What does compressive sensing mean for X-ray CT and comparisons with its MRI application

Emil Sidky
University of Chicago- Dept. of Radiology

work supported by the NIH
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects                           real data
  - sparsity-based sampling sufficiency      theoretical study
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects                           real data
  - sparsity-based sampling sufficiency              theoretical study
Standard introduction to CT

\[ \int_L \mu(r) \, d\ell = -\ln(I/I_0) \]

Radon transform
X-ray transform
Reality of CT

* object function is simplified:

\[ \mu(\vec{r}) \rightarrow \mu(\vec{r}, E, t) \]

* data model also simplified:
  - X-ray scatter
  - X-ray source beam-spectrum
  - detector physics
  - random processes
  ....

* CT is a digital instrument:
  finite number of samples
Overview of image reconstruction algorithms

* An algorithm consists only of a number of data processing steps

* Data/imaging models and their methods of solution help guide their design

* Trade-off (see Foundations of Image Science by Barrett and Myers)

- Simple model: easy to solve, model error is large
- Complex model: hard to solve, model error is small

* Practical I.R. algorithms evaluated on imaging task
  Theoretical I.R. research based on model solution
Implicit v. Explicit image reconstruction

\[ g = X(f) \]

(example: compressive sensing)
solved iteratively
non-linear
complex models can be devised
zoology of data models
need to reconstruct whole image

\[ f = X^{-1}(g) \]

(example: FBP)
one-shot processing
usually linear
modeling limited
models more uniform
can reconstruct point-by-point
Model zoology

\[ \vec{g} = X \vec{f} \]

Implicit / Iterative / CS

type of expansion elements:
- pixels, blobs, wavelets
number of expansion elements
ray sampling
measurement model
- line integration
  Siddon’s method, ray-tracing
area-weighted integration

\[ g(\theta_i, \xi_i) = \int_{L(\theta_i, \xi_i)} d\ell f(\vec{r}) \rightarrow \text{Radon/X-ray} \]

Explicit / FBP/ FDK
continuous object function
---
continuous data function
measurement model
- line integration
Full solution v. point-by-point

implicit

explicit
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects   real data
  - sparsity-based sampling sufficiency theoretical study
X-ray Imaging for Breast Cancer Screening

Goal: Early detection

Task: image asymptomatic women and decide to recall or not

Imaging: suspicious mass (tumor) or micro-calcification cluster (DCIS)
X-ray Imaging for Breast Cancer Screening

Digital mammography

resolution
depth: 6.0 cm
in-plane: 0.1 mm
X-ray Imaging for Breast Cancer Screening

Digital mammography

Digital breast tomosynthesis

resolution
depth: 6.0 cm
in-plane: 0.1 mm

resolution
depth: 1.0 mm
in-plane: 0.1 mm
X-ray Imaging for Breast Cancer Screening

Digital mammography

Digital breast tomosynthesis

Computed Tomography

resolution
depth: 6.0 cm
in-plane: 0.1 mm

resolution
depth: 1.0 mm
in-plane: 0.1 mm

resolution
depth: 0.3 mm
in-plane: 0.3 mm
X-ray Imaging for Breast Cancer Screening

design constraint: Equal X-ray dose

Digital mammography  Digital breast tomosynthesis  Computed Tomography

resolution
depth: 6.0 cm
in-plane: 0.1 mm

resolution
depth: 1.0 mm
in-plane: 0.1 mm

resolution
depth: 0.3 mm
in-plane: 0.3 mm
Mass imaging

Projection image

Digital breast tomosynthesis

in-plane

depth

Courtesy: Massachusetts General Hospital
GE prototype DBT scanner
Microcalcification imaging

Projection image

Digital breast tomosynthesis

2 cm

Courtesy: Massachusetts General Hospital
GE prototype DBT scanner
Breast computed tomography (bCT)

512-view, bCT simulation
FBP reconstruction

unregularized  Gaussian filtered
Breast computed tomography (bCT)

512-view, bCT simulation
FBP reconstruction

Can CS help?
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects                           real data
  - sparsity-based sampling sufficiency     theoretical study
Compressive sensing for CT with gradient magnitude sparseness

\[ \vec{f}^* = \arg\min \| \vec{f} \|_{TV} \text{ such that } X \vec{f} = \vec{g} \]

\[ \| \vec{f} \|_{TV} = \sum_i |\vec{\nabla} f_i| \]

specifying \( X \)
Siddon’s method
square pixels
different array sizes
Compressive sensing for CT with gradient magnitude sparseness (comparison with FT/MRI image model)

\[ \vec{g} = X \vec{f} \]

discrete Cartesian FT
consistent
discrete inverse
need \( N \times N \) samples
incoherence

discrete X-ray transform
may be inconsistent
no known direct discrete inverse
need \( 2N \times 2N \) samples?
partial incoherence
Inverse of the discrete X-ray transform?

1024x1024 discrete phantom  FBP applied to 2048x2048 data set
Incoherence

sparse image

Fourier transform

sparse image

fan-beam projection

source angle \( \phi \)

detector bin \( \xi \)
image gradient CS for CT

\[ \vec{f}^* = \arg\min \| \vec{f} \|_{TV} \mid \| X \vec{f} - \vec{g} \|^2 \leq \epsilon^2 \text{ and } f_{max} > \vec{f} > 0 \]

\[ \| \vec{f} \|_{TV} = \sum_i |\nabla f_i| \]

* data inconsistency ---> \( \epsilon > 0 \)
* no discrete inverse ---> challenge for algorithm development
* partial incoherence ---> no exact recovery theorems, RIP, NSP
  (we have performed extensive tests...)

Algorithm alternates POCS with TV-steepest descent
PMB 2008 - Sidky and Pan
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects real data
  - sparsity-based sampling sufficiency theoretical study
Evaluation of sparse-view reconstruction from flat-panel-detector cone-beam CT

Junguo Bian¹, Jeffrey H Siewerdsen³, Xiao Han¹, Emil Y Sidky¹, Jerry L Prince⁴, Charles A Pelizzari⁴ and Xiaochuan Pan¹,²

¹ Department of Radiology, The University of Chicago, Chicago, IL, USA
² Department of Radiation & Cellular Oncology, The University of Chicago, Chicago, IL, USA
³ Departments of Biomedical Engineering, Johns Hopkins University, Baltimore, MD, USA
⁴ Department of Electrical & Computer Engineering, Johns Hopkins University, Baltimore, MD, USA

PMB 2010

CS
algorithm

960-views
Robustness to model error
Algorithm-enabled Low-dose Micro-CT Imaging

Xiao Han, Student Member, IEEE, Junguo Bian, Student Member, IEEE, Diane R. Eaker, Timothy L. Kline, Student Member, IEEE, Emil Y. Sidky, Erik L. Ritman, and Xiaochuan Pan, Fellow, IEEE

TMI 2011
Is CS really new?

- Edge-preserving TV regularization used since early 1990s
  Constrained, TV-minimization equivalent to
  TV-penalized unconstrained optimization

- Sparsity and L1-relaxation exploited for contrast-enhanced vessel imaging
PMB 2002

An accurate iterative reconstruction algorithm for sparse objects: application to 3D blood vessel reconstruction from a limited number of projections

Meihua Li¹, Haiquan Yang² and Hiroyuki Kudo³

Figure 2. The cost function of ART method.

Figure 3. The $L_1$ norm cost function.

4-views!

$L_1$-relaxation
Contributions of CS

* Expanded thinking on optimization based image reconstruction
  Traditional iterative: minimize data fidelity + γ roughness penalty
  CS: Use penalty to break degeneracy of the solution space

* Novel rules for determining data sufficiency-based object sparsity

* Beating Nyquist Frequency??
  No.
  Nyquist is only one form of interpolation
  Use of interpolation, followed by FBP yields the continuous image
  CS yields only discrete representation of the image
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects real data
  - sparsity-based sampling sufficiency theoretical study
Focus on bCT: tradeoff between view-number and noise-per-view

1878-projections, 100 micron detector bins, low-intensity X-ray illumination

Courtesy XCounter
constrained, TV-minimization
First attempt: 100 micron pixel array
Second attempt: 25 micron pixel array

image array: 4096 x 4096
data samples: 1878 views x 1200 bins

undersampled!!
CS-algorithm modifications

- detector-coordinate
- Fourier upsampling
- constrain image spatial frequencies

Sidky et al. 2011 - arxiv.org/abs/1104.0909
Outline

* CT and image reconstruction background

* Application: mammography

* Compressive sensing in CT versus MRI

* Some results with real CT data

* Ongoing studies:
  - extremely small objects                           real data
  - sparsity-based sampling sufficiency     theoretical study
Preliminary investigation on sparsity-based data sufficiency

* Aiming for an empirical Donoho-Tanner type study

* Accurate, first order TV-minimization solver
  Jakob Joergensen - Danish Technical University
  T. Jensen et al. ( arxiv.org/abs/1105.3723 )

* Computer-generated breast phantom
Phantom

256x256 pixelized array
65536 unknowns

~10000 non-zero pixels

gradient magnitude
Sampling sufficiency study

objectives

\[ |\tilde{g} - X \tilde{f}|^2 + \alpha |f|^2 \quad \text{Tikhonov} \]
\[ |\tilde{g} - X \tilde{f}|^2 + \alpha |\nabla f|^2 \quad \text{CS} \]
\[ |\tilde{g} - X \tilde{f}|^2 + \alpha \|f\|_{TV} \]

\( \alpha \) extremely small \( \rightarrow \) data RMSE=10^{-5}

whole image error

necessary samples/sparsity \( \sim 2.5 \) ???

data: 32-512 views \( \times 512 \) bins

ROI error

What is fully sampled?
the group working on CS in CT

University of Chicago

Xiaochuan Pan    Emil Sidky

Students:
Junguo Bian
Xiao Han
Eric Pearson
Zheng Zhang
Adrian Sanchez

applied math experts:

Rick Chartrand
LANL

Jakob Joergensen
student at the Danish Technical Univ.
Fourier sampling problems

interpolation

extrapolation

"standard" CS

CS approach to an old problem

Chartrand, Sidky and Pan
math.lanl.gov/~rick/Publications/chartrand-2011-frequency.shtml
Papoulis-Gerchberg

band-limited signal

sampling interval
Papoulis-Gerchberg reversed

spatial frequency

sampling interval

compact support

space coordinate
Frequency extrapolation experiment

Problem: recover 4Kx4K FT sample grid from central set of 512x512 samples.
Frequency extrapolation method

\[ x^* = \arg\min_{i=1} \sum \varphi_p(|\nabla x|_i) + \lambda \| Ax - b \|_2^2 \]

\[ \varphi_p(t) = \begin{cases} 
\gamma |t|^2 & \text{if } |t| \leq \alpha \\
\gamma |t|^p/p - \delta & \text{if } |t| > \alpha 
\end{cases} \]

Chartrand ISBI 2009 for details
*efficient solver
Results: no frequency extrapolation

- inverse DFT
- zero pad
- zero pad and filter
Results: non-convex frequency extrapolation

- $p=1$
- $p=0.25$
- $p=-0.5$
Results: non-convex frequency extrapolation

$p=1$

$p=0.25$

$p=-0.5$