Mobile Facility Location

Sara Ahmadian, University of Waterloo
Joint work with Zachary Friggstad, Chaitanya Swamy

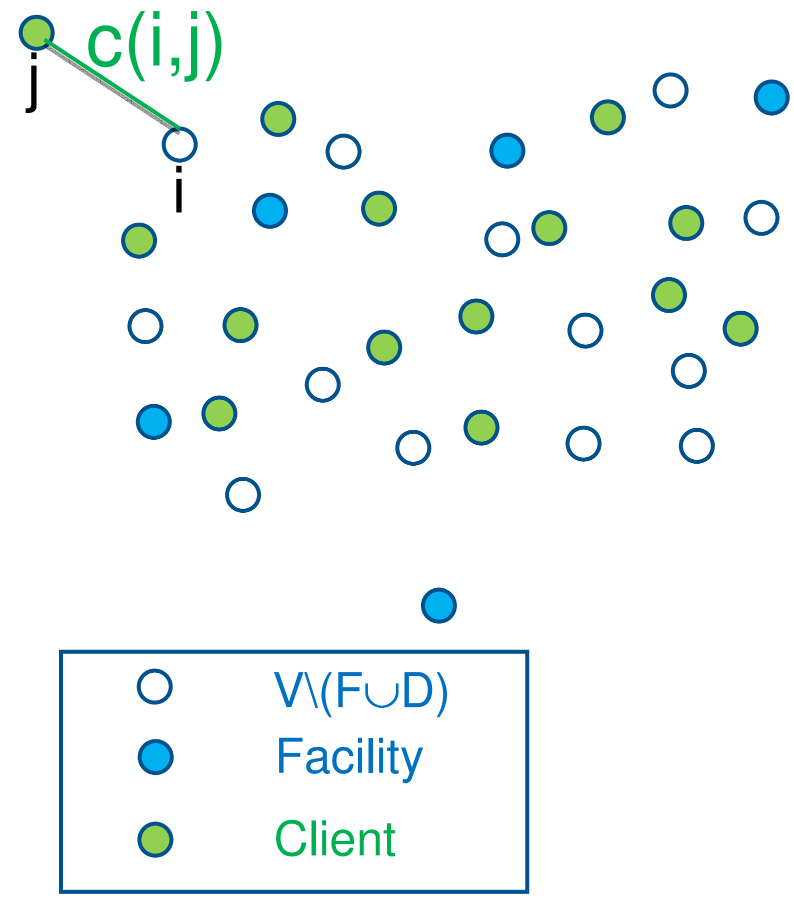
Mobile Facility Location(MFL)

- We are given:
  - $F \subseteq V$: k initial facility locations.
  - $D \subseteq V$: set of clients.

Located in common metric space G=(V,c).

- c(i,j) : cost of moving from point i to point j. We assume that c(i,j)s form a metric.

Sample Example:



Goal

- Find solution **S={s$_1$,s$_2$,…,s$_k$}**
  - Moves each facility i to location s_i in S incurring movement cost of $c(i,s_i)$:

$$\sum_{s_i \in S} c(i, s_i) = \sum_{i \in F} f_i$$

  - Assign each client to the closest location in S:

$$\sum_{j \in D} c(j, S) = \sum_{j \in D} c_j$$

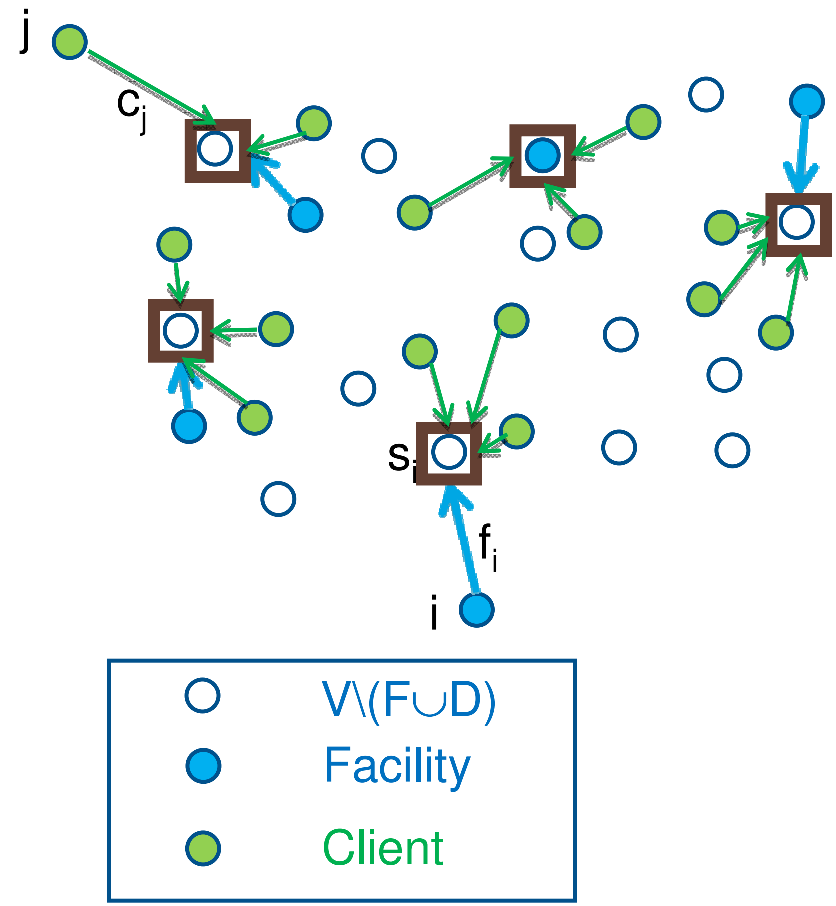
- Goal: minimize total cost

$$\sum_{i \in F} f_i + \sum_{j \in D} c_j$$

.

$$OPT = \sum_{i \in F} f_i^* + \sum_{j \in D} c_j^*$$

Sample Example:

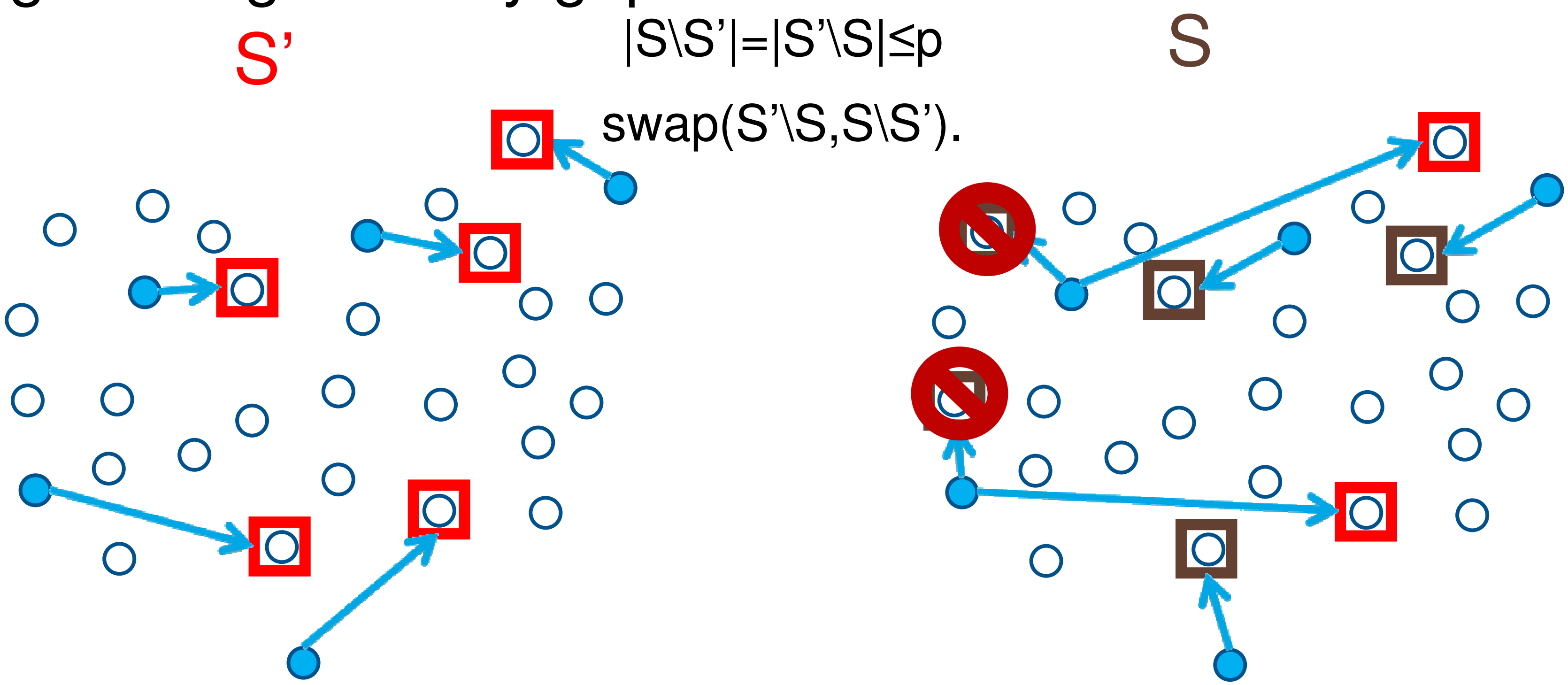


Our work

- We give (3+ε)-approximation for any constant ε>0 based on local-search (first combinatorial algorithm).
  - Previous best: 8-approximation due to Friggstad and Salavatipour [FS] based on LP-rounding.
- Extension to weighted generalization: the movement cost for facility i is w_i times the distance travelled by i.

Local Search

- At each step, swap in and swap out a fixed number (say p) of locations and rematch all initial locations to new set of locations. Without rematching, we may get a large locality gap.

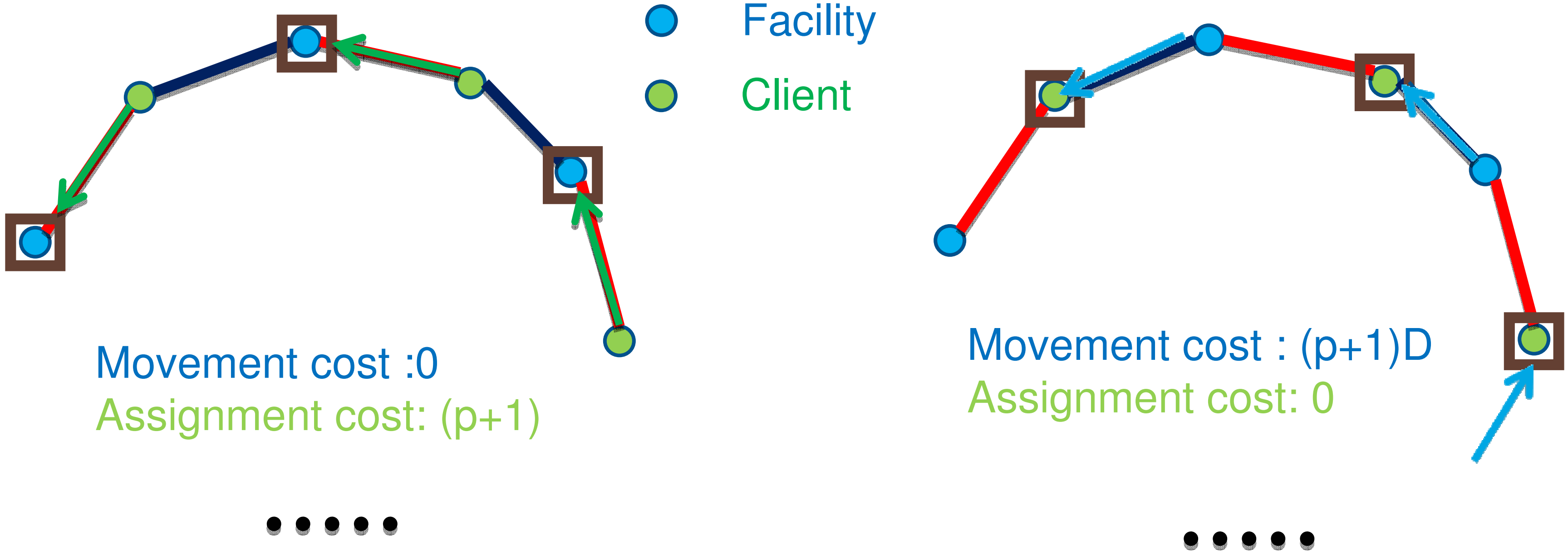


Analysis

- Generate inequalities using test swaps. Each individual inequality may have expensive terms but can amortize the cost by suitably combining the inequalities.
- Use “recursion” tree to identify a suitable collection of path swaps.

Different metrics for facilities and clients

- Local search has big locality gap if facilities and clients move in different metrics.



(1,∞) Red edge: assignment cost:1 and movement cost: ∞.

(∞,D) Blue edge: assignment cost: ∞ and movement cost: D.

Open Questions

- Better approximation ratio?
  - $1+\sqrt{3}$-approximation for k-median by Shi Li and Ola Svensson.
- Combinatorial algorithm for matroid median.
- What if we use approximate matching between initial locations and final locations instead of optimal matching?
- Reducing approximation ratio for single swap case.

Thank You

QUESTIONS?