Lecture I: A primer on power markets

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Overview of lectures

- Goal: give an introduction to the basics of modelling and pricing in energy
  - Focus on power, gas and weather:
    - spot, forwards and options

1. A primer on power
   - the case of NordPool
2. Stochastic volatility modelling in energy markets
   - Gas markets as the case
3. Stationary stochastic models
   - Spot power and weather markets
4. Option pricing in energy markets
   - Asian, spread and quanto options
5. Heath-Jarrow-Morton modelling of energy markets
   - Case study on NordPool data
For background and some of the theory in the course....
The NordPool market

- Stylized facts of power spot prices
- Stylized facts of electricity forwards

- The NordPool market organizes trade in
  - Hourly spot electricity, next-day delivery
  - Forward and futures contracts on the spot
  - European options on forwards

- Covers the Nordic region
  - Norway, Sweden, Denmark and Finland
  - Estonia and Lithuania

- Power production
  - Hydro, nuclear, coal, gas, wind, bio
• Power generation in the NordPool area in 2011: 377.4 TWh

<table>
<thead>
<tr>
<th>Energy source</th>
<th>Denmark</th>
<th>Finland</th>
<th>Norway</th>
<th>Sweden</th>
<th>Sum</th>
<th>Share</th>
<th>Class:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind power</td>
<td>8.9</td>
<td>0.5</td>
<td>1.3</td>
<td>6.1</td>
<td>16.7</td>
<td>4.4%</td>
<td>2</td>
</tr>
<tr>
<td>Other renewable</td>
<td>2.4</td>
<td>10.5</td>
<td>0.0</td>
<td>11.2</td>
<td>24.1</td>
<td>6.4%</td>
<td>2</td>
</tr>
<tr>
<td>Fossil fuels</td>
<td>21.8</td>
<td>24.2</td>
<td>4.8</td>
<td>5.4</td>
<td>56.1</td>
<td>14.9%</td>
<td>1</td>
</tr>
<tr>
<td>Nuclear power</td>
<td>0.0</td>
<td>22.3</td>
<td>0.0</td>
<td>58.0</td>
<td>80.3</td>
<td>21.3%</td>
<td>3</td>
</tr>
<tr>
<td>Hydropower</td>
<td>0.0</td>
<td>12.3</td>
<td>121.4</td>
<td>65.8</td>
<td>199.4</td>
<td>52.9%</td>
<td>2</td>
</tr>
<tr>
<td>Non-identifiable</td>
<td>0.0</td>
<td>0.7</td>
<td>0.0</td>
<td>0.0</td>
<td>0.7</td>
<td>0.2%</td>
<td>4</td>
</tr>
<tr>
<td>Production</td>
<td>33.1</td>
<td>70.4</td>
<td>127.4</td>
<td>146.4</td>
<td>377.4</td>
<td>100.0%</td>
<td></td>
</tr>
</tbody>
</table>
Elspot: the spot market

• A (non-mandatory) hourly market with physical delivery of electricity
  • About 70% of total production traded through NordPool (2010)
• Participants hand in bids before noon *the day ahead*
  • Volume and price for each of the 24 hours next day
  • Maximum of 64 bids within technical volume and price limits
• NordPool creates demand and production curves for each hour of the next day
• Graphical illustration of the power generation stack
- The **system price** is the equilibrium
  - Price for delivery of electricity at a specific hour next day
  - The *daily* system price is the average of the 24 hourly
- Reference price for the forward market
- A series of hourly prices from Friday 21–Friday 28 March, 2008
• Historical daily system price from 1998 until May 2013 (NOK/MWh)
• Local imbalances in production and demand
• Example: In winter Norway may be in deficit of electricity due to cold weather
  • Importing from Sweden mostly
Due to congestion (non-perfect transmission lines), *area prices* are derived

- Finland separate area
- Sweden and Denmark split into two
- Norway may be split into several areas (up to 5)

The area prices are the actual prices for the consumers/producers in the area in question
• Areas and area prices on March 28, 2008
The forward and futures market

• Contracts with “delivery” of electricity over a period
  • Financially settled: The money-equivalent of receiving electricity is paid to the buyer
  • The reference is the hourly system price in the delivery period

• Delivery periods
  • Next day or week (futures-style)
  • Monthly
  • Quarterly (earlier seasons)
  • Yearly

• Overlapping delivery periods (!)
• **Base load contracts**
  • Delivery over all hours in the period

• **Peak load contracts**
  • Delivery over *peak hours* only
  • Peak hours are from 8 to 20 every day
  • Weekends and holidays are excluded

• Also here the futures-style contracts have short delivery period

• **Contracts frequently called *swaps***
  • Fixed for floating spot price
• Monthly (base-load) forward prices up to June 1, 2013
• Dotted line is the system price
The NordPool market

Stylized facts of power spot prices

Stylized facts of electricity forwards

The case of freight rates forwards

- Supramax rates at Baltic Exchange
The option market

- European call and put options on electricity forwards
  - Quarterly and yearly delivery periods
- Low activity on the exchange
- OTC market for electricity derivatives huge
  - Average-type (Asian) options, quanto options, CfD’s, spread options
  - Flexible load contracts, other swing options....
Stylized facts of power spot prices
• Seasonality on different time scales
  • Yearly
  • Weekly
  • Intra-daily

• Plot of NordPool system (spot) price
• Mean-reversion of spot prices
  • Energy prices driven by supply and demand
  • Prices will revert towards an equilibrium level

• However, to what level?
  • A fixed long-term level?
  • A stochastic level?

• Plot of UK PX log-spot prices with running mean
• Mean reversion shows up in the autocorrelation function (ACF)
  • Assuming stationarity in prices

\[ \rho(\tau) = \text{corr} (S(t + \tau), S(t)) \]

• Empirically, ACF’s are often representable as sums of exponentials,
• This means that we have several scales of mean-reversion
  • Fast due to spikes
  • Medium and slow due to “normal” price variations
• Points towards several mean-reversion factors in dynamics
• **Empirical ACF of EEX spot prices**
  • Fitted with a sum of two exponentials
  • Multi-scale mean-reversion
• **Spikes** in spot electricity
• **Spike:** A large price increase followed by a rapid reversion back to normal levels
  • Happens within 2-3 days
  • May be of several magnitudes

• **Nord Pool price series**
• Zoom-in of the three biggest spikes in NordPool series
• Note the rapid reversion, and magnitude of the increase
• Spikes occur in winter at Nord Pool
  • Other markets may not have seasonality in spike occurrence
  • e.g. the German energy exchange EEX
• Spikes lead to highly leptokurtic spot price returns
• Example with UK electricity returns
  • Seasonality removed
  • Daily returns
  • Normal probability plot
• Returns are distinctively heavy tailed
  • Extreme events have much higher probability than the normal distribution can explain
• Small variations have higher probability than normal
• The effect of spikes....
  • ...but maybe also stochastic volatility?
  • Deseasonalized EEX logarithmic returns:
Stylized facts of power forward prices
Empirics of power forwards

- Every day at NordPool: Available forward contracts with different delivery periods
  - Given weeks, months, quarters, and years
  - Earlier: blocks (4 weeks) and seasons
  - Changed delivery periods over the years
- Desirable to have a \textit{structured set} of power forward prices for each day
- Why?
  - To facilitate a study of empirical properties of returns
Outline of procedure

- Find a *smooth* curve of forward prices for each day $t$

$$\hat{F}(t, T_1, T_2) \rightarrow F(t, T)$$

- $T$ will be the "delivery time", $t \leq T$
- Compute structured set of forward prices for each day $t$

$$F(t, T) \rightarrow \sum_{T=T_1}^{T_2} F(t, T)/(T_2 - T_1)$$
Constructing forward prices

- Idea is taken from interest-rate theory (forward rate/yield curve)
  - Adams and van Deventer (1994)
  - Benth, Koekebakker, Ollmar (2007)

- Suppose
  \[ F(t, T) = \Lambda(T) + \epsilon(T) \]

- \( \Lambda \) seasonal structure, \( \epsilon \) a polynomial spline, such that...
  1. ...implied forward prices match observed ones (or is in bid-ask spread when price does not exist)
  2. ...has ”least variability” in the sense minimizing
  \[
  \int_0^T [\epsilon''(t, T)]^2 \, dT
  \]
- Fourth order polynomial spline gives the solution
- Example: Data from Jan 2, 1997, seasonal structure given by a spot prognosis
Structured data set

- Available daily prices for the different power forward contracts
  - Collected from Nordic power exchange NordPool
  - Ranging from 1997 to 2005
- 1,750 daily forward curves (weekends and holidays excluded)
- Structured data
  - 7 weekly delivery contracts
  - 10 block contracts (4 weeks length of delivery)
  - 6 seasonal contracts (4 blocks, ie 16 weeks)
- Delivery starts sequentially, first week immediately
- All prices converted to NOK (most data in NOK, a few in Euro)
• Compute logreturns from forward prices

\[ r_i(t) = \ln \frac{F(t, T_{1i}, T_{2i})}{F(t - 1, T_{1i}, T_{2i})} \quad i = 1, \ldots, 23 \]

• General findings are:
  1. Distinct heavy tails across all segments
  2. No significant skewness
  3. Volatilities are in general falling with time to delivery (Samuelson effect)
  4. Significant correlation between different segments (idiosyncratic risk)
• Expected logreturn (left) and volatility (right)
• PP-plot to illustrate the heavy tails
  • NIG = normal inverse Gaussian distribution
• 45° line will be a perfect fit
The normal inverse Gaussian (NIG) distribution

- A normal mean-variance mixture model:
  - Let $Z$ be inverse Gaussian distributed
    \[ f_{IG}(z) = \frac{\delta}{\sqrt{2\pi}} z^{-3/2} \exp \left( \delta \gamma - \frac{1}{2} \left( \delta^2 z^{-1} + \gamma^2 z \right) \right), \quad z > 0 \]
  - Conditional distribution of $X$ is normal:
    \[ X|Z \sim \mathcal{N}(\mu + \beta Z, Z) \]

- $X$ is NIG with parameters $\alpha, \beta, \mu$ and $\delta$, where
  \[ \alpha = \sqrt{\gamma^2 + \beta^2} \]
• Density function

\[ f_{\text{NIG}}(x) = c \exp(\beta(x - \mu)) \frac{K_1\left(\alpha \sqrt{\delta^2 + (x - \mu)^2}\right)}{\sqrt{\delta^2 + (x - \mu)^2}} \]

• \( K_1(x) \) modified Bessel function of the third kind with index one. Normalizing constant \( c \) is known

• \( \mu \) location, \( \beta \) asymmetry, \( \delta \) scale ("volatility"), \( \alpha \) steepness
  • smaller \( \alpha \) yields steeper distribution, and thus fatter tails

\[ \delta > 0, 0 \leq |\beta| < \alpha \]

• Moment generating function (MGF): for \(-\alpha - \beta \leq \theta \leq \alpha - \beta\)

\[ M_{\text{NIG}}(\theta) = \exp\left(\theta \mu + \delta \sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + \theta)^2}\right) \]
- Fitting NIG and normal to logreturns of forwards by maximum likelihood
- Example: first week
• *Shape triangle*: measure the "distance" from normality

\[ \chi = \frac{\beta}{\alpha}, \quad \xi = \left(1 + \delta \sqrt{\alpha^2 - \beta^2}\right)^{-1/2} \]

• Domain for \((\chi, \xi)\): \(|\chi| < \xi < 1\)
• \(\chi\) measures the *asymmetry*
  • \(\chi = 0\) symmetric NIG distribution
• \(\xi\) measures the *steepness* (or, *tail heaviness*)
  • \(\xi \to 1\) gives Cauchy distribution, \(\xi \to 0\) gives normal distribution
• Shape triangle parameters for the forwards
  • Cannot reject symmetric NIG distribution at 5% significant level in any segment
  • The ”steepness” $\xi$ larger than 0.8 in all segments
Correlation between returns

- Correlation between returns for different contracts
  - Various length of delivery
  - Various time to delivery

- Specialize to weekly contracts, over three years
  - Reconstruct weekly-delivery prices from smooth forward curve
  - Total 156 weekly-delivery forward prices every day

- Calculate the empirical correlation as a function of distance between delivery
  - Naturally measured in terms of weeks
  - Study by Frestad (2009)
- Plot of log-correlation as a function of years between delivery
- Correlation decreases in general with distance between delivery
  - ...but in a highly complex way
• A more recent study by Andresen, Koekebakker and Westgaard (2010)
• Analysis of base load quarter contracts constructed from NordPool data
  • The "forward curve" 1 January, 2006
• Observed Samuelson effect
  • Volatility of forwards decrease with time to maturity
  • Reflection of the mean-reversion of the forward price
  • The influence becomes insignificant in the long end of the market

• Plot of Nordpool quarterly contracts, empirical volatility
• Study of the correlation structure of quarterly contracts in NordPool
Summary so far

- Forward curve moves as a stochastic process parametrized by time-to-delivery $x$
  - Also by length-of-delivery
- Increasing volatility as we approach delivery
  - Samuelson effect
- Strong dependencies between maturity times
  - High degree of idiosyncratic risk in the market
- Non-Gaussian distributed logreturns
  - NIG seems to fit nicely
Some remarks on modelling and pricing of electricity

- Situation similar to that of fixed-income markets
- Spot price ↔ short rate of interest
  - Power spot is non-storable!
  - Cannot create portfolios in the spot
- Forward contracts ↔ forward rates
  - ... or at least zero-coupon bonds
- Modelling problem:
  - Spot modeling, to price forwards
  - What is the link between spot and forwards (Lectures II and III)?
  - HJM-approach, that is, direct modeling of forward prices (Lecture V)
Home work...

Ex. 1 Let $X(t)$ be the solution to an Ornstein-Uhlenbeck SDE

$$dX(t) = (\mu - \alpha X(t)) \, dt + \sigma \, dB(t)$$

Find $X(s)$ given $X(t)$, $s \geq t \geq 0$.

Ex. 2 Compute the conditional expectation

$$\mathbb{E}[\exp(X(T)) \mid \mathcal{F}_t]$$

with $T \geq t$. 
References

- Adams and van Deventer (1994). Fitting yield curves and forward rate curves with maximum smoothness. J. Fixed Income 4
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