Lecture IV: Option pricing in energy markets

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Introduction

• OTC market huge for energy derivatives
• Highly exotic products:
  • Asian options on power spot
  • Various (cross-commodity) spread options
  • Demand/volume triggered derivatives
  • Swing options
• Payoff depending on spot, indices and/or forwards/futures
• In this lecture: Pricing and hedging of (some) of these exotics
Example of swing options

- Simple operation of a gas-fired power plant: income is
  \[
  \int_t^T e^{-r(s-t)} u(s) (P(s) - G(s)) \, ds
  \]

- \( P \) and \( G \) power and gas price resp, in Euro/MWh.
  - Heating rate is included in \( G \)... 
- \( 0 \leq u(s) \leq 1 \) production rate in MWh
  - Decided by the operator

- Value of power plant

  \[
  V(t) = \sup_{0 \leq u \leq 1} \mathbb{E} \left[ \int_t^T e^{-r(s-t)} u(s) (P(s) - G(s)) \, ds \mid \mathcal{F}_t \right]
  \]

- More fun if there are constraints on production volume....
  - Maximal and/or minimal total production
  - Flexible load contracts, user-time contracts
• Tolling agreement: virtual power plant contract
  • Strip of European call on spread between power spot and fuel
  • Fuel being gas or coal

\[ V(t) = \int_t^\tau e^{-r(s-t)}E\left[ \max(P(s) - G(s), 0) \mid \mathcal{F}_t \right] ds \]

• Spark spread, the value of exchanging gas with power
  • Dark spread, crack spread, clean spread....
German (EEX) spark spread in 2011

- Green: EEX power (Euro/MWh)
- Blue: Natural gas (Euro/MWH)
- Red: Spark spread, with efficiency factor (heat rate) of 49.13%
Example: Asian options

- European call option on the average power spot price

\[
\max \left( \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) \, du - K, 0 \right)
\]

- Traded at NordPool around 2000
  - "Delivery period" a given month
  - Options traded until \( \tau_1 \), beginning of "delivery"
Example: Energy quanto options

- Extending Asian options to include volume trigger
- Sample payoff

\[
\max \left( \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) \, du - K_P, 0 \right) \times \max \left( \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} T(u) \, du - K_T, 0 \right)
\]

- \( T(u) \) is the temperature at time \( u \)
  - in a location of interest, or average over some area (country)
- Energy quantos on:
  - gas and temperature (demand)
  - or power and wind (supply)
  - Dependency between energy price and temperature crucial
Spread options (tolling agreements)
• Spread payoff with exercise time $\tau$

$$\max (P(\tau) - G(\tau), 0)$$

• $P, G$ bivariate geometric Brownian motion $\rightarrow$ Margrabe’s Formula
  • Introducing a strike $K \neq 0$, no known analytic pricing formula
• Our concern: valuation for exponential non-Gaussian stationary processes
  • Exponential Lévy semistationary (LSS) models
• Recall definition of LSS process from Lecture III

\[ Y(t) = \int_{-\infty}^{t} g(t - s)\sigma(s) \, dL(s) \]

• \( L \) a (two-sided) Lévy process (with finite variance)
• \( \sigma \) a stochastic volatility process
• \( g \) kernel function defined on \( \mathbb{R}_+ \)
• Integration in semimartingale (Ito) sense
  • \( g(t - \cdots) \times \sigma(\cdot) \) square-integrable
• \( Y \) is stationary whenever \( \sigma \) is
  • Prime example: \( g(x) = \exp(-\alpha x) \), Ornstein-Uhlenbeck process
• Bivariate spot price dynamics

\[
\ln P(t) = \Lambda_P(t) + Y_P(t) \\
\ln G(t) = \Lambda_G(t) + Y_G(t)
\]

• \( \Lambda_i(t) \) seasonality function, \( Y_i(t) \) LSS process with kernel \( g_i \) and stochastic volatility \( \sigma_i, i = P, G \)
  - The stochastic volatilites are assumed independent of \( U_P, U_G \)
• \( L = (U_P, U_G) \) bivariate (square integrable) Lévy process
  - Denote cumulant (log-characteristic function) by \( \psi(x, y) \).
• We suppose that spot model is under \( Q \)
  - Pricing measure
Fourier approach to pricing

- To compute the expected value under $Q$ for the spread:
- Factorize out the gas component

$$\Lambda_P(\tau)\mathbb{E} \left[ e^{Y_G(\tau)} \left( e^{Y_P(\tau)-Y_G(\tau)} - h \frac{\Lambda_G(\tau)}{\Lambda_P(\tau)} \right)^+ \mid \mathcal{F}_t \right]$$

- Apply the tower property of conditional expectation, conditioning on $\sigma_i$,
  - Recall being independent of $L$
  - $\mathcal{G}_t = \mathcal{F}_t \vee \{\sigma_i(\cdot), i = P, G\}$

$$\Lambda_P(\tau)\mathbb{E} \left[ \mathbb{E} \left[ e^{Y_G(\tau)} \left( e^{Y_P(\tau)-Y_G(\tau)} - h \frac{\Lambda_G(\tau)}{\Lambda_P(\tau)} \right)^+ \mid \mathcal{G}_t \right] \mid \mathcal{F}_t \right]$$
• For inner expectation, use that $Z$ is the density of an Esscher transform for $t \leq \tau$

\[ Z(t) = e^{\int_{-\infty}^{t} g_G(\tau-s)\sigma_G(s) \, dU_G(s) - \int_{-\infty}^{t} \psi_G(-ig_G(\tau-s)\sigma_G(s)) \, ds} \]

• $\psi_G(y) = \psi(0, y)$, the cumulant of $U_G$.
• The characteristics of $L$ is known under this transform

• This "removes" the multiplicative term $\exp(Y_G(\tau))$ from inner expectation

• Finally, apply Fourier method
• Define, for $c > 1$,

$$f_c(x) = e^{-cx} \left( e^x - h \frac{\Lambda_G}{\Lambda_P} \right)^+$$

• $f_c \in L^1(\mathbb{R})$, and its Fourier transform $\hat{f}_c \in L^1(\mathbb{R})$

• Representation of $f_c$:

$$f_c(x) = \frac{1}{2\pi} \int_{\mathbb{R}} \hat{f}_c(y) e^{iyx} \, dy$$

• Gives general representation for a random variable $X$

$$\mathbb{E}[f(X)] = \frac{1}{2\pi} \int_{\mathbb{R}} \hat{f}_c(y) \mathbb{E} \left[ e^{i(y-ic)X} \right] \, dy$$
Theorem

Suppose exponential integrability of L. Then the spread option has the price at $t \leq \tau$

$$C(t, \tau) = e^{-r(\tau-t)} \frac{\Lambda_P(\tau)}{2\pi} \int_{\mathbb{R}} \hat{f}_c(y) \Phi_c \left( Y_P(t, \tau), Y_G(t, \tau) \right) \Psi_{c,t,\tau}(y) \, dy$$

where, for $i = P, G$,

$$Y_i(t, \tau) = \int_{-\infty}^{t} g_i(\tau - s) \sigma_i(s) \, dU_i(s)$$

$$\Phi_c(u, v) = \exp \left( (y - ic)u + (1 - (iy + c))v \right)$$

and

$$\Psi_{c,t,\tau}(y) = \mathbb{E} \left[ e^{\int_{t}^{\tau} \psi((y-ic)g_P(\tau-s)\sigma_P,((c-1)i-y)g_G(\tau-s)\sigma_G(s)) \, ds} \mid \mathcal{F}_t \right]$$
• Note: spread price not a function of the current power and gas spot, but on $Y_i(t, \tau), i = P, G$

• Recalling theory from Lecture III: $Y_i(t, \tau)$ is given by the logarithmic forward price...

$$\ln f_i(t, \tau) = X_i(t, \tau, \sigma_i(t)) + Y_i(t, \tau)$$

• No stochastic volatility, $\sigma_i = 1$: $X_i$ is a deterministic function
Some remarks on hedging

- Power spot not tradeable, gas requires storage facilities
- Alternatively, hedge spread option using forwards!
- But incomplete model, so only partial hedging possible
  - Quadratic hedging, for example
  - May also depend on stochastic volatility, making model ”more incomplete”
- In real markets: forwards on power and gas deliver over a given time period
  - Further complication, as we cannot easily express spread in such forwards
  - Further approximation of partial hedging strategy
Asian options
Asian options

- Options on the average spot price over a period
  - Traded at NordPool up to around 2000 for "monthly periods"
- Recall payoff function

$$\max\left(\frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) \, du - K, 0\right)$$

- Geometric LSS spot model:

$$\ln S(t) = \Lambda(t) + Y(t)$$

- $Y$ an LSS process with kernel $g$ and stochastic volatility $\sigma$
Pricing requires simulation

- Propose an efficient Monte Carlo simulation of the path of an LSS process

Suppose that $g_{\lambda}(u) := \exp(\lambda u)g(u) \in L^1(\mathbb{R})$ and its Fourier transform is in $L^1(\mathbb{R})$

$$Y(t) = \frac{1}{2\pi} \int_{\mathbb{R}} \hat{g}_{\lambda}(y) \hat{Y}_{\lambda,y}(t) \, dy$$

- $\hat{Y}_{\lambda,y}(t)$ complex-valued Ornstein-Uhlenbeck process

$$\hat{Y}_{\lambda,y}(t) = \int_{-\infty}^{t} e^{(iy-\lambda)(t-s)}\sigma(s) \, dL(s)$$
• Paths of Ornstein-Uhlenbeck processes can be simulated iteratively

\[ \hat{Y}_{\lambda,y}(t+\delta) = e^{(iy-\lambda)\delta} \hat{Y}_{\lambda,y}(t) + e^{(iy-\lambda)\delta} \int_t^{t+\delta} e^{(iy-\lambda)(t-s)} \sigma(s) dL(s) \]

• Numerical integration (fast Fourier) to obtain paths of \( Y \)
  • Extend \( g \) to \( \mathbb{R} \) if \( g(0) > 0 \)
  • Let \( g(u) = 0 \) for \( u < 0 \) if \( g(0) = 0 \).
  • Smooth out \( g \) at \( u = 0 \) if singular in origo

• Error estimates in \( L^2 \)-norm of the paths in terms of time-stepping size \( \delta \)
- Asian call option on $Y$ over $[0, 1]$, with strike $K = 5$
- $Y$ BSS-process, with $\sigma = 1$, $Y(0) = 10$, and kernel (modified Bjerksund model)

$$g(u) = \frac{1}{u + 1} \exp(-u)$$
Issues of hedging

• Let \( F(t, \tau_1, \tau_2) \) be forward price for contract delivering power spot \( S \) over \( \tau_1 \) to \( \tau_2 \): At \( t = \tau_2 \),

\[
F(\tau_2, \tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) \, du
\]

• Asian option: call option on forward with exercise time \( \tau_2 \)
• In power and gas, forwards are traded with delivery period
  • Hence, can price, but also *hedge* using these
  • Analyse based on forward price model rather than spot!
• Problem: many contracts are *not* traded in the settlement period
  • Can hedge up to time \( \tau_1 \)
  • ..but not all the way up to exercise \( \tau_2 \)
Example: quadratic hedging

- Hedge option with payoff $X$ at exercise $\tau_2$, using $\psi(s)$ forwards
- Assume Levy (jump) dynamics for the forward price
  - Martingale dynamics
- Can only trade forward up to time $\tau_1 < \tau_2$

$$V(t) = V(0) + \int_0^{t \wedge \tau_1} \psi(s)dF(s)$$

$$+ \mathbf{1}_{\{t > \tau_1\}} \psi(\tau_1)(F(t, \tau_1, \tau_2) - F(\tau_1, \tau_1, \tau_2))$$

- Predictable strategies $\psi$ being integrable with respect to $F$. 
• Minimize quadratic hedging error

\[ \mathbb{E}[(X - V(\tau_2))^2] \]

• Solution:
  - Classical quadratic hedge up to time \( \tau_1 \),
  - thereafter, use the constant hedge

\[ \psi_{\min} = \frac{\mathbb{E}[X(F(\tau_2) - F(\tau_1)) | \mathcal{F}_{\tau_1}]}{\mathbb{E}[(F(\tau_2) - F(\tau_1))^2 | \mathcal{F}_{\tau_1}]} \]
Example: geometric Brownian motion

- $X$ call option with strike $K$ at time $\tau_2$
- $t \mapsto F(t, \tau_1, \tau_2)$ geometric Brownian motion with constant volatility $\sigma$
  - We suppose the forward is tradeable only up to time $\tau_1 < \tau_2$
- Quadratic hedge
  - $N$ is the cumulative standard normal distribution function
    \[
    \psi(t) = \begin{cases} 
    N(d(t)), & t \leq T_1 \\
    \psi_{\text{min}}, & t > T_1 
    \end{cases}
    \]
  - $\psi_{\text{min}}$ is given by
    \[
    \psi_{\text{min}} = \frac{F(\tau_1)e^{\sigma^2(\tau_2 - \tau_1)}N(\sigma \sqrt{\tau_2 - \tau_1} + d(\tau_1)) - (K + F(\tau_1))N(d(\tau_1)) + K N(d(\tau_1) - \sigma \sqrt{\tau_2 - \tau_1})}{F(\tau_1)(e^{\sigma^2(\tau_2 - \tau_1)} - 1)}
    \]
  - $d(t)$ is given by
    \[
    d(t) = \frac{\ln(F(t, \tau_1, \tau_2)/K) + 0.5\sigma^2(\tau_2 - t)}{\sigma \sqrt{\tau_2 - t}}
    \]
• **Empirical example:**
  - Annual vol of 30%, $\tau_1 = 20$, $\tau_2 = 40$ days
  - ATM call with strike 100
  - Quadratic hedge jumps 1.8% up at $\tau_1$ compared to delta hedge
Quanto options
• Recall payoff of an energy quanto option

\[
\max \left( \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u) \, du - K_P, 0 \right)
\times \max \left( T_{\text{index}}(\tau_1, \tau_2) - K_T, 0 \right)
\]

• \( T_{\text{index}}(\tau_1, \tau_2) \) temperature index measured over \([\tau_1, \tau_2]\)
  • CAT index, say, or HDD/CDD

• Consider idea of viewing the contract as an option on two forwards
  • Product of two calls,
  • One on forward energy, and one on temperature (CAT forward)

• Main advantages
  • Avoid specification of the risk premium in the spot modelling
  • Can price ”analytically” rather than via simulation
Case study: bivariate GBM

- Consider bivariate GBM model

\[
dF_P(t, \tau_1, \tau_2) = \sigma_P(t, \tau_1, \tau_2)F_P(t, \tau_1, \tau_2) \, dW_P(t) \\
dF_T(t, \tau_1, \tau_2) = \sigma_T(t, \tau_1, \tau_2)F_T(t, \tau_1, \tau_2) \, dW_T(t)
\]

- \(\sigma_P, \sigma_T\) deterministic volatilities, \(W_P, W_T\) correlated Brownian motions

- May express the price of the quanto as a ”Black-76-like” formula
• Price of quanto at time $t \leq \tau_1$ is

$$C(t) = e^{-r(\tau_2-t)} \left\{ F_P(t)F_T(t)e^{\rho \sigma_P \sigma_T} N(d^{**}, d^{***}) \right.$$

$$- F_P(t)K_T N(d^{*}, d^{**}) - F_T(t)K_P N(d^{*}, d^{*})$$

$$+ K_P K_T N(d_P, d_T) \right\}$$

where

$$d_i = \frac{\ln(F_i(t)/K) - 0.5\sigma_i^2}{\sigma_i}, \quad d_i^{**} = d_i + \sigma_i, i = P, T$$

$$d_i^* = d_i + \rho \sigma_j, \quad d_i^{**} = d_i + \rho \sigma_j + \sigma_i, i, j = P, T, i \neq j$$

• $N(\chi, \gamma)$ bivariate cumulative distribution function with correlation $\rho$, equal to the one between $W_P$, and $W_T$

• $\sigma_P$ and $\sigma_T$ integrated volatility

$$\sigma_i^2 = \int_t^{\tau_2} \sigma_i^2(s, \tau_1, \tau_2) \, ds, \quad i = P, T$$
Empirical study of US gas and temperature

- Temperature index in quanto is based on Heating-degree days

\[ T_{\text{index}}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(c - T(u), 0) \, du \]

- \( F_T(t, \tau_1, \tau_2) \) HDD forward
- HDD forward prices for New York
  - Use prices for 7 first delivery months
- NYMEX gas forwards, monthly delivery
  - Use prices for coming 12 delivery months
- 3 years of daily data, from 2007 on
• Approach modelling of \( F(t, \tau_1, \tau_2) \) by \( F(t, \tau) \), forward with fixed maturity date
  • Choose the maturity date \( \tau \) to be middle of delivery period
  • Price dynamics only for \( t \leq \tau \!

• Two factor structure (long and short term variations)

\[
dF_i(t, \tau) = F_i(t, \tau) \left\{ \gamma_i \, dW_i + \beta_i e^{-\alpha_i(\tau-t)} \, dB_i \right\} \quad i = G, T
\]

• Estimate using Kalman filtering
  • \( W \) and \( B \) strongly negatively correlated for both gas and temperature
  • \( W \)'s negatively correlated , \( B \)'s positively
• Compute quanto-option prices from our formula
  • The period $\tau_1$ to $\tau_2$ is December 2011
  • Current time $t$ is December 31, 2010
  • Use market observed prices at this date for $F_G(t), F_T(t)$
• Prices benchmarked against independent gas and temperature
  • Quanto option price is equal to the product of two call options prices, with interest rate $r/2$

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<tr>
<td>independence</td>
<td>470</td>
<td>164</td>
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• Note: long-dated option, long-term components most influential
  • These are negatively correlated, approx. $-0.3$
Conclusions

- European-style options can be priced using transform-based methods
  - Example: spread options
- Path-dependent options require simulation of LSS processes
  - Suggested a method based on Fourier transform
  - Paths simulated via a number of OU-processes
- Considered "new" quanto option
  - Priced using corresponding forwards
  - Case study from US gas and temperature market
- Discussed hedging based on minimizing quadratic hedge error
  - Particular consideration of no-trading constraint in delivery period
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