Down a Rabbit Hole:
From Network Design to Network Modelling

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Based on multiple projects with:
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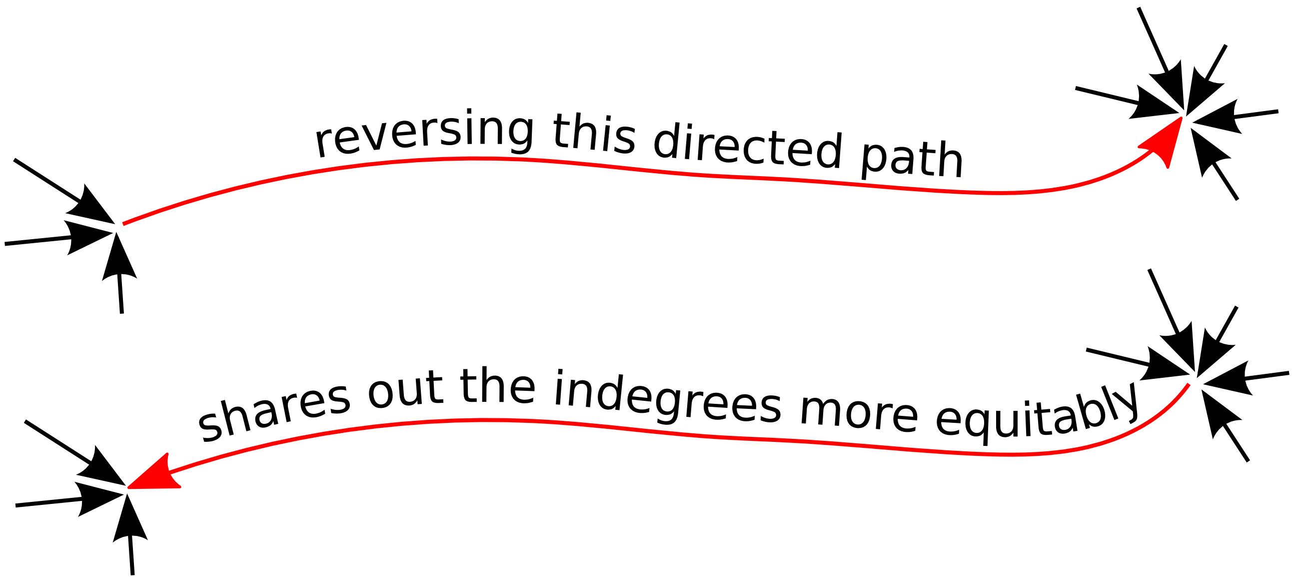
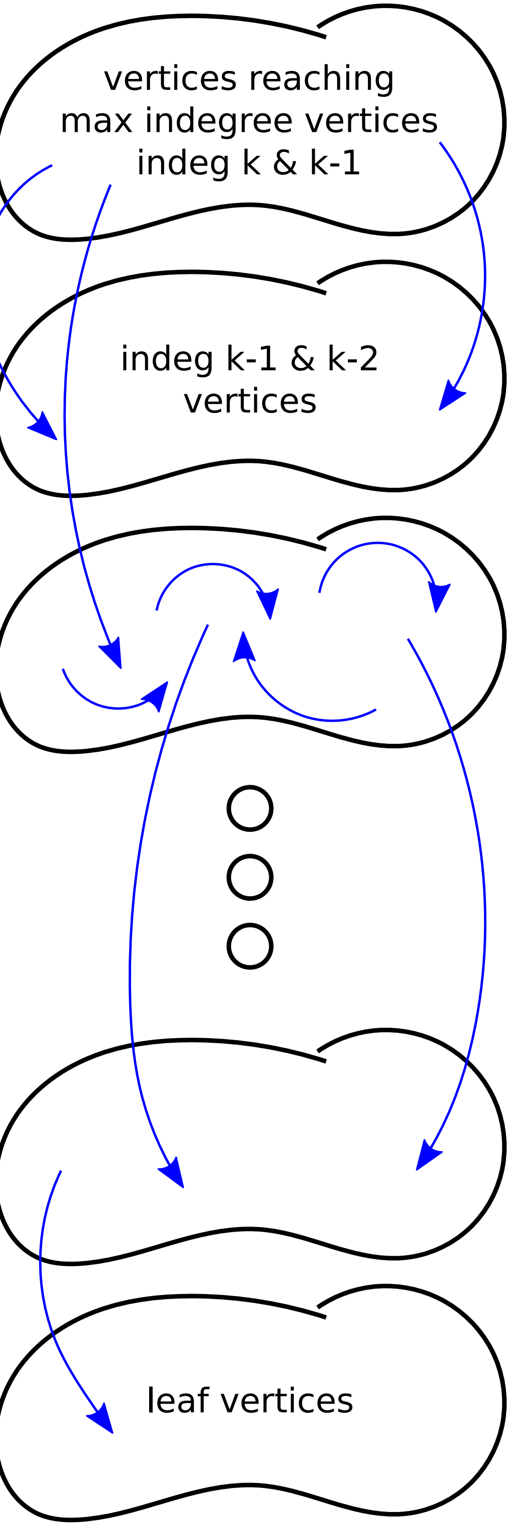




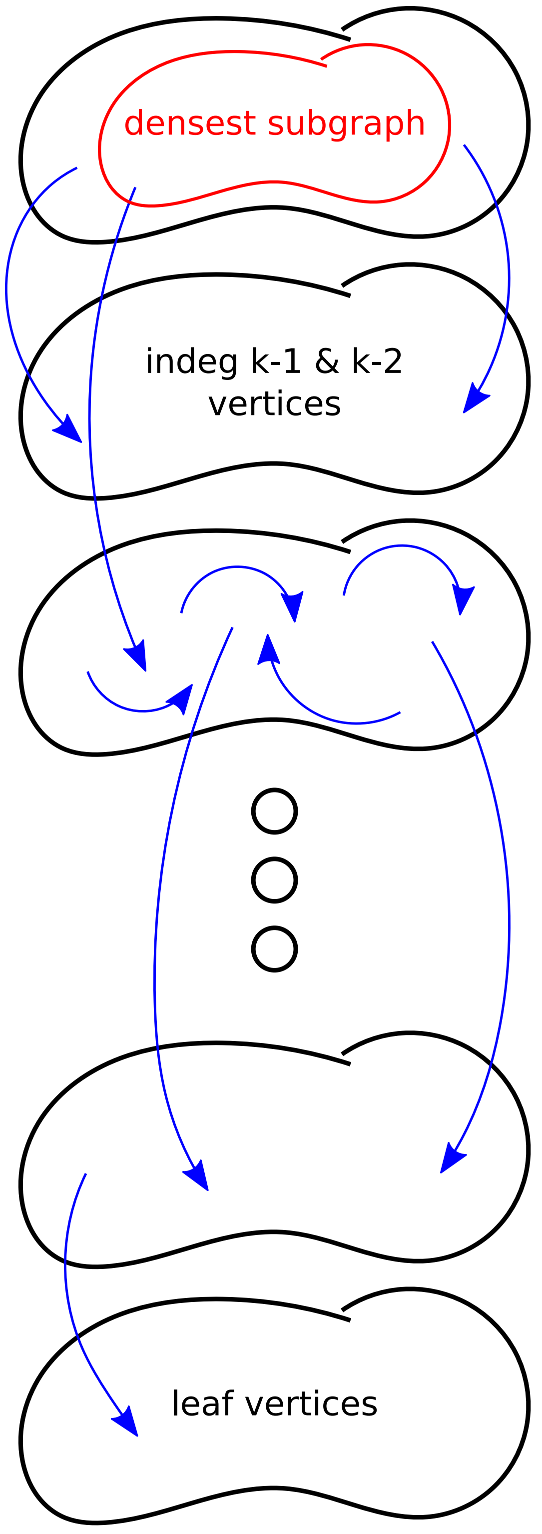
Start with any orientation.
Repeatedly reverse low-to-high indegree directed paths.

Results in an indegree sequence that is lexicographically minimum.

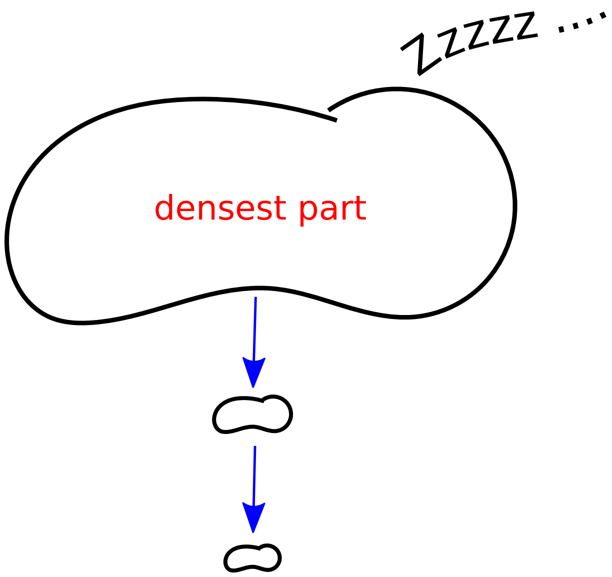
And a natural decomposition of the vertices.

Density Decomposition

- reflects areas of decreasing density
- unique



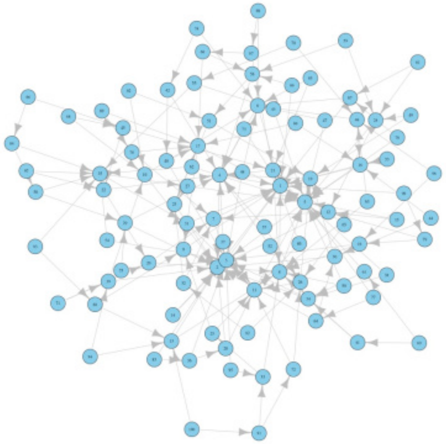
Network models have boring density decompositions



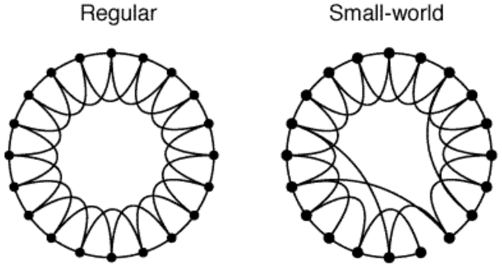
$G_{n,p}$:
Almost everything in densest part.

(pn)

Preferential attachment:
All but O(1) in densest part.



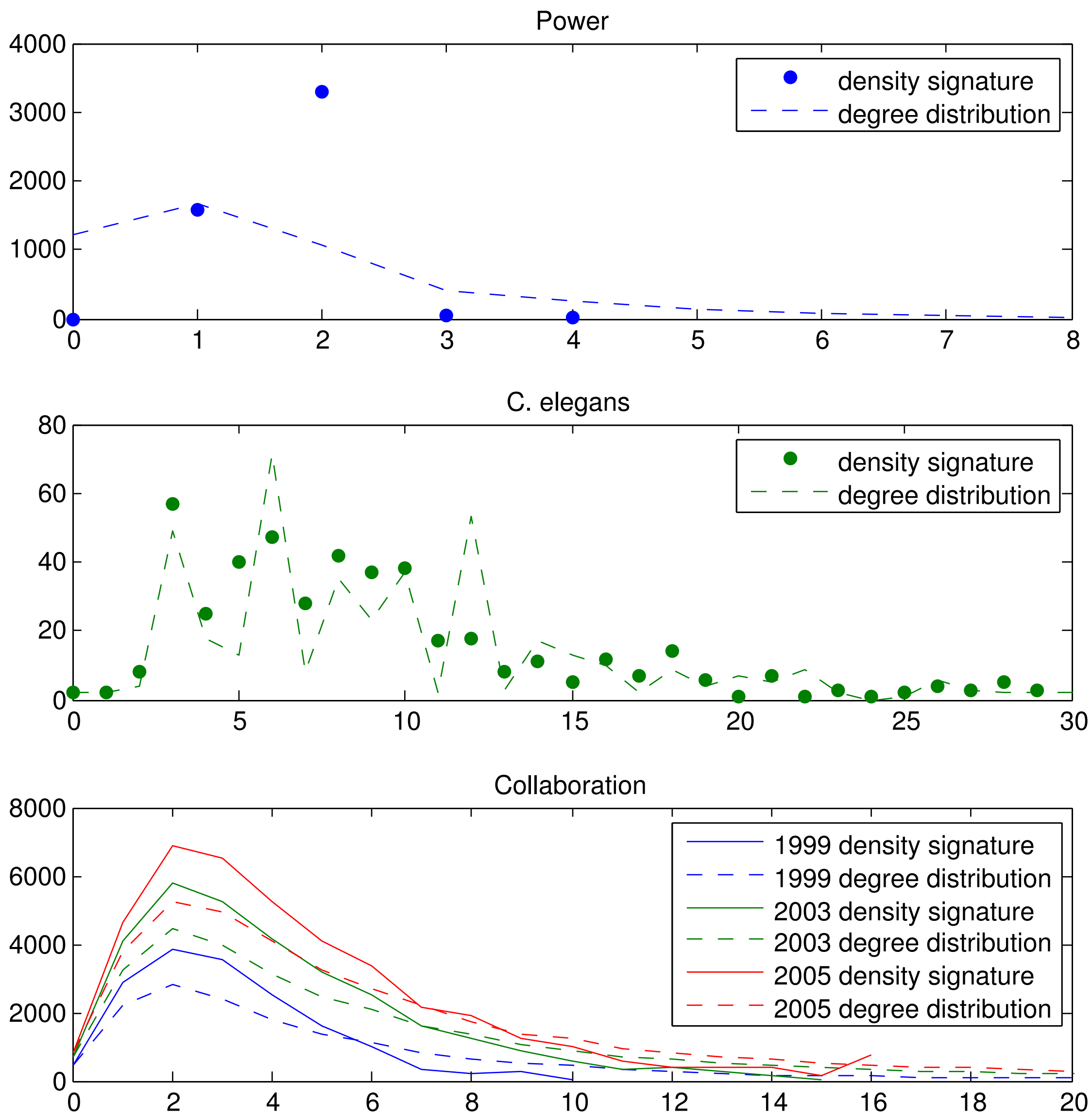
Small world:
All in densest part.

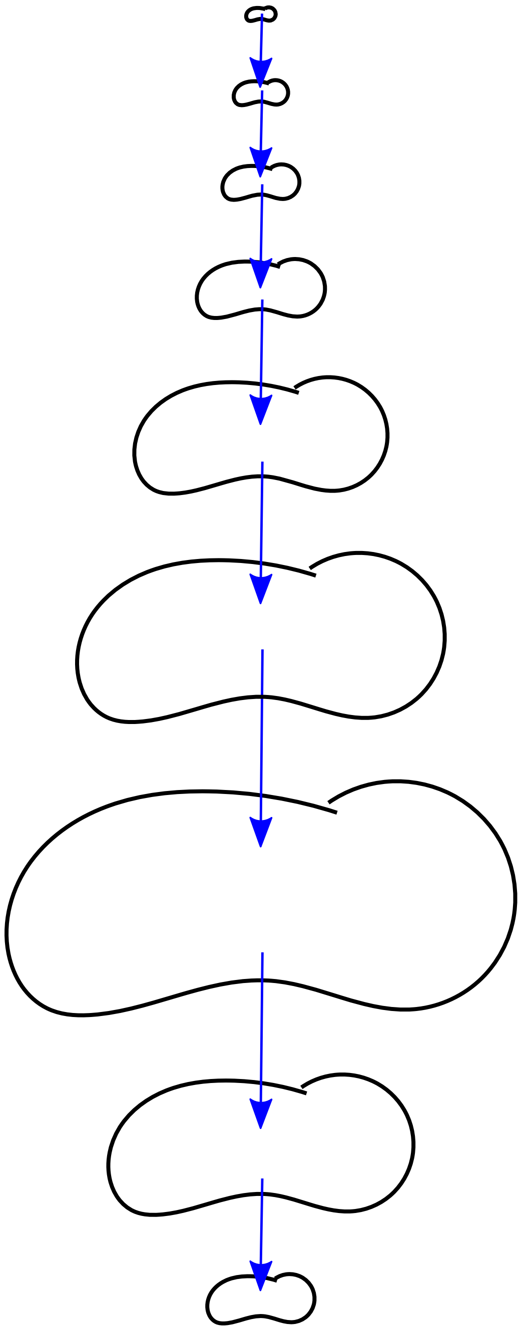


How realistic are network models?

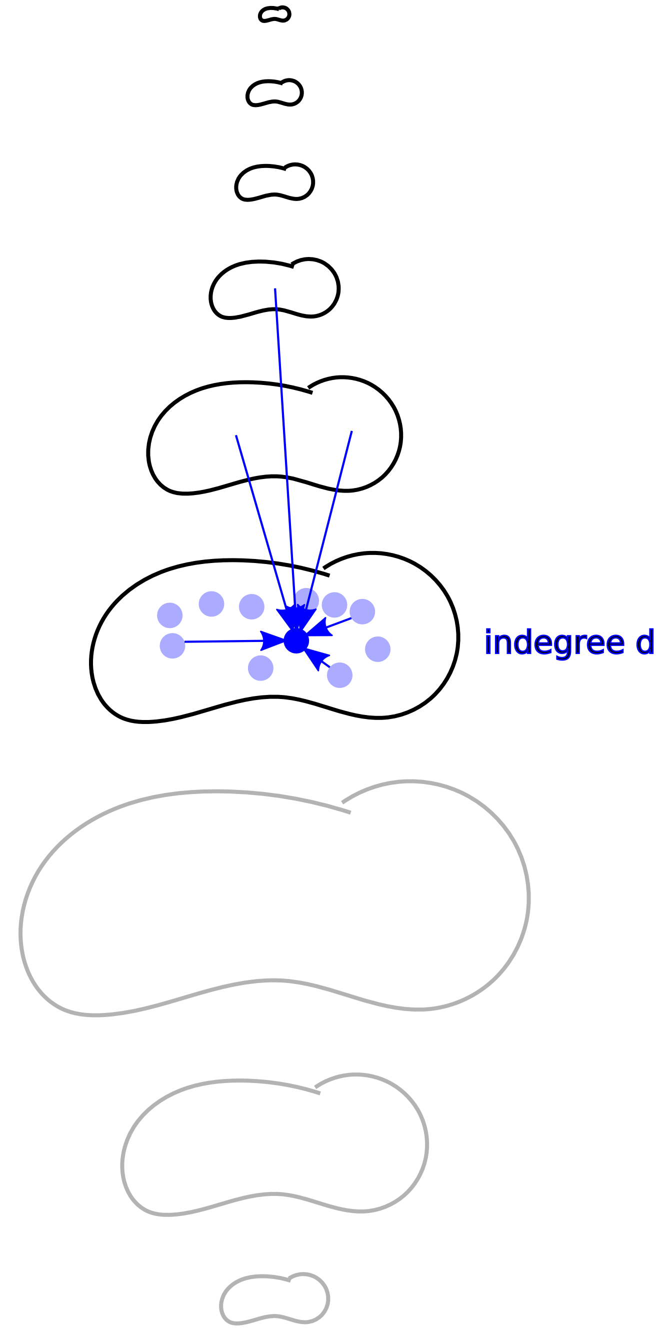
	Some accepted characteristic properties			
	Degree Distribution	Diameter	Clustering (how many neighbors are neighbors)	**Density Decomposition**
$G_{n,p}$	normal	low	very low	**trivial**
Preferential attachment	power law	low	very low	**trivial**
Small world	regular/normal	high to low	high to low	**trivial**
Real networks	debatable	low (usually)	high (usually)	**non-trivial**

Real networks have interesting density decompositions



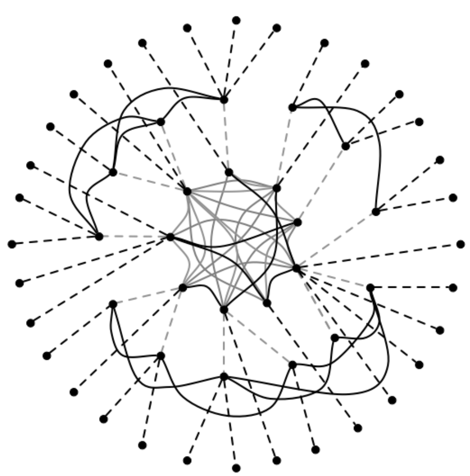


Real networks have interesting density decompositions



Compared to synthetic networks, real networks have
↳ many more edges within layers
↳ "small world" behaviour within layers
↳ "preferential attachment" behaviour between layers

Real networks are preferentially attached small worlds?



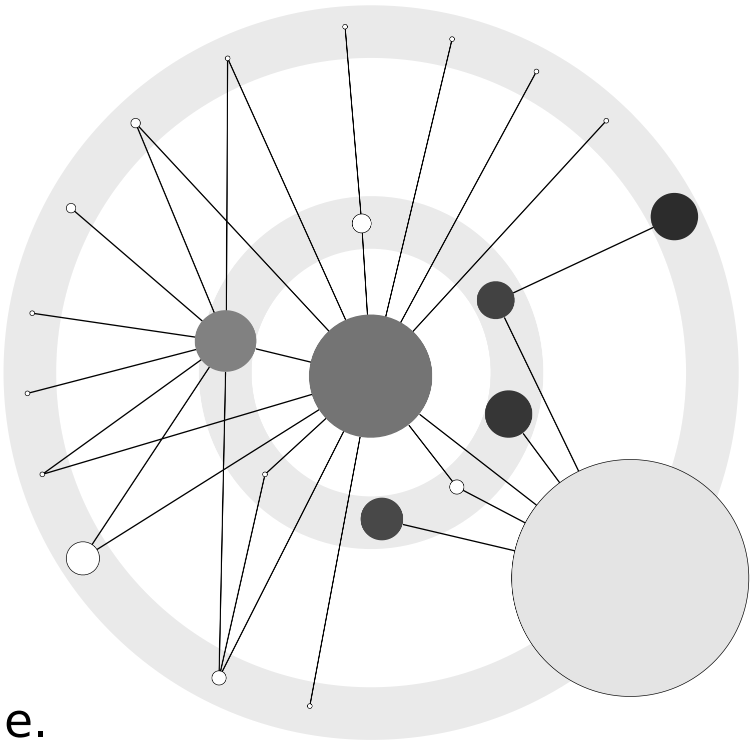
Case study: collaboration networks

Longitudinal AMS co-authorship network spanning 20 years.

Density decomposition is stable:
people move slowly up through the layers over time, and
people join at a layer near their coauthors.

High-impact people are in denser layers?

Case study: power grids

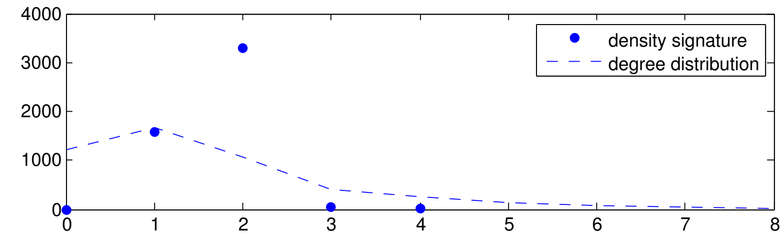


Goals:

↳ Generate realistic synthetic networks.
↳ Determine structural properties to identify:
- where add new resources to grid
- where to add resiliency
- ease smart-grid operation

Observations:

↳ Grid is low treewidth (Western US < 15).
↳ Transmission layer is very low treewidth (2-4).
↳ Density decomposition splits roughly by voltage.



Open theoretical questions?

- ↳ Prove that most of $G_{n,p}$ is densest.
- ↳ Prove a relationship between degree distribution and density decomposition.
- ↳ Give a single number that captures the triviality of a density signature.
- ↳ Give a distance measure between density signature and degree distribution.
- ↳ Is there a **simple** network model that captures the density decompositions of real networks?
- ↳ Is there a **simple, temporal** network model that captures the evolution of a real network?
- ↳ Faster decomposition algorithm? Is O(m log n) possible?
- ↳ Why does the density decomposition tell us more about a graph than the k-cores decomposition?
- ↳ Algorithms for modifying/augmenting a built network.