Feedback of zonal flows on Rossby-wave turbulence driven by small scale instability

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Workshop on Wave Interactions and Turbulence
Fields Institute 23 May 2013
Coexistence of large scale coherent structures and small scale turbulence in geophysical flows

Zonal turbulence on Jupiter (NASA)

Eddy-resolving simulation of Earth's oceans (Earth Simulator Center/JAMSTEC)
Introduction: Rossby wave turbulence
Generation of large scales by modulational instability
Feedback of large scales on small scales

Zonal Flows and turbulence in magnetised plasmas

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Plasma turbulence (L. Villard)

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EPL 96, 25001 (2011)
Outline

1. Introduction: Rossby wave turbulence
2. Generation of large scales by modulational instability
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Charney-Hasegawa-Mima equation driven by small scale "instability"

\[(\partial_t - \mathcal{L}) (\Delta \psi - F \psi) + \beta \partial_x \psi + J[\psi, \Delta \psi] = 0.\]

Operator $\mathcal{L}$ has Fourier-space representation

\[\mathcal{L}_k = \gamma_k - \nu k^{2m}\]

which mimics a small scale instability (and hyperviscosity).

- $\gamma_k$ peaked around $(k_f, 0)$.
- Choose $k_f^2 \sim F$.
- Forcing excites meridionally propagating Rossby waves with wavelength comparable to deformation radius.
Initialise vorticity field with white noise of very low amplitude. What are the dynamics? The following scenario was proposed by Zakharov and coworkers (1990’s):

- Waves having $\gamma_k > 0$ initially grow exponentially.
- When amplitudes get large enough, nonlinearity initiates cascades.
- Inverse cascade transfers energy to large scales leading to zonal jets.
- Jets shear small scale waves generating negative feedback. "Switches off" the forcing.

This talk: is this what really happens?
Weak and strong turbulence regimes

Nonlinearity measured by dimensionless amplitude:

\[ M = \frac{\psi_0 k^3}{\beta} \]

where \( k \) typical scale, \( \psi_0 \) is typical amplitude. Two limits:
- \( M \gg 1 \): Euler limit.
- \( M \ll 1 \): Wave turbulence limit.

Wave turbulence limit is analytically tractable:
- Evolution of turbulence spectrum described by a closed kinetic equation.
- Kinetic equation has exact stationary solutions describing cascades.
Introduction: Rossby wave turbulence

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Wave turbulence kinetic equation

\[
\frac{\partial n_k}{\partial t} = 4\pi \int \left| V_{q,r}^k \right|^2 \delta(k - q - r) \delta(\omega_k - \omega_q - \omega_r) \times 
\left[ n_q n_r - n_k n_q \text{sgn}(\omega_k \omega_r) - n_k n_r \text{sgn}(\omega_k \omega_q) \right] dq \, dr + \gamma_k n_k.
\]

where

\[
\omega_k = -\frac{\beta k_x}{k^2 + F}
\]

\[
V_{q,r}^k = \frac{i}{2} \sqrt{\beta} |k_x q_x r_x| \left( \frac{q_y}{q^2 + F} + \frac{r_y}{r^2 + F} - \frac{k_y}{k^2 + F} \right).
\]

Problem: stationary Kolmogorov solution describing the inverse cascade is non-local.
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Small scale Rossby waves are unstable to large scale modulations (Lorenz 1972, Gill 1973)

- In wave turbulence limit, $M \ll 1$, unstable perturbations concentrate on resonant manifold:
  \[
  p = q + r \\
  \omega(p) = \omega(q) + \omega(p-).
  \]
  
- Perturbations with fastest growth rate become close to zonal for $M \ll 1$. 

\[ p = (1, 0). \]
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Linear stage of modulational instability

Growth of $|\psi_q|^2$ compared to predictions of linear stability.

Zonal velocity profile:

$$U_Z(y) = \frac{1}{2\pi} \int_0^{2\pi} v_x(x, y) \, dx$$
Generation of jets by modulational instability

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Distortion of small scales by large scales in weakly nonlinear regime

Modulational instability generates large scales directly from small scale waves (non-locality).

Subsequent evolution of the small scales can be described by a nonlocal turbulence theory:

- Assume major main contribution to collision integral for small scales comes from modes $q$ having $q \ll k$ (scale separation).
- Taylor expand in $q$. Leading order equation for small scales is an anisotropic diffusion equation in $k$-space:
  \[
  \frac{\partial n_k}{\partial t} = \frac{\partial}{\partial k_i} S_{ij}(k_1, k_2) \frac{\partial n_k}{\partial k_j}.
  \]

- Diffusion tensor, $S_{ij}$, depends on structure of the large scales: more intense zonal flows give faster diffusion.

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More on the spectral diffusion approximation

- Diffusion tensor, $S_{ij}$ turns out to be \textit{degenerate}.
- A change of variables shows that diffusion is along 1-d curves:
  \[
  \Omega = \frac{\beta k^2 k_x}{k^2 + F} = \text{const}
  \]
- \textit{Wave-action}, $n_k$, of a small scale wavepackets is conserved by this motion.
- Energy, $\omega_k n_k$, of small scale wavepackets is not conserved.
Wave-action diffuses along the $\Omega$ curves from the forcing region until it is dissipated at large wave-numbers.
Mechanism for saturation of the large scales

- Energy lost by the small scales is transferred to the large scale zonal flows which grow more intense.
- More intense large scales results in a larger diffusion coefficient.
- Increases the rate of dissipation of the small scale wave-action → negative feedback.

Prediction:

Growth of the large scales should suppress the small scale instability which turns off the energy source for the large scales leading to a saturation of the large scales without large scale dissipation.
Numerical observations of feedback loop

- Feedback loop is very evident in weakly nonlinear regime.
- Scaling arguments allow one to predict the saturation level of the large scales in terms of the small scale growth rate:

\[
[U (\nabla U)]_{LS} \sim \gamma \beta \quad M \ll 1
\]

\[
[U (\nabla U)]_{LS} \sim \gamma U \quad M \gg 1
\]

These estimates are reasonably supported by numerics - especially the cross-over from the weak to strong turbulence regimes (see inset).
Conclusions

- The CHM equation driven by “instability” forcing behaves differently than one might expect.
- There is no inverse cascade but rather direct generation of large scales by modulational instability (this could change with broadband forcing).
- Evolution of subsequent turbulence is non-local in scale and partially tractable analytically in the wave turbulence limit.
- A negative feedback loop is set up whereby the growth of the small scales is suppressed by the large scales leading to a saturation of the large scales without any large scale dissipation mechanism.
References


- C. Connaughton, S. Nazarenko and B. Quinn, *The life-cycle of drift-wave turbulence driven by small scale instability*, EPL 96, 25001 (2011)