A History of Options
From the Middle Ages to Harrison and Kreps

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Risk Instruments in the Middle Ages

• Casualty, credit, and market risks associated with shipments of goods, notably by ships on the Mediterranean.
• Insurance contracts were common for casualty and credit risks, as early as 1350 in Palermo, with the two kinds written separately.
• A popular contract was a conditional sale (sort of a put option) where insurer agreed to purchase ship or cargo if it failed to arrive.
• For market risk a merchant could hedge two ways: forward transactions (sometimes with advance payment) or derivative contracts.
• In 1400s the Cerchi bank of Florence bought and sold call options.
• In 1500s a derivative called a premium transaction was popular: at settlement either buyer or seller could cancel by paying a premium.
• Trading in derivatives took place in context of widespread gambling.
policy would assert the opposition, not only of the Home Rulers, but of a large body of English Radicals. The slightest possibility with which the House of Commons responded to Mr Gladstone's appeal has completely dispelled these delusions, and no Continental statesman, any longer believes that England is ready to pay for peace whatever price King Louis sets on it. In

one sense it is true that Mr Gladstone's speech has failed to achieve the end he defined the issue between England and Russia, and defined it in such terms as to make the issue between peace and war hang upon its determination. But we cannot help feeling that a similar declaration, equally final, in its explanations, and equally momentous in its tone, if made a month or six weeks ago, might have had an eminently pacific tendency. The source of our present trouble has been traced to the inconsiderate belief in the inveterate belief in Russia that we had no serious purpose in the matter of the Afghan frontier. This belief has certainly been strengthened in which we have shown the Government has treated each successive stage of the controversy. It was probably a mistake, in the first instance, to have dispatched the English Commissioners before any working basis had been laid down for the operation of the Commission. It was clearly a mistake to have allowed Sir Peter Landeau to be kept waiting for months for hiscollaborator, and the special envoy was sent from St Petersburg to this country, and received at the Foreign Office. It was, again, an act of weakness not to have insisted on the withdrawal of the Russian officers from the positions which they had taken up in dangerous condition to the Afghans, at a condition precedent to the continuance or resumption of negotiations. All these symptoms of apparent irresolution and half-measures were not only noted by St Petersburg and taken advantage of. We welcome, there-fore, Mr Gladstone's speech, as proving conclusively that there is a new attitude has been adopted, which, though it may be too late to avert the impending catastrophe of war, is at least worthy of the dignity of a great nation in an hour of difficulty and peril.

THE VIRTUES AND VICES OF OPTIONS.

A rather marked feature in the Stock Exchanges recently has been the revival of "option" dealing. In early years only, a considerable amount of business was habitually transacted in "options," especially in Canada, but more recently this species of speculation had dwindled down to very restricted dimensions. But at no period has it ever been so popular as it is on the continental bourses, and on the stock exchanges across the Atlantic. At Paris, and on all the German bourses, there is a vast amount of speculation constantly carried on by means of options, not separate from, but auxiliary to direct operations for the rise or fall. In New York, "options" and "privileges" are also a very favourite form of speculation, and the means for deriving it in have been abundant, it is evidenced by the fact that Mr Russell Sage, the well-known associate of Mr John Gooch who was, until the collapse of May, 1894, one of the most influential and most powerful manipulators in Wall Street, has been a great dealer in "stock privileges." It is difficult to understand why options, as far as not far from being a medium of speculation, but in view of the very mean principles of business, may be as well to show their advantages and disadvantages from an outside standpoint.

An "option" is the price paid for the right to demand, or to deliver a certain amount of stock at a given price within a certain definite period. The prices given for this "option" may, of course, range infinitely, according to the supposed value of the elements of which it is composed. The right to demand a stock is termed the "call," and the right to deliver it is the "put." For instance, one may pay 50, or 100 cents for the "call" of a month hence of 3000 Empire, 1874, which right may or may not be exercised. And a "put" would be exactly the converse of this. It is possible to buy the double privilege of both "put and "call," but the price asked is usually so heavy as to be practically prohibitive. Now, the idea of the

The Economist 1885

“The Virtues and Vices Of Options”

INDIAN CHIPS.

(From our Special Correspondent.)

Two headings of this letter afford an explanation for my pretested silence. I have nothing before me but fragments. I commenced the year by promoting a series of letters on Indian railway projects. I had hardly finished my account of the Hindi line, and was about to commence a trinitite on the Nagpur-Bengal line, when railway enterprises began to stagger under the bare covert of a storm of aks on the further frontier of Afghanistan. Within the last few weeks the prosecution of the railway works on the Euphrates has been suddenly stopped, and parties of Indian plateayers have actually
Excerpts from the 1885 Economist Article

• “At Paris, and on all the German bourses, there is a vast amount of speculation constantly carried on by means of options… In New York options or ‘privileges’ are also a favourite form of speculation…”

• “From the standpoint of business morality, two things may be adduced in connection with options, one for and one against. In the first place, they foster a form of speculation which already flourishes too abundantly. … On the other hand, used by experienced speculators, options are generally great safeguards against unexpected and violent movements in prices…”
Louis Bachelier

- Born in 1879 Le Havre
- Sorbonne 1892
- Thesis defense 1900
- Sorbonne lecturer
- World War I army
- 1919-1937 professor at Besancon, Dijon, and Rennes
- Died in 1946
- Bachelier Finance Society founded in 1996
THÉORIE
DE
LA SPÉCULATION,
PAR M. L. BACHELIER.

INTRODUCTION.

Les influences qui déterminent les mouvements de la Bourse sont innombrables, des événements passés, actuels ou même escomptables, ne présentant souvent aucun rapport apparent avec ses variations, se répercutent sur son cours.

A côté des causes en quelque sorte naturelles des variations, interviennent aussi des causes factices : la Bourse agit sur elle-même et le mouvement actuel est fonction, non seulement des mouvements antérieurs, mais aussi de la position de place.

La détermination de ces mouvements se subordonne à un nombre infini de facteurs : il est dès lors impossible d’en espérer la prévision mathématique. Les opinions contradictoires relatives à ces variations se partagent si bien qu’au même instant les acheteurs croient à la hausse et les vendeurs à la baisse.

Le Calcul des probabilités ne pourra sans doute jamais s’appliquer aux mouvements de la cote et la dynamique de la Bourse ne sera jamais une science exacte.

Mais il est possible d’étudier mathématiquement l’état statique du marché à un instant donné, c’est-à-dire d’établir la loi de probabilité des variations de cours qu’admet à cet instant le marché. Si le marché, en effet, ne prévoit pas les mouvements, il les considère comme étant
Poincare’s Report

Thesis Committee
Paul Appell
Joseph Boussinesq
Henri Poincare

The original document is held at the Centre historique des Archives Nationales in Paris, classification number AJ/16/5537.
Accomplishments in Thesis

• Assumed price fluctuations over small time intervals are independent of present and past price levels
• Applied central limit theorem to deduce price increments are independent and normally distributed (so the price process is Brownian motion as the diffusion limit of a random walk!)
• Used lack-of-memory (Markov) property to derive (what is now called) the Chapman-Kolmogorov equation
• Established connection with heat equation
• Simple formula for the price of at the money calls
• Recognized concept of arbitrage
• Work was cited and used by Kolmogorov in 1931 and by Doob (the “father” of martingales)
Other Option Research Prior to 1950’s

none
Bachelier “discovered” by Samuelson

In early 1950s Jimmy Savage sent postcards to various economists, including Samuelson, about Bachelier

Samuelson said, “In the early 1950s I was able to locate by chance this unknown book, rotting in the library of the University of Paris, and when I opened it up it was if a whole new world was laid out before me.”

Samuelson had been giving thought to option pricing, so he commissioned the translation by James Boness.

Inventor of the option terms “American” and “European”
Research in 1964 book edited by Paul Cootner
*The Random Character of Stock Market Prices*

- By this time people were using geometric Brownian motion models of stock market prices
- People like Boness, Samuelson, and Sprenkle were calculating the expected discounted payoff of European puts and calls, but they were all using different choices for the discount factor and the stock’s appreciation rate
- The mathematician McKean, in an appendix to Samuelson’s paper, studied a free boundary problem pertaining to the pricing of an American put (optimal stopping time = optimal early exercise time)
Work by Sheen Kassouf and Edward Thorp

- Two young professors at University of California, Irvine
- Developed empirical formula
- Recognized and introduced concepts of *hedge ratio* and *dynamic hedging*
- Thorp learned of Cootner’s book, and based on his empirical work he set the stock’s appreciation rate equal to the riskless interest rate, arriving at the BS formula
- He used the Black-Scholes formula for profitable trading but he couldn’t prove why it was correct
Original Derivation of Black-Scholes Equation

Derivation of PDE was primarily due to Black. He focused on a portfolio of the form

\[ V = QS + C, \]

where

- \( V \) = portfolio value
- \( Q \) = stock position
- \( S \) = stock price
- \( C \) = price of European call

Using a Taylor series expansion he figured out how to use dynamic hedging so that this portfolio will have zero beta at each point in time.
Recall the Capital Asset Pricing Model (CAPM):

\[ E[R_P] = R + \beta(E[R_M] - R), \]

where

\[ R_p = \text{return of an arbitrary portfolio} \]
\[ R = \text{return of a riskless investment} \]
\[ R_M = \text{return of the market portfolio} \]
\[ \beta = \text{beta of arbitrary portfolio with respect to market} \]

Hence for Black’s zero-beta portfolio, over any time period:

\[ E[R_V] = R \]

More Taylor series calculations led to the famous Black-Scholes pde
In spite of Black’s Harvard PhD in applied mathematics, it took a while to find a solution of the pde

Since the stock’s appreciation rate $\mu$ did not appear in the pde, they set it equal to the riskless interest rate $r$, used an expression derived by Sprinkle (1961), and (voila!) they had the solution to the pde and its boundary condition.

The first draft working paper was dated October 1970.

The paper was eventually published in the May/June 1973 issue of the *Journal of Political Economy*, but this was after an earlier rejection by this journal as well as a rejection by the *Review of Economics and Statistics*.

Robert Merton made important subsequent contributions that were published in a 1973 issue of the *Bell Journal*. Merton’s paper was accepted before the BS paper, and Merton asked the *Bell Journal* editor to hold up publication of his paper until a journal accepted and published the one by Black and Scholes.
Black, Merton, Scholes and Samuelson were all together at MIT

Merton recognized how to use dynamic hedging to achieve a portfolio of the form

$$V = QS + C$$

that is actually riskless, not just zero-beta

This enabled him to derive the pde in a more rigorous fashion

Robert Merton
Merton’s Derivation of the Black-Scholes PDE

Assumptions:

Stock price: \[ \frac{dS}{S} = \mu dt + \sigma dW \]
Call price: \[ C = c(S_t, t) \]
Call’s boundary condition: \[ c(S_T, T) = \max\{0, S_T - K\} \]
Riskless interest rate: \[ r \] (with continuous compounding)
Portfolio value: \[ V = QS + C \]

It follows that: \[ dV = QdS + dC \]

Applying Ito’s lemma to \( c(S_t, t) \):

\[
dC = \frac{\partial c}{\partial S} dS + \frac{\partial c}{\partial t} dt + \frac{1}{2} \frac{\partial^2 c}{\partial S^2} \sigma^2 S^2 dt
\]
Substituting this in $dV = QdS + dC$:

$$dV = QdS + \frac{\partial c}{\partial S} dS + \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt$$

$$= \left[ Q + \frac{\partial c}{\partial S} \right] dS + \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt$$

This becomes, after setting $Q = -\frac{\partial c}{\partial S}$:

$$dV = \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt$$

This describes the dynamics of a deterministic, riskless portfolio, so its return, namely $dV/V$, must always equal the riskless interest $rdt$.
In other words:

\[
dV = rV dt = r(QS + C) dt = r \left( -\frac{\partial c}{\partial S} S + C \right) dt
\]

\[
= \frac{\partial c}{\partial t} dt + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2} dt
\]

Finally, dropping the common factor \(dt\) we get the Black-Scholes pde:

\[
\frac{\partial c}{\partial t} = rc - rS \frac{\partial c}{\partial S} - \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 c}{\partial S^2}
\]

This and the boundary condition \(c(S_T,T) = \max\{0,S_T - K\}\) are solved to obtain the famous Black-Scholes formula for the call option price.
Next Major Development: Risk Neutral Valuation

- By working with the BS model and a stock price model featuring Poisson jumps, John Cox and Steve Ross introduced the concept of risk neutrality.
- They hypothesized (but did not prove) that in some generality the price of an option can be computed with preferences (which we call probabilities) such that expected returns for both the stock and the option are equal to returns under the riskless rate.
- For stock the preferences should satisfy: \( \mathbb{E}[S_T/S_t | S_t] = \exp\{r(T-t)\} \)
- For European option with price \( H \) satisfying \( H_T = h(S_T) \) they said:

\[
\mathbb{E}[H_T / H_t | S_t] = e^{r(T-t)} \iff H_t = e^{-r(T-t)} \mathbb{E}[h(S_T) | S_t]
\]
Remarks About the Cox-Ross Results

- The generality and “why” are unclear
- There was no mention of martingales
- Harrison and Kreps were greatly stimulated by the Cox-Ross paper, for they said, “…Cox and Ross provide the following key observation. If a claim is redundant in a world with one stock and one bond, then its value can be found by first modifying the model so that the stock earns at the riskless rate, and then computing the expected (discounted) value of the claim. They analyze two examples, and in each case they determine the correct modification by the following procedure. First, using the technique of Black and Scholes, they derive an analytical expression (e.g., a pde) that the value of the claim must satisfy. Having observed that one model parameter (e.g., the geometric BM appreciation rate) does not appear in this relationship, they then adjust the value of the parameter so that the stock earns at the riskless rate.”
The Harrison-Kreps Results

- Recognizing that the Cox-Ross equation $E[S_T/S_t|S_t] = \exp\{r(T-t)\}$ is the same as $E[e^{rT}S_T|S_t] = e^{rt}S_t$, they were led to the idea that the risk neutral probabilities (i.e., Cox-Ross preferences) must be such that the discounted price processes are martingales.

- This led to the notion of equivalent martingale measures.

- Another important notion is viability; this is approximately the same as the absence of arbitrage opportunities.

- Key Result #1: the model is viable if and only if there exists an equivalent martingale measure.

- Still another notion: a redundant claim is one which can be replicated by some portfolio involving the stocks and bank account.

- Key Result #2: in a viable model a claim is redundant iff it has the same expectation under every equivalent martingale measure.

- Key Result #3: if a claim is redundant, then its arbitrage value is that common expectation.
Remarks About the Harrison-Kreps Results

- They greatly increased both the understanding and the generality of the risk neutral approach
- They opened the door to the relevance of martingale theory and stochastic integration
- They applied their results to the case where a collection of stock prices is a vector diffusion process, but did not proceed further
- Their assumptions about trading strategies were somewhat restrictive and thus limited the generality of their results
References


The End