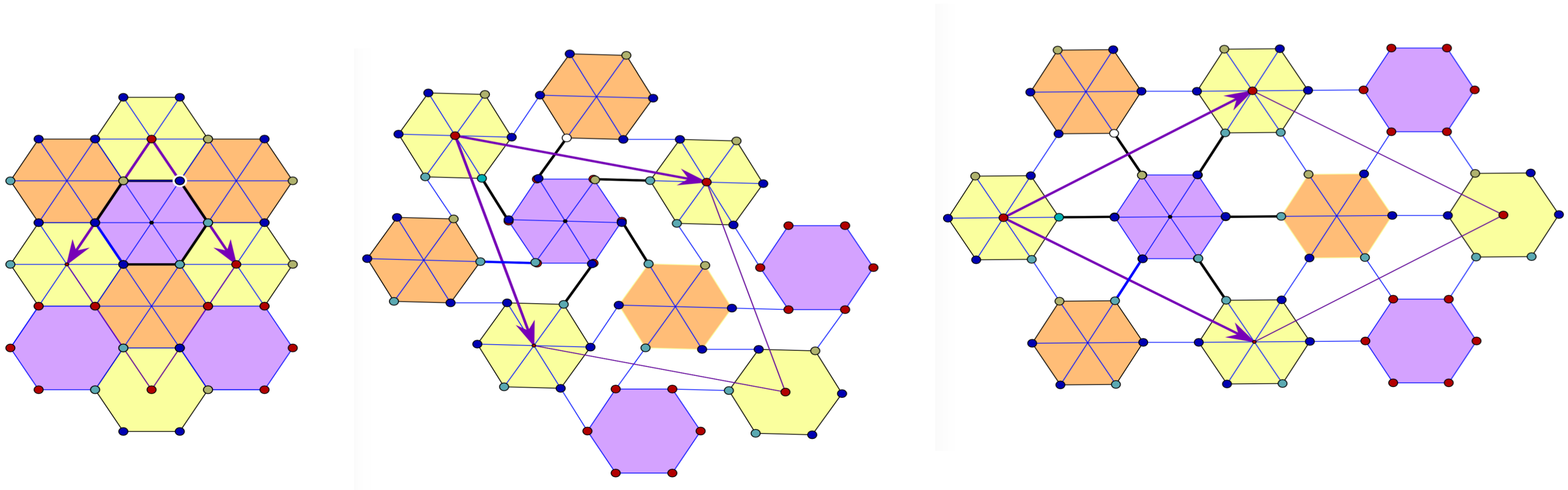
expressing the variation of the Gram matrix for a chosen set of generators of the periodicity lattice. Infinitesimally, we obtain a linear (i.e. tangent) map:

Infinitesimal deformations (F) → Sym(dxd)

The image of this linear map, intersected with the positive semidefinite cone in Sym(dxd), defines a spectrahedral cone.

Thus: infinitesimal auxetic cones are linear pre-images of spectrahedral cones.

Auxetic deployment



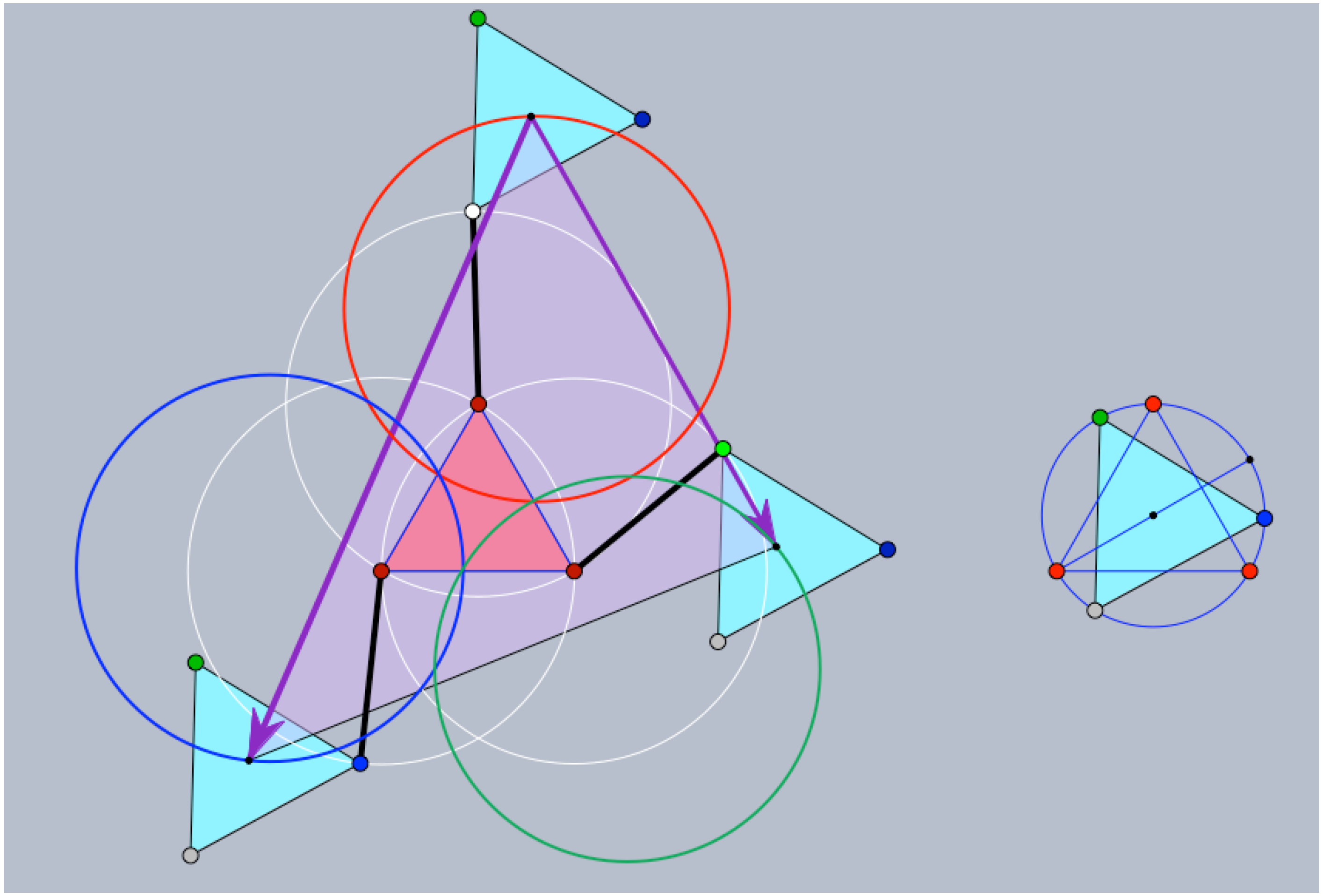
This deployment deformation is auxetic since the Gram map goes to increasing scalar multiples of the Gram matrix for the tiling.

Animation 1 here.

The framework with squares (3 dof) and the framework with triangles (4 dof) present genuine possibilities of auxetic deployment design.

We will focus on the framework with articulated triangles.

Geometric underpinnings

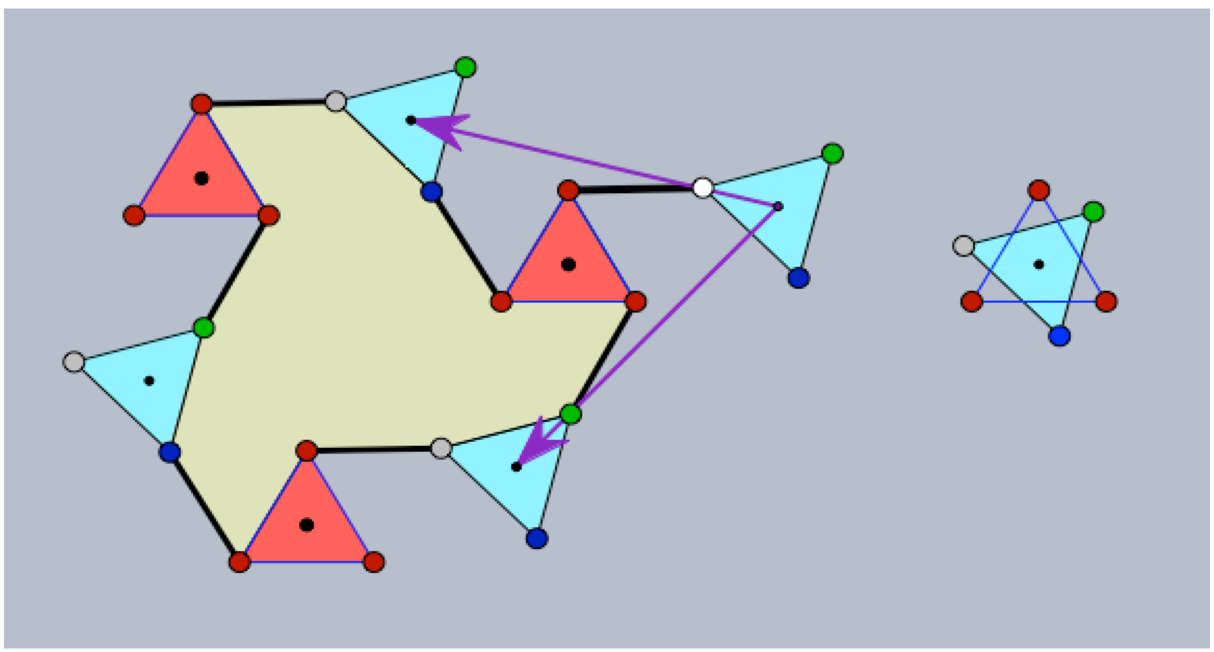


Use animation 2 here.

Typology of growth

EXPANSION implies AUXETIC BEHAVIOR implies AREA INCREASE.

We consider the unit cell area function on a 2-parameter subfamily where the threefold rotational symmetry of the framework is preserved.

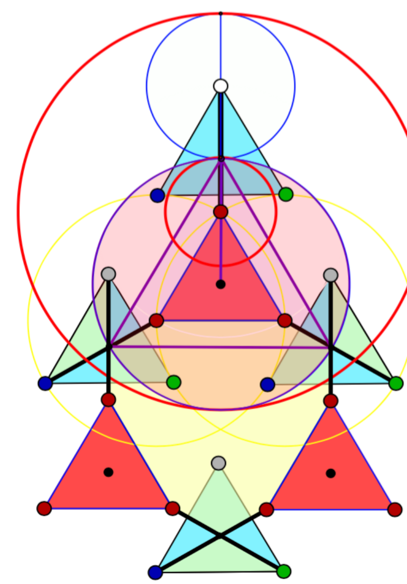


Animation 3 here.

Properties (on 2-torus)

- The Gram map gives scalar multiples of the tiling Gram matrix, hence area increasing deformations are auxetic.

- The area function is a Morse function on the 2-torus.
  It has six critical points: the absolute maximum,
  three critical points of index one
  and two absolute minima
  (for vanishing area).



A critical configuration of index one.

Selected References

Geometric Theory

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Proc. Roy. Soc. A 466 (2010), 2633-2649.

Borcea and Streinu: Geometric auxetics,
Proc. Roy. Soc. A 471 (2015), 20150033.

Borcea and Streinu: Liftings and stresses for planar periodic frameworks,
Discrete and Computational Geometry, 53 (2015), 747-782.

Borcea and Streinu: Periodic auxetics: Structure and design,
Quarterly Journal of Mechanics and Applied Mathematics,
71 (2018), 125-138.

Borcea and Streinu: Periodic tilings and auxetic deployments,
Mathematics and Mechanics of Solids (2021), Vol. 26(2) 199-216.

Thank you