Topological spines, minimal realisations and cohomology of strictly developable simple complexes of groups

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joint work with Tomasz Prytuła

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Outline

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Outline

1. Motivation
2. Davis complex
3. Bestvina complex
4. Generalisations
5. Applications

The setting

General problems

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- X is a model for classifying space $E_{\mathfrak{F}}G$.

Classifying space for a family of subgroups

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- $E_{\mathfrak{F}}G$ is denoted by $\underline{E}G$ and $\underline{\underline{E}}G$ when $\mathfrak{F}$ is the family of finite and virtually cyclic subgroups, respectively.

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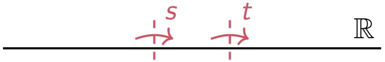

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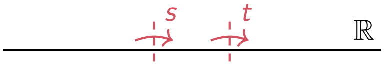

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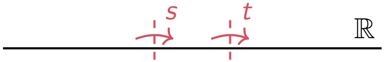
1. 

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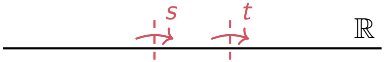
1. 

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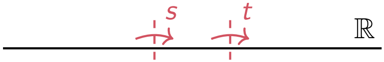
1. 

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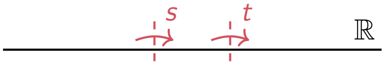
1. 

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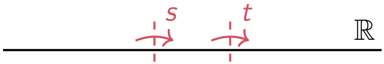
1. 

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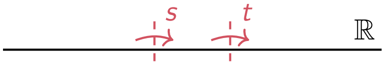
1. 

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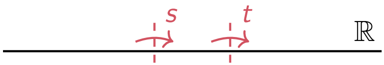
1. 

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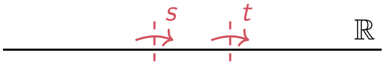
1. 

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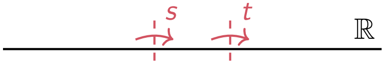
1. 

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Right-Angled Coxeter groups

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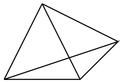

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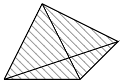

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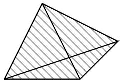

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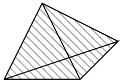

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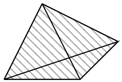

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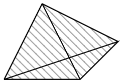

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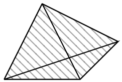

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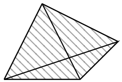

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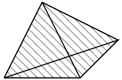

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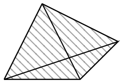

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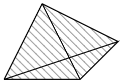

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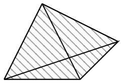

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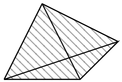

$$W = W_L = \langle s_i \in V(L) \mid s_i^2 = e, s_i s_j = s_j s_i \text{ iff } \{s_i, s_j\} \in E(L) \rangle$$

Examples

L	Δ^n	$(\Delta^n)^{(0)}$	$L_1 * L_2$	$L_1 \sqcup L_2$
W_L	$(\mathbb{Z}/2)^{n+1}$	$(\mathbb{Z}/2)^{*(n+1)}$	$W_{L_1} \times W_{L_2}$	$W_{L_1} * W_{L_2}$

$\underline{E}W = \Sigma_W = \Sigma$ - Davis complex

Davis complex for a Coxeter group

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Example

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$D_\infty = W_L$ where

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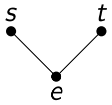
$CL =$ 

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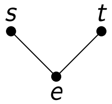

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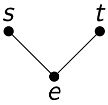
$CL =$ 

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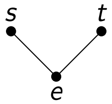
$CL =$ 

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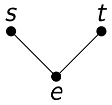

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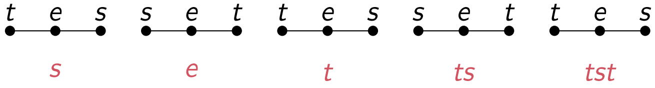

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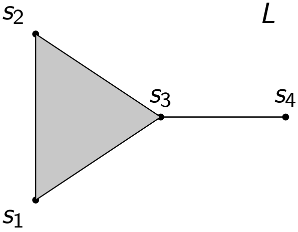

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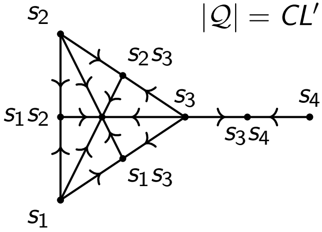

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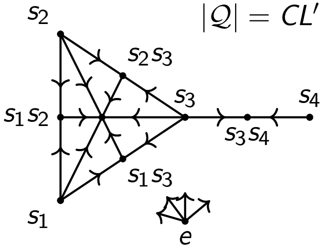

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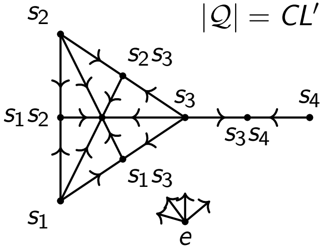

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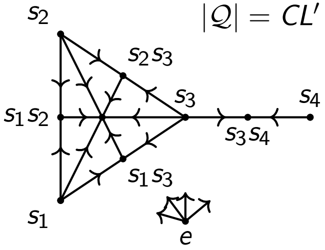

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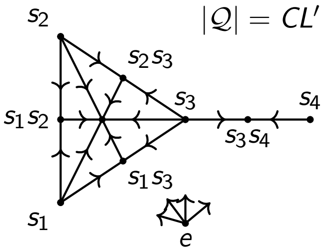

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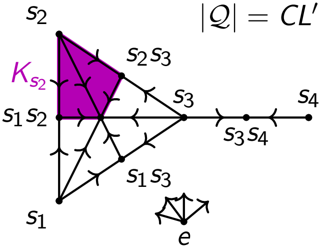

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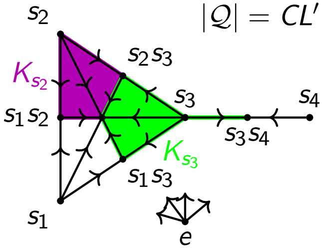

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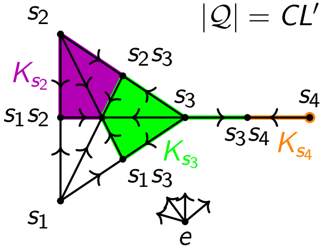

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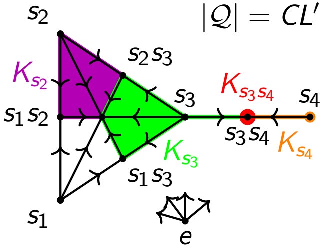

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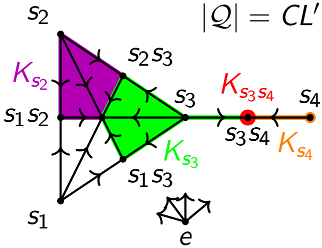

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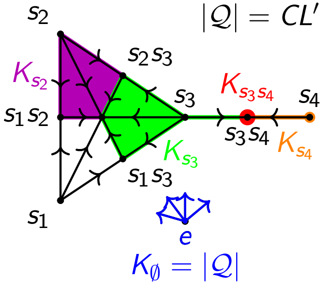

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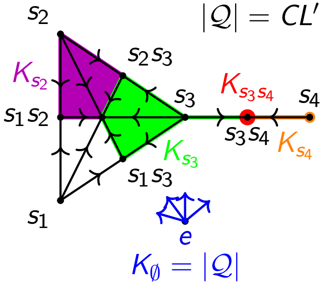

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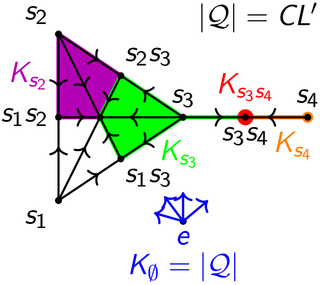

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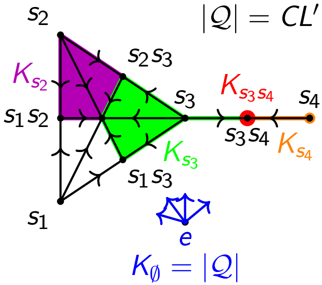

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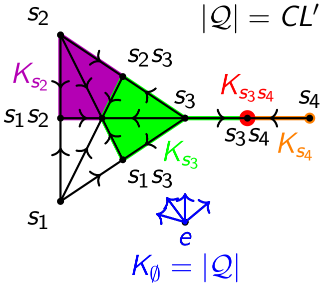

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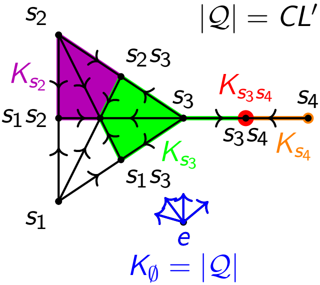

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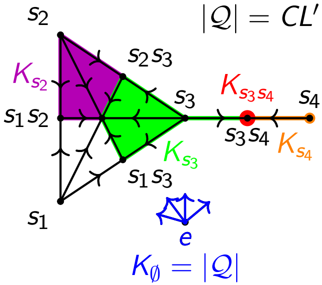

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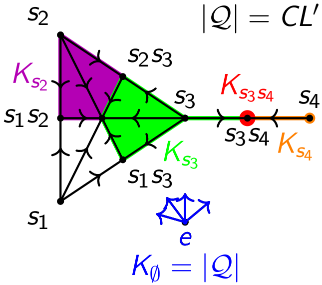

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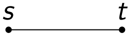
$L =$ 

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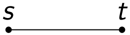
$L =$ 

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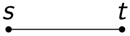
$L =$ 

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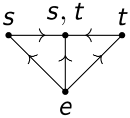
$|\mathcal{Q}| =$ 

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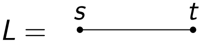

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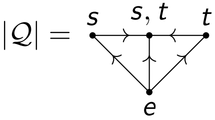

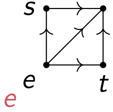

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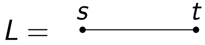

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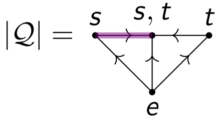

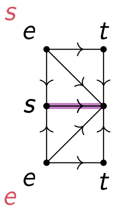

Davis complex for a Coxeter group

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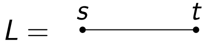

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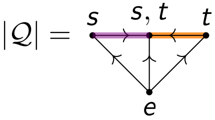

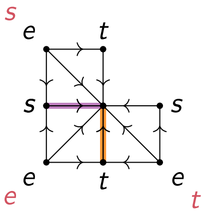

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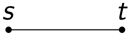
$L =$ 

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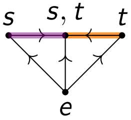
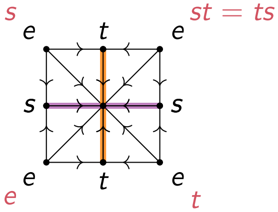
$|\mathcal{Q}| =$ 



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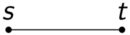
$L =$ 

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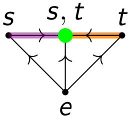
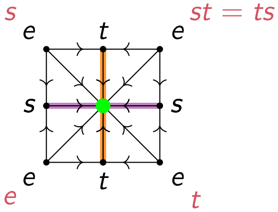
$|\mathcal{Q}| =$ 



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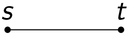
$L =$ 

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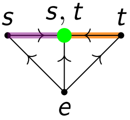
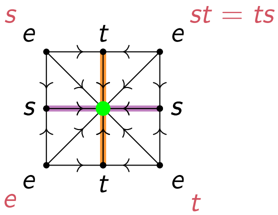
$|\mathcal{Q}| =$ 



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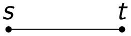
$L =$ 

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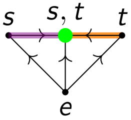
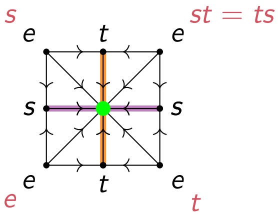
$|\mathcal{Q}| =$ 



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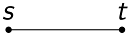
$L =$ 

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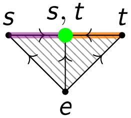
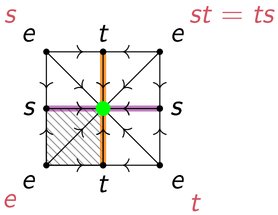
$|\mathcal{Q}| =$ 



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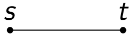
$L =$ 

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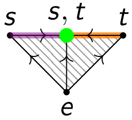
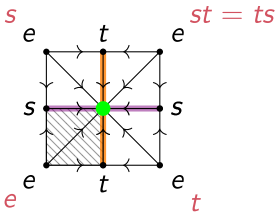
$|\mathcal{Q}| =$ 



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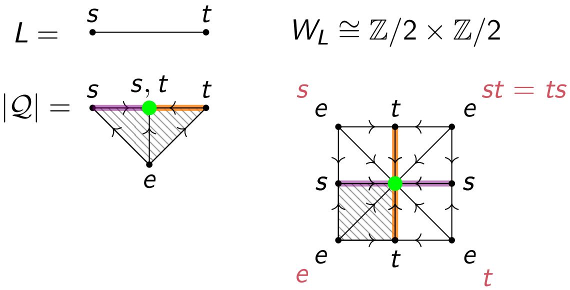

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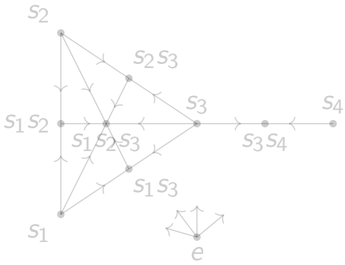

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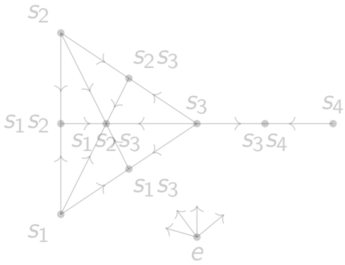

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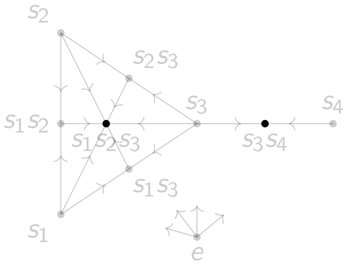

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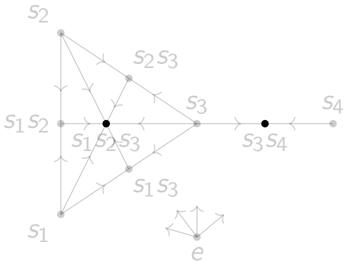

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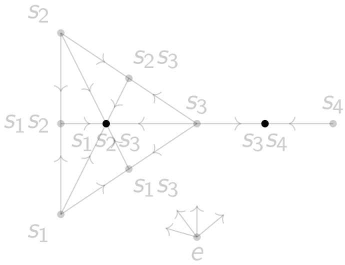

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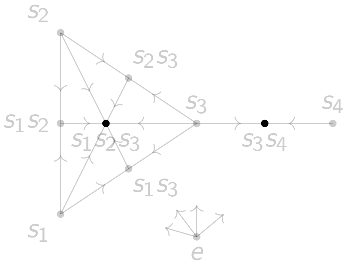

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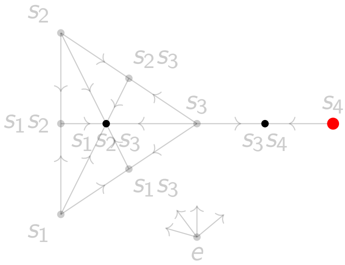

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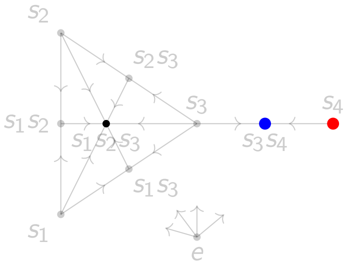

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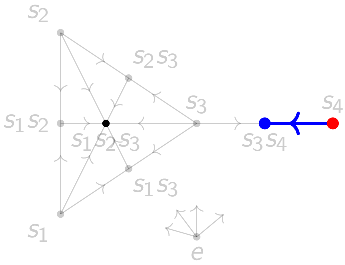

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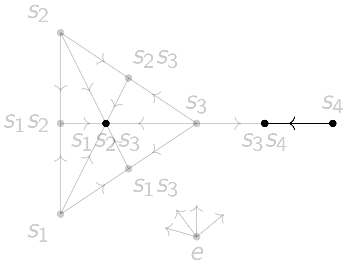

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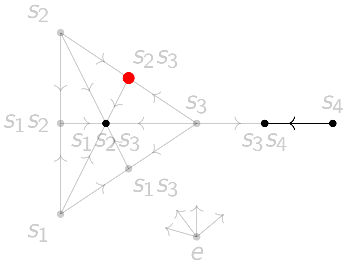

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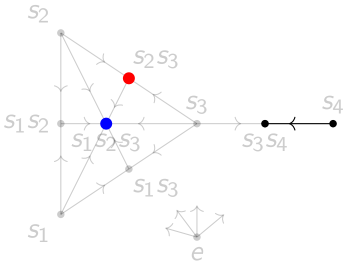

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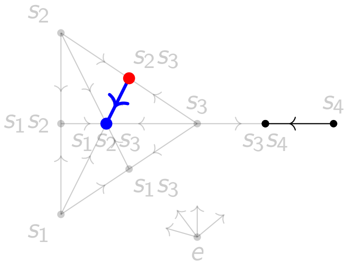

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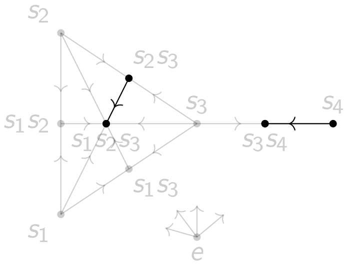

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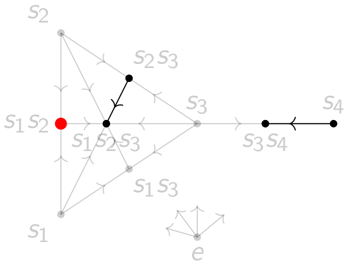

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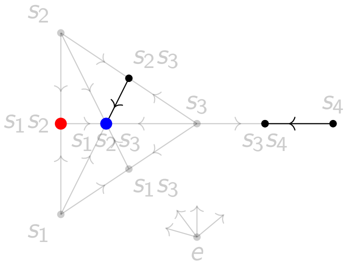

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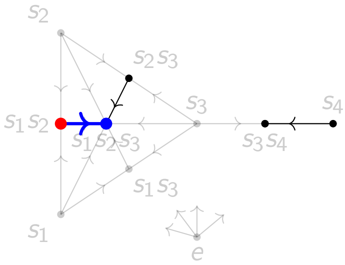

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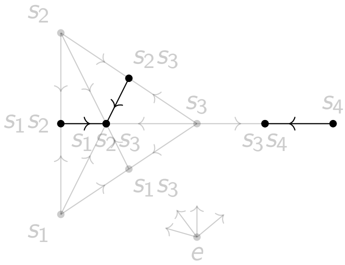

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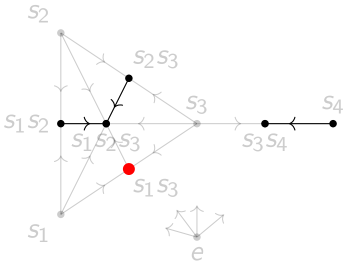

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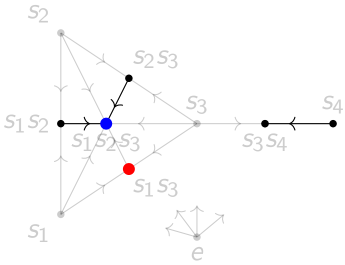

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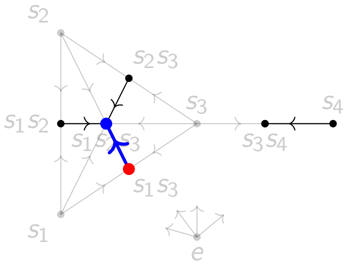

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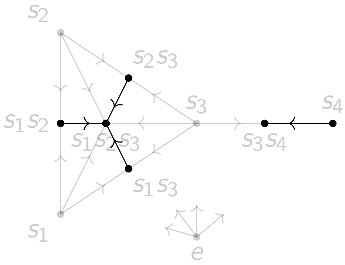

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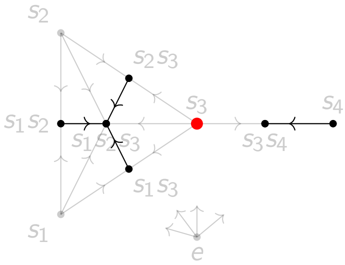

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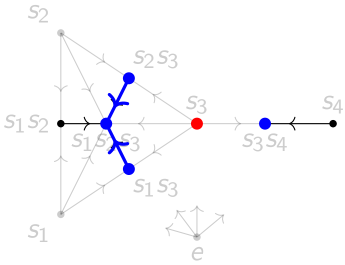

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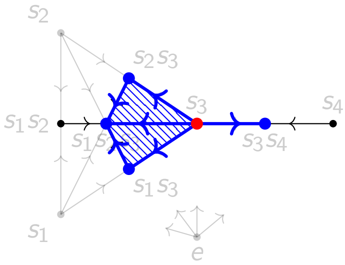

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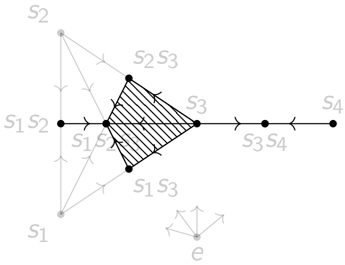

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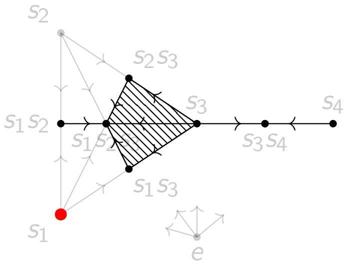

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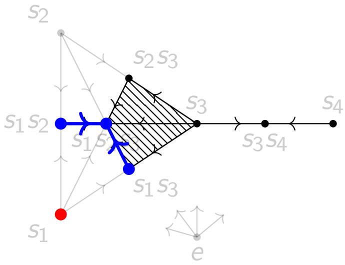

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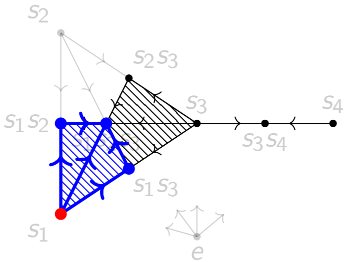

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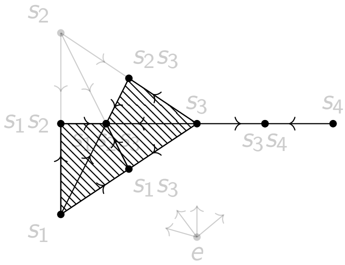

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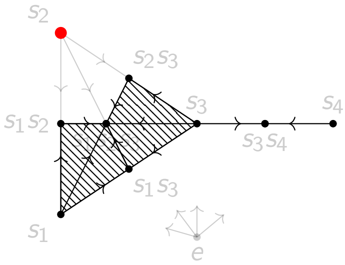

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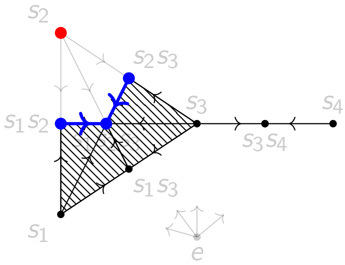

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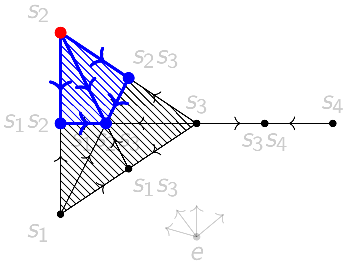

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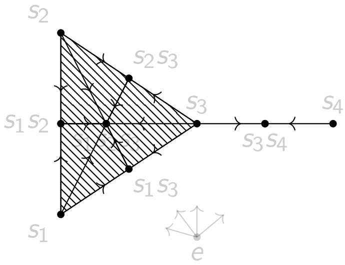

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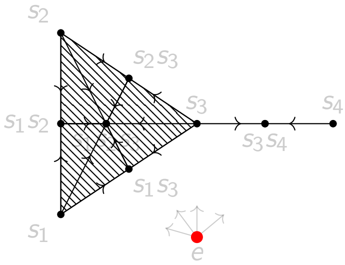

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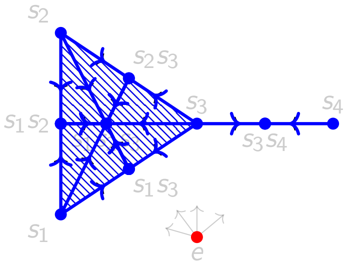

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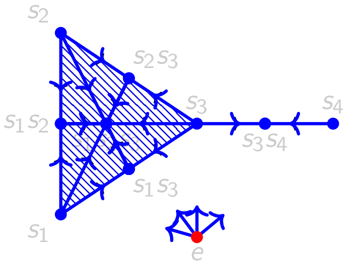

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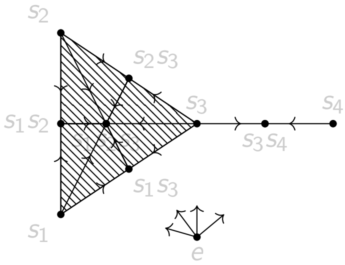

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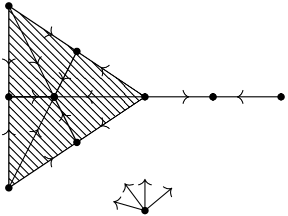

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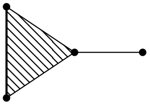

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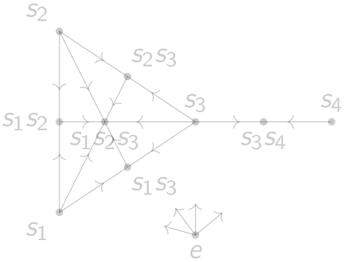

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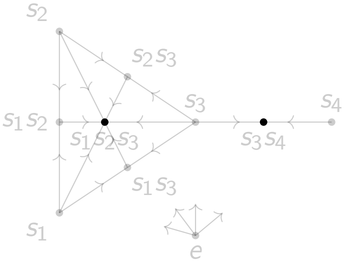

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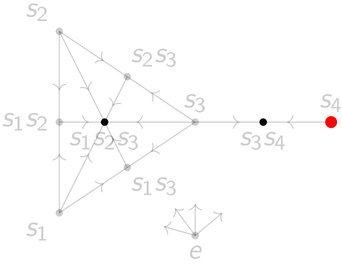

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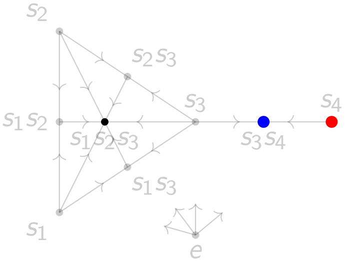

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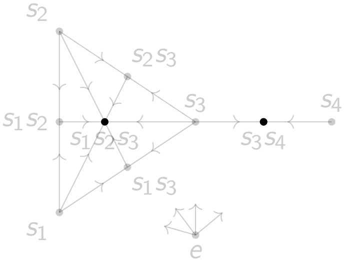

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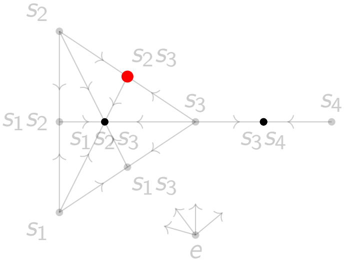

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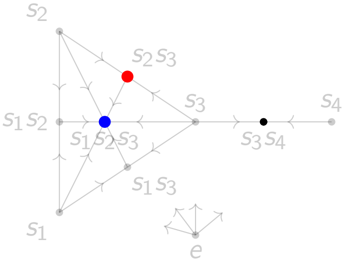

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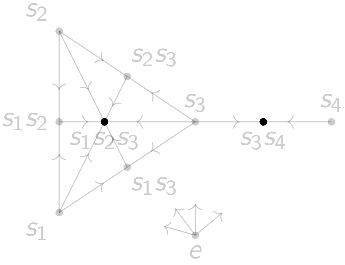

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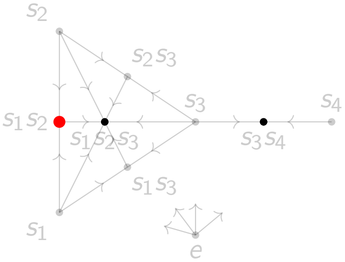

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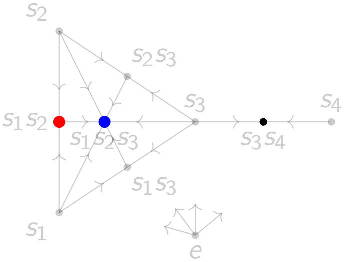

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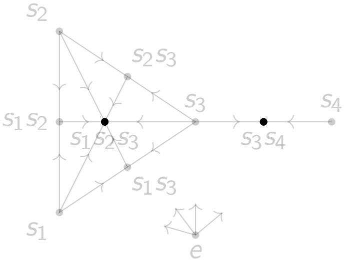

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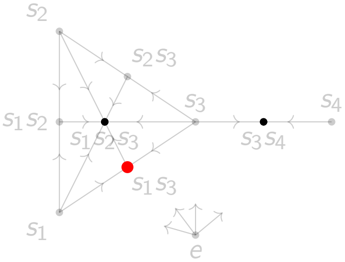

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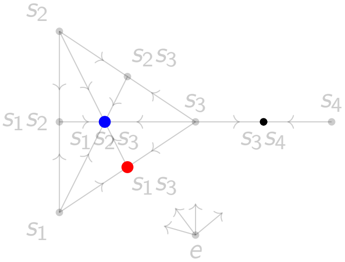

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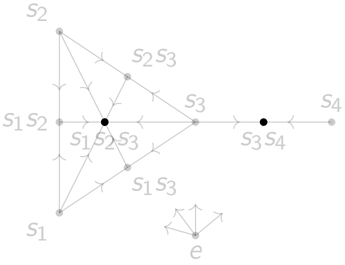

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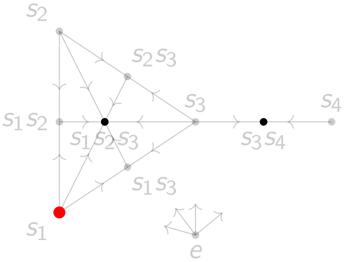

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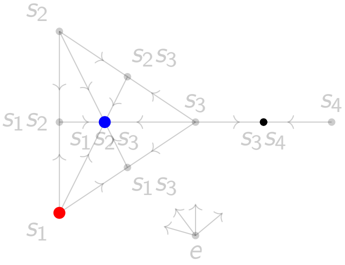

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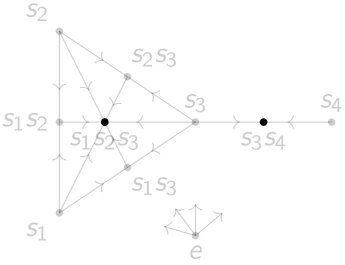

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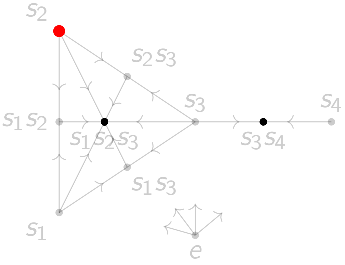

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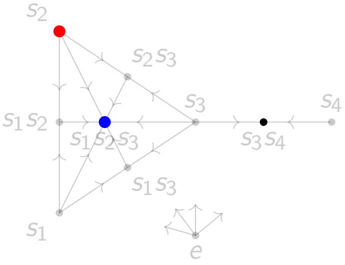

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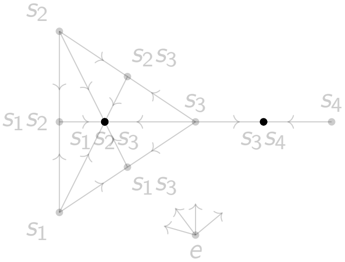

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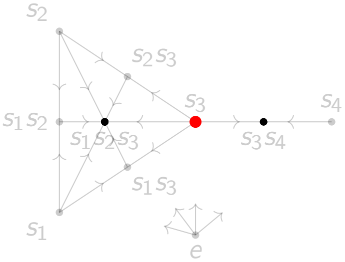

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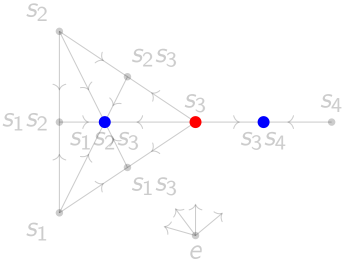

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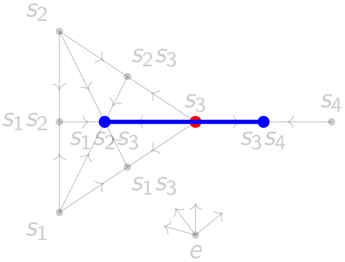

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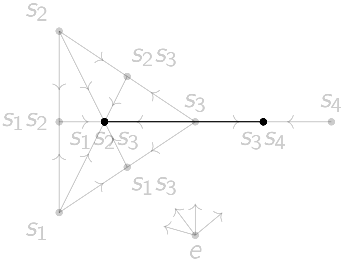

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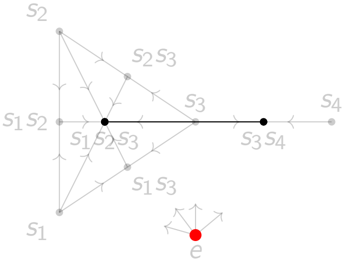

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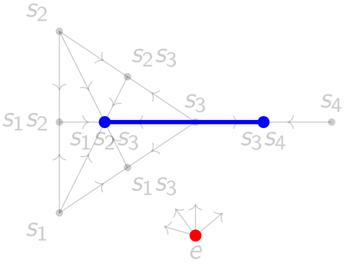

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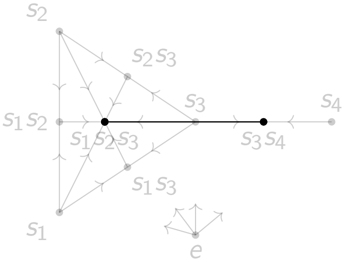

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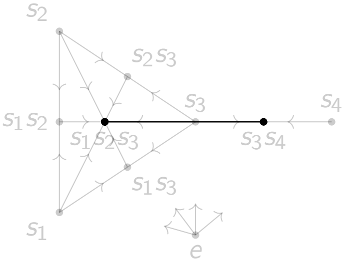

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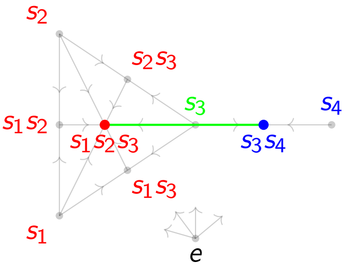

Construction of B_W

Example

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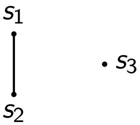
Example L



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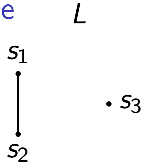

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Construction of B_W

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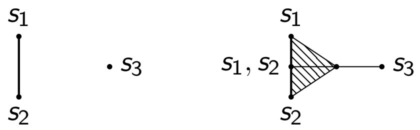
L $\qquad\qquad$ CL'



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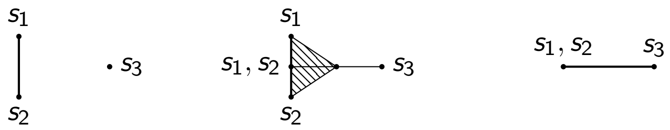

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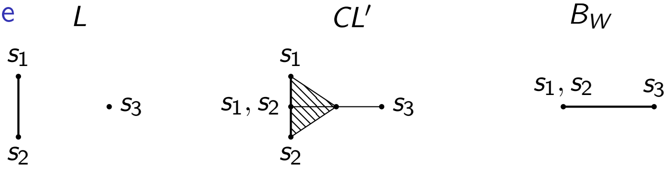

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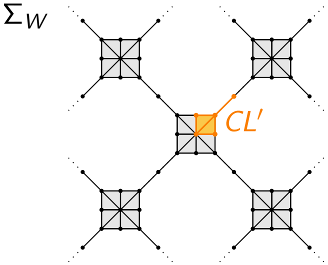

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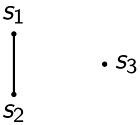

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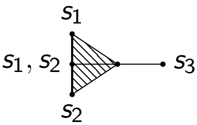

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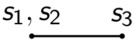

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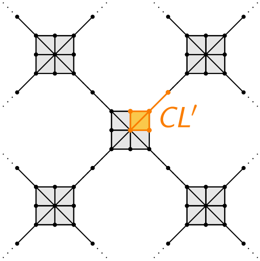
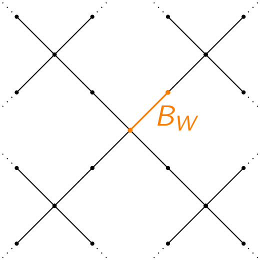


$\widetilde{B}_W$



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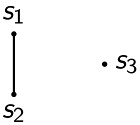

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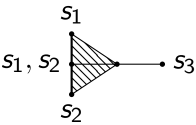

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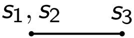

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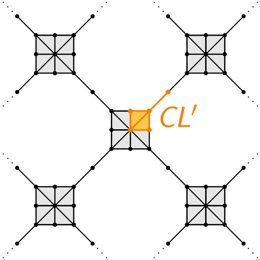

$\simeq_W$

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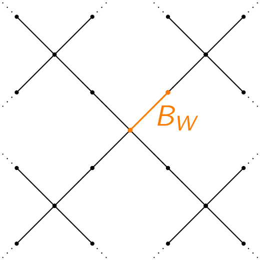

Simple complexes of groups

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Conjecture

Let G be a group and $\mathfrak{F}$ be a family of subgroups. Then $\mathrm{cd}_{\mathfrak{F}} G \leq 1$ if and only if G acts on a tree with stabilisers generating $\mathfrak{F}$.

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Corollary

If G is virtually torsion-free, then

$$\mathrm{vcd} G \leq \mathrm{vcd} W + \max\{\mathrm{vcd} P \mid P \text{ is parabolic}\}.$$

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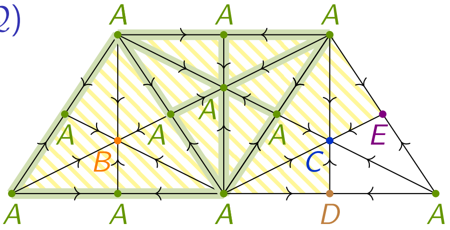

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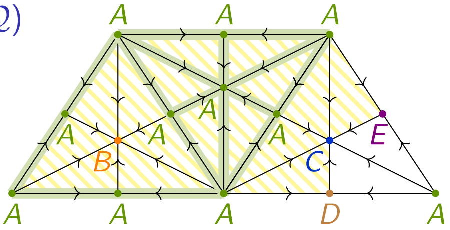

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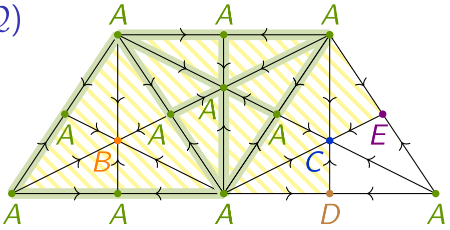

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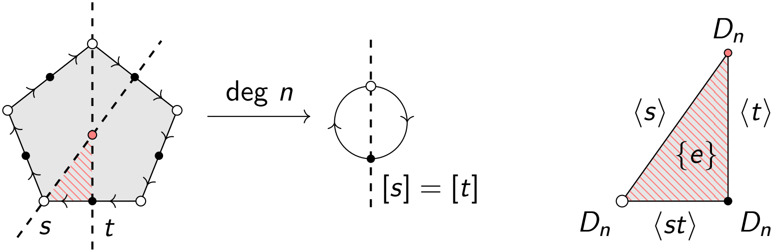

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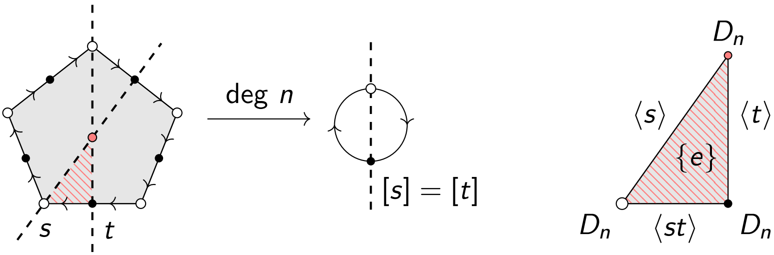

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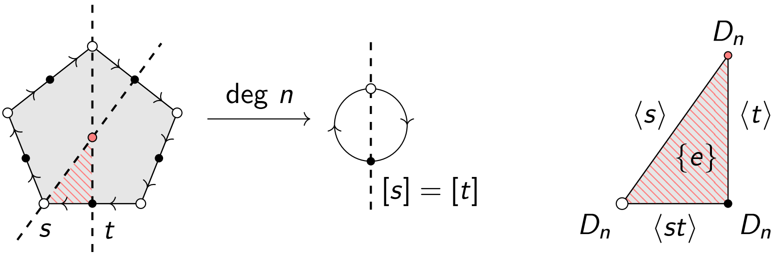

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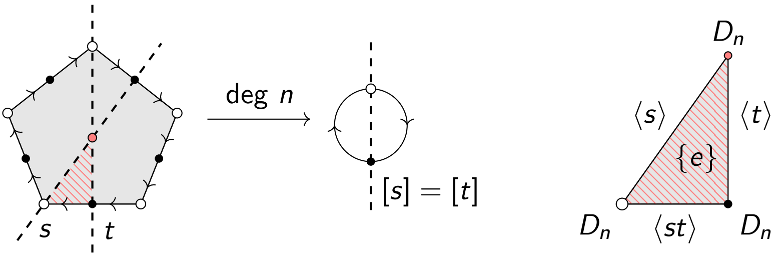

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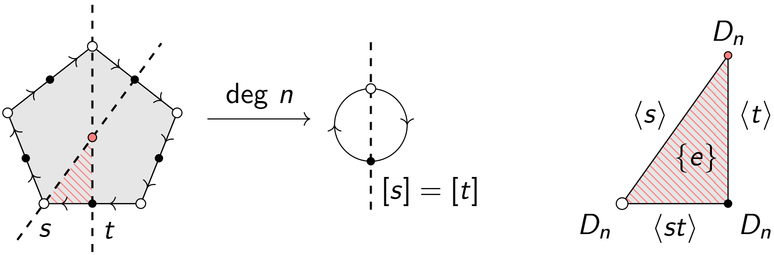

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$$G = W_L \rtimes F \;\Rightarrow\; \mathrm{vcd}\, G = 4 \quad \text{and} \quad \underline{\mathrm{cd}}\, G = 5.$$

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THANK YOU