
Plenary Talks

J.Chuzhoy (Toyota Technological Institute at Chicago)

Polynomial Bounds for the Grid-Minor Theorem

Abstract: One of the key results in Robertson and Seymour's seminal work on graph minors is the Grid-Minor Theorem (also known as the Excluded Grid Theorem). The theorem states that any graph of treewidth at least k contains a grid minor of size $f(k)$ for some function f . This theorem has found many applications in graph theory and algorithms. The best current quantitative bound, due to recent work of Kawarabayashi and Kobayashi, and Leaf and Seymour, shows that $f(k) = \Omega(\sqrt{\log k / \log \log k})$, while the best known upper bound implies that $f(k) = O(\sqrt{k / \log k})$. In this talk, we present the first polynomial relationship between treewidth and grid-minor size, by showing that $f(k) = \Omega(k^\delta)$ for some fixed constant $\delta > 0$, and also describe an efficient algorithm to construct such a minor.

Joint work with Chandra Chekuri.

N.Harvey (University of British Columbia)

Spectrally Thin Trees

Abstract: Thin trees are intriguing objects in graph theory that relate to foundational topics, such as nowhere-zero flows and the asymmetric traveling salesman problem (ATSP). A spanning subtree T of a graph G is called α -thin if every cut in T contains at most an α -fraction of G 's edges in that cut.

Asadpour, et al., gave an algorithm that constructs an $O(\log n / k \log \log n)$ -thin tree in any graph with n vertices and edge-connectivity k . Improving this to $O(1/k)$ would imply a constant-factor approximation algorithm for ATSP.

We define a stronger notion of thinness that is naturally motivated by spectral sparsification of graphs. A spanning subtree T of G is called α -spectrally-thin if T 's Laplacian matrix is upper-bounded by α times G 's Laplacian matrix in the Lowner ordering.

We give a deterministic, polynomial-time algorithm to construct an $O(\log n / c \log \log n)$ -spectrally thin tree in any graph with n vertices and for which the effective conductance across each edge is at least c .

Remarkably, this can be improved to $O(1/c)$ if one does not require an efficient algorithm. The recent breakthrough results of Marcus, Spielman and Srivastava imply the existence of a $O(1/c)$ -spectrally-thin tree.

This is joint work with Neil Olver (MIT / Vrije Universiteit).

L.C.Lau (The Chinese University of Hong Kong)

Analysis of Spectral Partitioning Through Higher Order Spectral Gap

Abstract: Let $\phi(G)$ be the minimum conductance of an undirected graph G , and let $0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n \leq 2$ be the eigenvalues of the normalized Laplacian matrix of G . We prove that for any graph G and any $k \geq 2$, $\phi(G) = O(k)\lambda_2/\sqrt{\lambda_k}$, and this performance guarantee is achieved by the spectral partitioning algorithm. This improves Cheeger's inequality, and the bound is optimal up to a constant factor for any k . The result shows that the spectral partitioning algorithm is a constant factor approximation algorithm for finding a sparse cut if λ_k is a constant for some constant k . This provides some theoretical justification to its empirical performance in image segmentation and clustering problems. The analysis can be extended to other graph partitioning problems, including multi-way partition, balanced separator, and maximum cut.

T.Rothvoss (Massachusetts Institute of Technology)

Approximating Bin Packing within $O(\log OPT \log \log OPT)$ bins

Abstract: For bin packing, the input consists of n items with sizes s_1, \dots, s_n in $[0, 1]$ which have to be assigned to a minimum number of bins of size 1. The seminal Karmarkar-Karp algorithm from 1982 produces a solution with at most $OPT + O(\log^2 OPT)$ bins.

We provide the first improvement in now 3 decades and show that one can find a solution of cost $OPT + O(\log OPT * \log \log OPT)$ in polynomial time. This is achieved by rounding a fractional solution to the Gilmore-Gomory LP relaxation using the Entropy Method from discrepancy theory. The result is constructive via algorithms of Bansal and Lovett-Meka.