FIELDS

Industrial Problem-Solving Workshop on Medical Imaging

Problem # 2

Rapid Modeling of Internal Structures of Deformable Organs (i.e. Liver)

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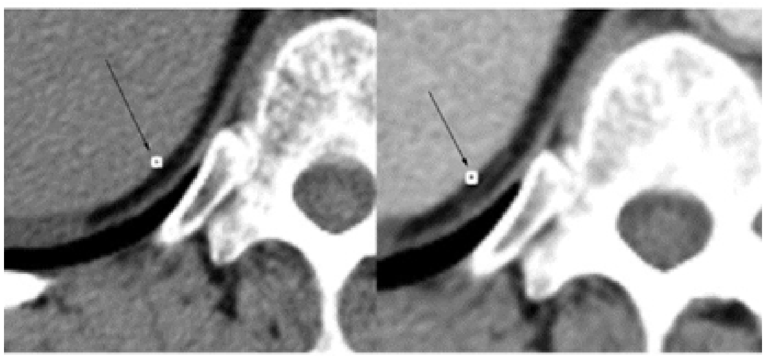
Problem definition

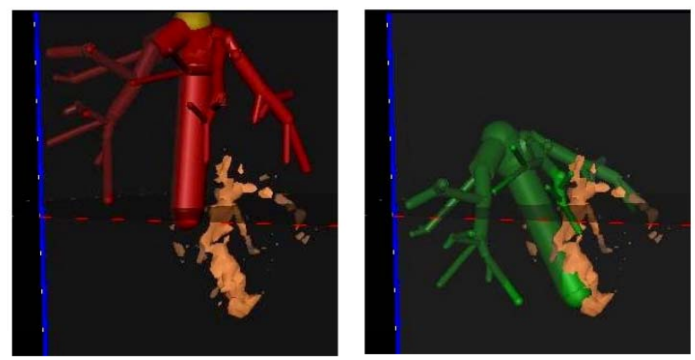
❖ Accurate estimation of deformation of the soft organ's internal structures between two images acquired at different conditions

What information are used?!

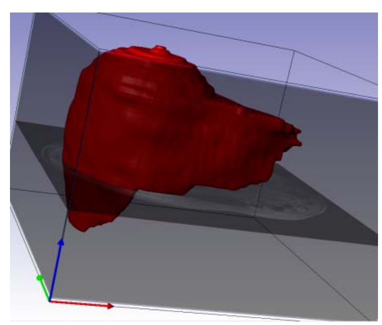
Point landmarks [1]



Vessel segments and curves [1]



Surface information



Problem definition

- ❖ **Application:** Tumor treatment using high-intensity focused ultrasound (HIFU) thermal procedures.
- Real-time image guidance is required for targeting.
- Intraoperative imaging systems:
  - Low resolution and quality images
  - Long acquisition time
  - Compatibility with other equipments in the operation room.
- ✓ **Solution:** Updating preoperative treatment plans and physical deformation models based on intraoperatively acquired images.
- ✓ **The idea:** Employing boundary conditions (i.e. curves of vessels and bifurcations of vessels, landmarks, and surface information) to constrain the solution of the deformable model of organ.

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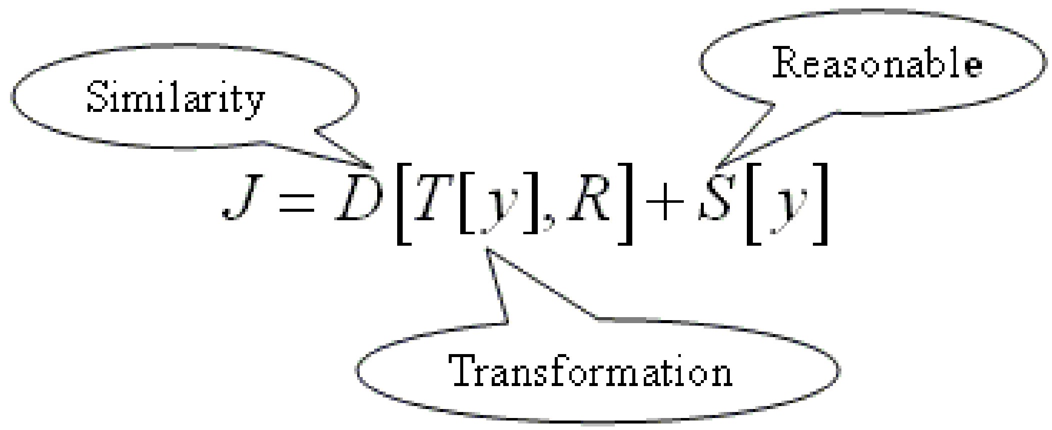
Problem definition

- ❖ Registering internal structures and surface information of the two set of images acquired from the soft organ.

➢ *Image Registration*

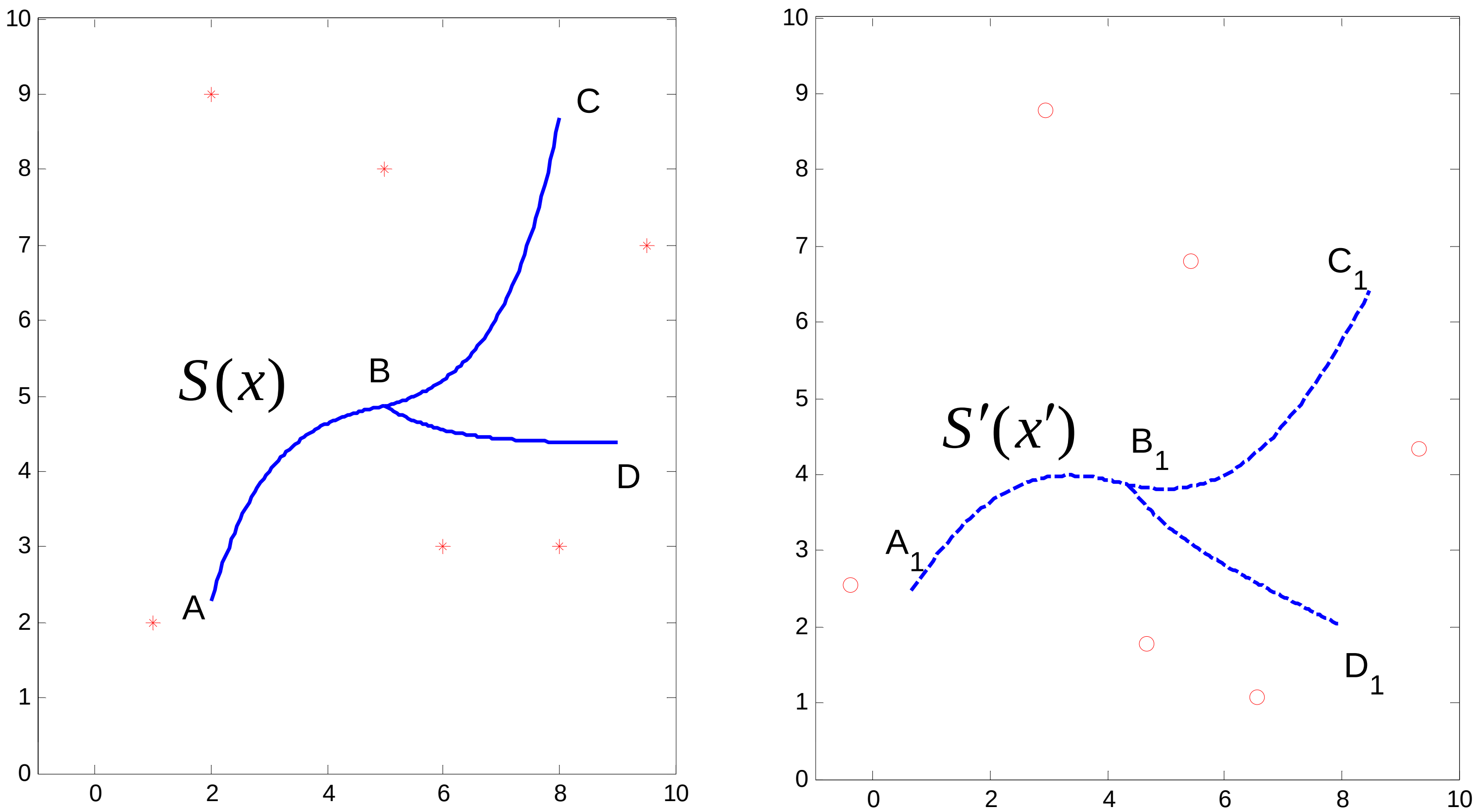
Given a reference image *R* and a template image *T*, find a reasonable transformation *y*, so that the transformed image *T[y]* is similar to *R* [2].



$$J = D\left[T[y], R\right] + S\left[y\right]$$

Problem 1

❖ **Modeling internal structures in 2D images using landmarks and vessel segments with known end points**



Problem 1

- **Solution**

Landmarks: $\left(x_l, x_l\right)$ and $\left(x'_l, y'_l\right)$ for $l = 1, 2, \ldots, n$

Vessel Curves: $y = S(x)$ and $y' = S'(x')$

Displacement Function: $\left(u(x, y) \quad , \quad v(x, y)\right)$

Landmarks: $\left(x'_l, y'_l\right) \Leftarrow \left(x_l + u(x_l, y_l), y_l + v(x_l, y_l)\right)$

Any point on the curve: $\left(x', y'\right) \Leftarrow \left(x + u(x, y), y + v(x, y)\right)$

Problem 1

❖ **Solution**

The cost function to be minimized for landmarks matching:

$$J_1(u,v)=\sum_{l=1}^{n}\left\{\left(x_l'-x_l-u(x_l,y_l)\right)^2+\left(y_l'-y_l-v(x_l,y_l)\right)^2\right\}$$

The cost function to be minimized for curve matching:

$$\left(x,S(x)\right) \;\Rightarrow\; \left(x+u\left(x,S(x)\right),S(x)+v\left(x,S(x)\right)\right)$$

This point corresponds to

$$\left(x,S(x)\right) \;\Rightarrow\; \left(x+u\left(x,S(x)\right) \quad , \quad S'\left(x+u\left(x,S(x)\right)\right)\right)$$

$$J_2(u,v)=\int_{x_1}^{x_2}\left|S'\left(x+u\left(x,S(x)\right)\right)-S(x)-v\left(x,S(x)\right)\right|^2 dx$$

Problem 1

❖ **Solution**

The total cost function to be minimized for landmarks and curve matching:

$$J(u,v) = w_1 J_1(u,v) + w_2 J_2(u,v)$$

Having the variational model in optimization model form:

$$u = Ax + By + x_0 + \sum_{i=1}^{N} a_i \varphi_i(x,y)$$

$$v = Cx + Dy + y_0 + \sum_{i=1}^{N} b_i \varphi_i(x,y)$$

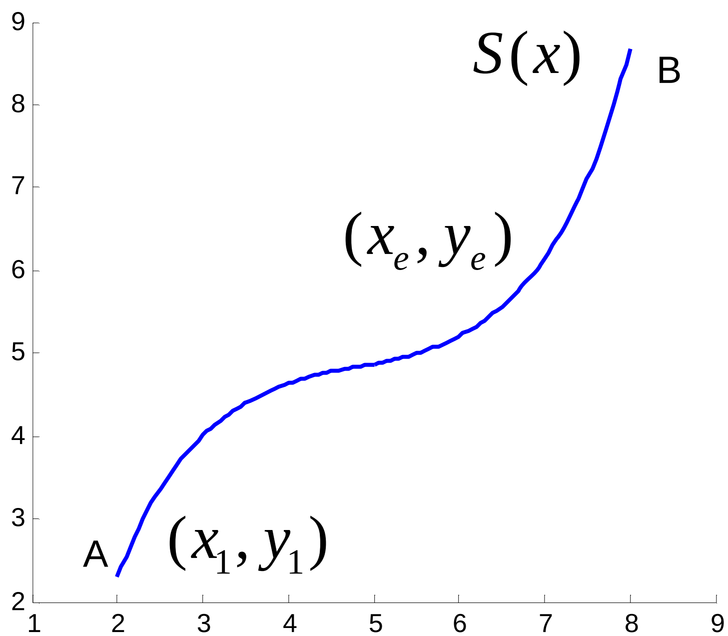
$\varphi_i \; (i = 1, 2, \ldots, N)$ are radial basis functions.

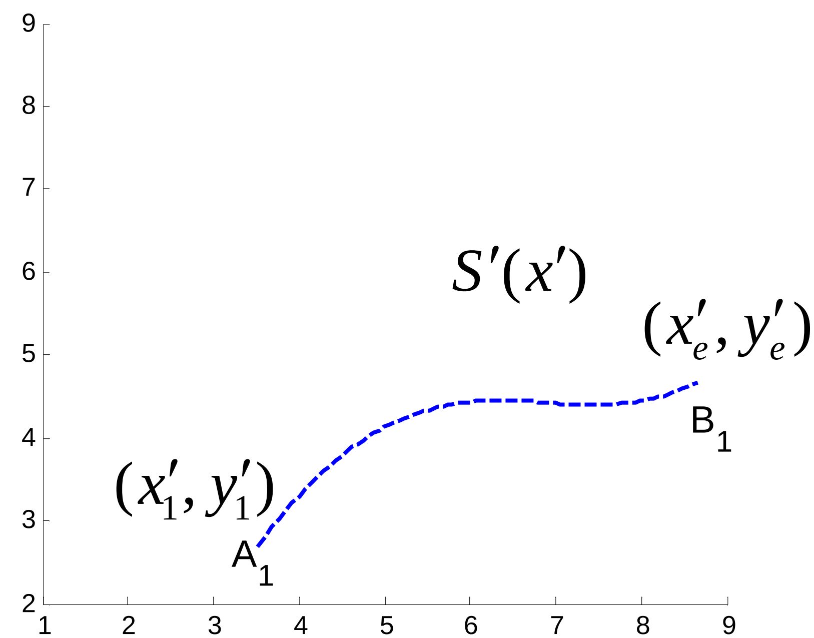
The cost function in terms of unknown parameters

$$\min \quad J\left(A, B, C, D, x_0, y_0, a_1, \ldots, a_N, b_1, \ldots, b_N\right)$$

Problem 2

❖ **Modeling internal structures in 2D images using landmarks and vessel segments with one end point.**





$$A \rightarrow A_1$$

$$B \rightarrow ?!$$

Problem 2

- **Solution**

(x_e, y_e) is unknown point to be found and needs to be added to the optimization function as follows:

$$J_2(x_e,u,v) = \left(x'_e - x_e - u\left(x_e, S(x_e)\right)\right)^2 + \left(y'_e - y_e - v\left(x_e, S(x_e)\right)\right)^2$$

$$+ \int_{x_1}^{xe} \left| S'\left(x + u\left(x, S(x)\right)\right) - S(x) - v\left(x, S(x)\right) \right|^2 dx$$

Problem 3

❖ **Modeling internal structures in 3D image**

➢ **Landmarks:** The same as 2D images

$$J_1(u,v,w) =$$

$$\sum_{l=1}^{n}\left\{\left(x_l' - x_l - u(x_l, y_l, z_l)\right)^2 + \left(y_l' - y_l - v(x_l, y_l, z_l)\right)^2 + \left(z_l' - z_l - v(x_l, y_l, z_l)\right)^2\right\}$$

➢ **Curves:**

Parametric cubic representation: piecewise spline

$$x(t) = a_x t^3 + b_x t^2 + c_x t \ + d_x$$

$$y(t) = a_y t^3 + b_y t^2 + c_y t \ + d_y$$

$$z(t) = a_z t^3 + b_z t^2 + c_z t \ + d_z$$

Problem 4

- **Registration based on surface matching of the object in two set of images**

$$\left(x, y, S(x, y)\right) \qquad \left(x', y', S'(x', y')\right)$$

$$\left(u(x, y, z), v(x, y, z), w(x, y, z)\right)$$

Problem 4

❖ **The cost function to be minimized:**

$\left(x, y, S(x,y)\right)$ $\qquad$ $\left(x', y', S'(x',y')\right)$

$$J(u,v,w) =$$

$$\int_{\Omega} \left| S'\left(x + u\left(x,y,S(x,y)\right), y + v\left(x,y,S(x,y)\right)\right) - S(x,y) - w\left(x,y,S(x,y)\right) \right|^2 dx\,dy$$

Another solution: deformable surface modeling using active contours [3]

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Impact

❖ **FEM-based image registration.**

➢ **Intensity based** [4]

$$\min \; J(u) = D\left(T\left[u\right], R\right) + \alpha \frac{1}{2} u^T K u; \quad \alpha \in \Re_+$$

$$\frac{\partial J(u)}{\partial u} = \frac{\partial D\left(T\left[u\right], R\right)}{\partial u} + \alpha K u = 0$$

$$Ku = f(u) = -\frac{1}{\alpha} \frac{\partial D\left(T\left[u\right], R\right)}{\partial u}$$

- Multimodality
- Local minima

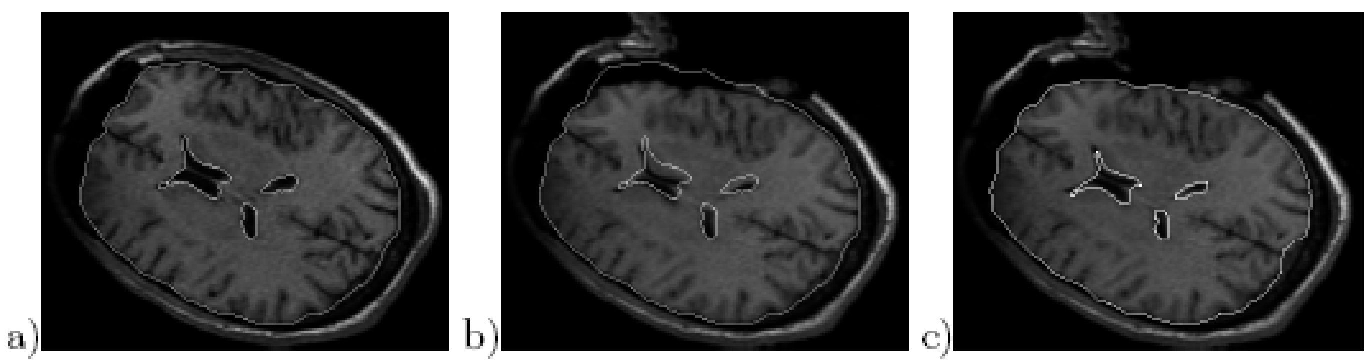
Impact

❖ **FEM-based image registration.**

➢ **Feature based**

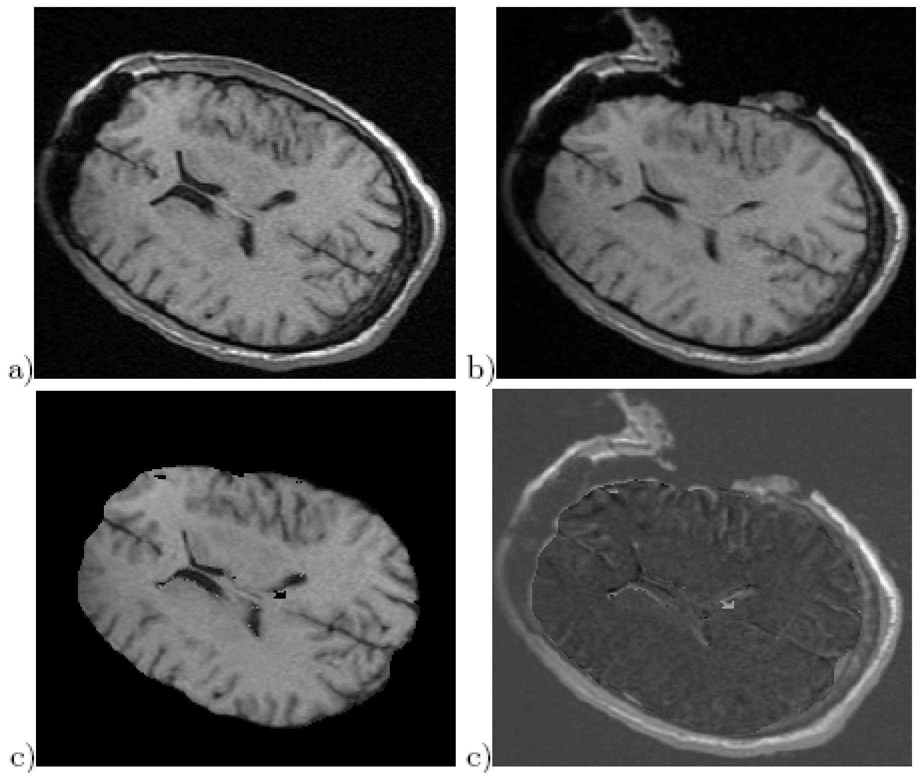
Internal structures and surface information of the deformable organ provides necessary boundary condition to solve the FEM-based deformation equation.

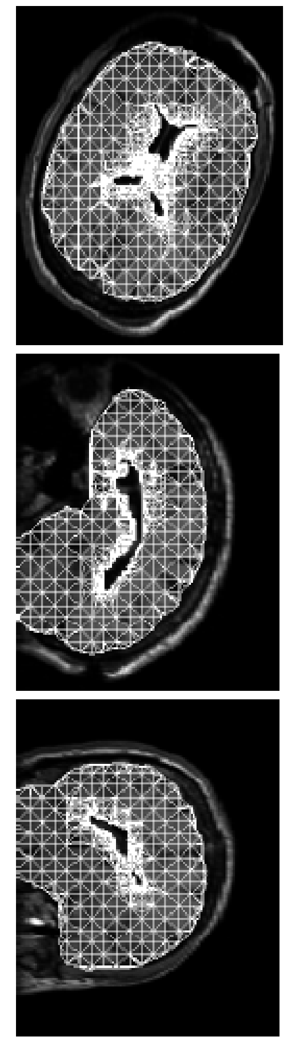
Example: Linear finite element model of the brain shift and deformation [3].



Impact

- **FEM-based image registration.**





References

[1] Dietlind Zühlke, Sven Arnold, Gernoth Grunst, Peter Wißkirchen, “Intra-interventional registration of 3D ultrasound to models of the vascular system of the liver” GMS CURAC 2007, Vol. 2(1)

[2] J. Modersitzki, Numerical Methods for Image Registration. New York:Oxford, 2004.

[3] M. Ferrant, A. Nabavi, B. Macq, F. Jolesz, R. Kikinis, and S. Warfield, “Registration of 3D Intraoperative MR Images of the Brain Using a Finite-Element Biomechanical Model,” IEEE Trans. on Medical Imaging, vol. 20, no. 12, pp. 1384–1397, 2001.

[4] B. Marami, S. Sirouspour, and D. Capson, “Model-Based Deformable Registration of Preoperative 3D to Intraoperative Low-Resolution 3D and 2D Sequences of MR Images,” in MICCAI 2011, pp. 460–467, 2011.

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Motivation

- Modelling a three-dimensional image of the liver is computationally expensive (and has challenging physics)
- Image data arrives in several two-dimensional cross sections (slices) as the body is scanned
- Each slice is identified by unique markers (landmarks and curves)
- If all of the markers from one slice map to all be on a transformed slice then we can consider a series of two-dimensional transformations

Setup

- Landmarks
  - The N distinguishing points on image
- Curves
  - The κ distinguishing curves (vessels) on image
- Assume a transformation from "pre-image" (X, Y) to "post-image" $(\tilde{X}, \tilde{Y})$ via a transformation $(\bar{X}, \bar{Y})$

$$\bar{X} = X + \sum_j a_j \phi_j(X, Y),$$

$$\bar{Y} = Y + \sum_j b_j \phi_j(X, Y),$$

- ϕ_j are radial basis functions (RBF)

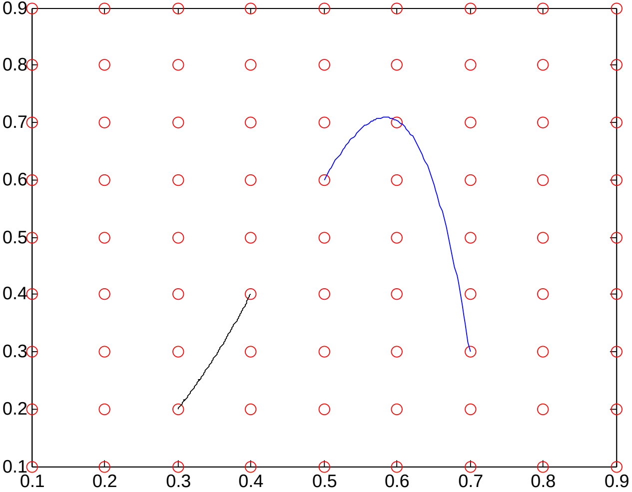
$$\phi_j(X, Y) = \exp\left(-\frac{(X - X_j)^2 + (Y - Y_j)^2}{\sigma}\right)$$

- (X_j, Y_j) are the L allocation points to form a basis set $(L \leq N)$

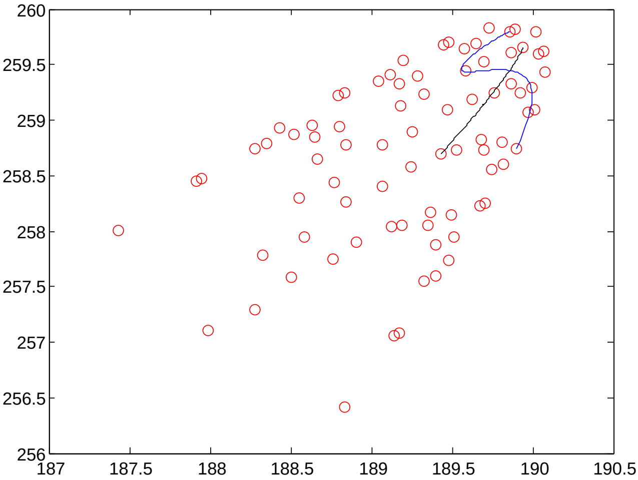
Setup

- For curves we assume there exists a mapping for curve points on the pre-image (xx, yy) given by $yy = S(xx)$
- Similarly there exists a map $\tilde{yy} = \tilde{S}(\tilde{xx})$ for the curve points on the post-image $(\tilde{xx}, \tilde{yy})$
- Often the form of S and $\tilde{S}$ will be through an interpolation (cubic splines)
- The curves have the requirement that the endpoints are landmarks

Pre Image



Post Image



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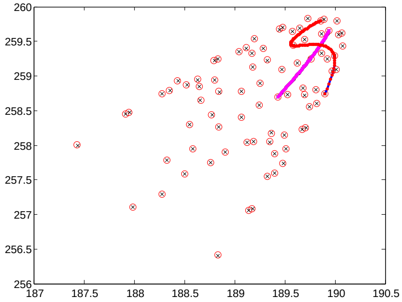
Optimization

- We wish to determine the coefficients for the RBF that transforms the pre-image into the post-image
- We determine these by minimizing an error function composed of matching landmarks and curves
- Therefore we consider
  1. Landmark Error
     - We wish all terms of the form $\tilde{X} - \bar{X}$ to be small so that the landmarks are close
  2. Curve Error
     - We wish the curve points to be close as well i.e. $\tilde{xx} - \bar{xx}$ and $\tilde{S}(\tilde{xx}) - \bar{S}(xx)$ are small
     - The predicted curve $\bar{S}(xx) = S(xx) + \sum_j b_j \phi_j(xx, S(xx))$

Solution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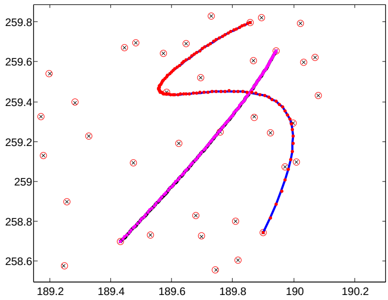
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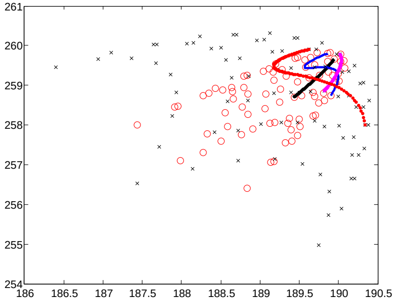
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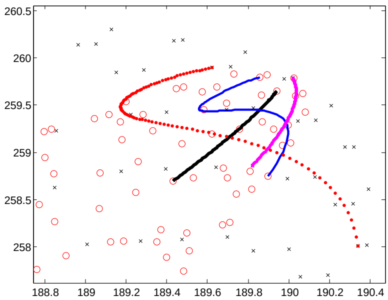
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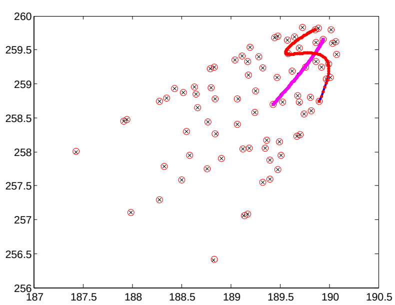
Constrained Optimization

- There are many local minima to this problem with large energies
- In fact, there are many local minima with small energies that don't represent the "true" solution (lack of global minimizer?)
- **Idea:** Build in a constraint for each curve that forces the area under the post-image curve to match that of the predicted curve

$$\int_{\tilde{x}x_1}^{\tilde{x}x_2} (\tilde{S}(\tilde{x}x) - S(xx) - \sum_j b_j \phi_j) \, \mathrm{d}\tilde{x}x = 0$$

Solution



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Conclusions

- Created image data
- Performed an optimization to recover RBF coefficients that transform pre-image to post-image
- Included integral constraints to reduce set of minimizers

Future Work

- Apply to real landmark data
- Consider weightings carefully
  - Center of mass type penalty system
  - Voronoi diagrams
- Allow for curve endpoints to move