

$(3, g)$ Hamiltonian bipartite graphs for a range of vertices and Notations for representing Hamiltonian trivalent graphs and Hamiltonian trivalent bipartite graphs

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Abstract

We present new notations for representing Hamiltonian trivalent graphs and Hamiltonian trivalent bipartite graphs that have advantages over the LCF Notation.

We introduce a notation for representing Hamiltonian trivalent bipartite graphs called D3 chord index graph Notation and a notation for representing Hamiltonian trivalent graphs called E3 chord index graph Notation. We define a parameter called symmetry factor b for a Hamiltonian trivalent graph with $2m$ vertices, where $b|m$ that reflects the extent of rotational symmetry. We describe algorithms to construct D3 chord index graphs and E3 chord index graphs. We introduce criteria to check whether a D3 chord index graph or a E3 chord index graph is trivalent. We also show the advantages of D3 chord index graph notation and E3 chord index graph notation over the LCF notation and Extended LCF Notation. We present some mathematical theorems for the condition of a D3 chord index graph representing a $(3, g)$ Hamiltonian bipartite graph. We show that the known $(3, 6)$, $(3, 8)$, $(3, 10)$, $(3, 12)$ cages and the known $(3, 14)$ record graph with 384 vertices can be represented with D3 chord index graph notation. We have independently obtained a $(3, 14)$ record graph with 384 vertices which can be represented with D3 chord index notation that is isomorphic to the known $(3, 14)$ record graph with 384 vertices found by Exoo.

We present a range of $(3, g)$ Hamiltonian bipartite graphs for a range for vertices for even values of girth g satisfying $6 \leq g \leq 16$ and present a new methodology to analyze the trivalent cage problem for even girth. The motivation for this work is one of the open problems for the Cage Problem mentioned in [1]. We quote the open problem from [1], "Finding an infinite family of trivalent graphs with large girth g and order 2^{cg} for $c < 3/4$."

Thus, these results shed new light on open questions in the Cage Problem.

References

- [1] Geoffrey Exoo and Robert Jajcay, Dynamic Cage Survey, The Electronic Journal Of Combinatorics 18 (2011), DS16.
- [2] Vivek S. Nittoor and Reiji Suda, Notations For Representing Hamiltonian Trivalent Bipartite Graphs And Hamiltonian Trivalent Graphs, Draft.

- [3] Vivek S. Nittoor and Reiji Suda, $(3, g)$ Hamiltonian Bipartite Graphs For A Range Of Vertices, Draft.