# Reducing the structural group by using stabilizers in general position

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 $\begin{array}{lll} & \text{Octonion algebra} \\ & \text{Albert algebra} \\ & \text{Freudenthal triple system} \end{array} \begin{array}{lll} 3 \leq \operatorname{ed}(G_2) \leq 3 \\ 5 \leq \operatorname{ed}(F_4) \leq 7 \\ 7 \leq \operatorname{ed}(E_7) \leq 17 \end{array}$ 

# Key techniques for finding upper bounds

Let G be a smooth linear algebraic group over a field F, and V a linear representation.

#### Theorem 1

If V is generically free that  $ed(G) \leq \dim V - \dim G$ .

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#### "Theorem 2"

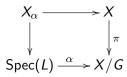
If the stabilizers of points in general position (in V) are all conjugate to the same  $H \subset G$  subgroup, then

$$H^1(L, N_G(H)) \rightarrow H^1(L, G)$$

is surjective for all fields L/F. Hence,  $ed(G) \le ed(N_G(H))$ .

## Classifying torsors

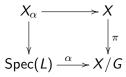
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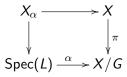
A *G*-torsor is **classifying** if for all infinite fields L/F and all *G*-torsors *T* over *L*, the following set is *dense* in X/G:

$$\{ \alpha \in (X/G)(L) \mid X_{\alpha} \cong T \}$$

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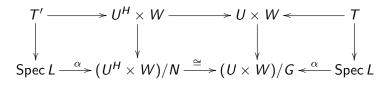
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#### Example

If  $G \subset GL_n$ , then the *G*-torsor  $GL_n \rightarrow GL_n / G$  is classifying.

Assume  $W \to W/G$  is a classifying *G*-torsor induced from a generically free representation, and  $U \subset V$  is an open dense on which the stabilizers are conjugate to *H*.

If T is any G-torsor over L, then we have an N-torsor T' over L:



Then  $[T'] \mapsto [(G \times T')/N] = [T].$ 



$$\begin{array}{ll} 5 \leq \operatorname{ed}(F_4) & \leq \operatorname{ed}(N_{F_4}(\operatorname{Spin}_8)) & \text{Theorem 2} \\ & \leq \operatorname{ed}(N_{N_{F_4}}(\operatorname{Spin}_8)(\operatorname{SL}_3)) & \text{Theorem 2} \\ & \leq 17 - 10 = 7 & \text{Theorem 1} \end{array}$$

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### $E_n$ will denote the split simply connected group

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## Thanks!

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