Prospect Theory, Partial Liquidation and the Disposition Effect

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The Problem

• Consider an agent with prospect theory preferences who seeks to liquidate a portfolio of (divisible) claims -
  * how does the agent sell-off claims over time?
  * how does prospect theory alter the agent’s strategy vs (rational) expected utility?
  * is the strategy consistent with observed behavior eg. disposition effect?
• Examples of claims might include stocks, executive stock options, real estate, managerial projects,...
Prospect Theory (Kahneman and Tversky (1979))

- Utility defined over gains and losses relative to a reference point, rather than final wealth
- Utility function exhibits concavity in the domain of gains and convexity in the domain of losses (“S shaped”)
- Steeper for losses than for gains, a feature known as loss aversion
- Non-linear probability transformation whereby small probabilities are overweighted
• The agent has prospect theory preferences denoted by the function \( U(z); z \in \mathbb{R} \)

(I) Piecewise exponentials: (Kyle, Ou-Yang and Xiong (2006))

\[
U(z) = \begin{cases} 
\phi_1(1 - e^{-\gamma_1 z}) & z \geq 0 \\
\phi_2(e^{\gamma_2 z} - 1) & z < 0 
\end{cases}
\]  

where \( \phi_1, \phi_2, \gamma_1, \gamma_2 > 0 \).

Assume \( \phi_1 \gamma_1 < \phi_2 \gamma_2 \) so that \( U'(0-) > U'(0+) \)

(II) Piecewise power: (Tversky and Kahneman (1992))

\[
U(z) = \begin{cases} 
z^{\alpha_1} & z \geq 0 \\
-\lambda(-z)^{\alpha_2} & z < 0 
\end{cases}
\]  

where \( \alpha_1, \alpha_2 \in (0, 1) \) and \( \lambda > 1 \).

Locally infinite risk aversion, \( U'(0-) = U'(0+) = \infty \).
The Disposition Effect

- Many studies find that investors are reluctant to sell assets trading at a loss relative to the price at which they were purchased.
- For large datasets of share trades of individual investors, Odean (1998) (and others) “finds the proportion of gains realized is greater than the proportion of realized losses.”
- Disposition effects have also been found in other markets - real estate, traded options and executive stock options.
- Reluctance of managers to abandon losing projects “throwing good money after bad.”
• Prospect theory has long been recognized as one potential way of understanding the disposition effect
• Intuition that more likely to sell when \textit{ahead} (concave) and wait/gamble when \textit{behind} (convex)
• Shefrin and Statman (1985) give intuition and one period numerical eg., we provide mathematical model
• Other recent models include Kyle, Ou-Yang and Xiong (2006), Barberis and Xiong (2008, 2008) but each of these results in a "strong" disposition effect whereby the agent \textit{never} sells at a loss
**Price Dynamics**

- Let $Y_t$ denote the asset price. Work on a filtration $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$ supporting a BM $W = \{W_t, t \geq 0\}$ and assume $Y_t$ follows a time-homogeneous diffusion process with state space $\mathcal{I} \subseteq \mathbb{R}$ and

$$dY_t = \mu(Y_t)dt + \sigma(Y_t)dW_t \quad Y_0 = y_0$$

with Borel functions $\mu : \mathcal{I} \rightarrow \mathbb{R}$ and $\sigma : \mathcal{I} \rightarrow (0, \infty)$.

We assume $\mathcal{I}$ is an interval with endpoints $-\infty \leq a_{\mathcal{I}} < b_{\mathcal{I}} \leq \infty$ and that $Y$ is regular in $(a_{\mathcal{I}}, b_{\mathcal{I}})$. 

The Optimal Stopping Problem - Indivisible Claims

• Agent chooses when to receive payoff $h(Y_\tau)$, $h$ non-decreasing. Let $y_R$ denote the reference level. Interpret $y_R$ as price paid, hence “breakeven” level.

• Agent’s objective is:

$$V_1(y) = \sup_{\tau} \mathbb{E}[U(h(Y_\tau) - y_R)|Y_0 = y], \ y \in I$$  \hspace{1cm} (3)

where $U(.)$ is increasing
Heuristics

- Approach is to consider stopping times of the form “stop when price $Y$ exits an interval” and choose the “best” interval.
- The key is to transform into natural scale via $\Theta_t = s(Y_t)$ where scale function $s(.)$ is such that the scaled price $\Theta_t$ is a (local) martingale.

Define

$$g_1(\theta) := U(h(s^{-1}(\theta)) - y_R)$$

...value of the game if the asset is sold immediately
Figure 1: Stylized representation of the function $g_1(\theta)$ as a function of transformed price $\theta$, where $\theta = s(y)$. 
Proposition 1  On the interval \((s(a_I), s(b_I))\), let \(\bar{g}_1(\theta)\) be the smallest concave majorant of \(g_1(\theta) := U(h(s^{-1}(\theta)) - y_R)\).

(i) Suppose \(s(a_I) = -\infty\). Then

\[ V_1(y) = U(h(b_I) - y_R); \quad y \in (a_I, b_I) \]

(ii) Suppose \(s(a_I) > -\infty\). Then

\[ V_1(y) = \bar{g}_1(s(y)); \quad y \in (a_I, b_I) \]

Proposition 2  The solution to problem (3) with $h(y) = y$, $dY = \mu dt + \sigma dW$, and $U(z)$ is given by piecewise exponential $S$-shape, consists of four cases:

(I): If $\mu \geq 0$, the agent waits indefinitely

(II) If $\mu < 0$ and $\mu/\sigma^2 > -\frac{1}{2}\gamma_2$ and $|\mu|/\sigma^2 < \frac{1}{2}\frac{\phi_1}{\phi_2}\gamma_1$, the agent stops at and above a level $\bar{y}_u^{(1)} > y_R$ given by:

$$\bar{y}_u^{(1)} = y_R - \frac{1}{\gamma_1} \ln \left( \left( \frac{2\mu}{2\mu - \gamma_1 \sigma^2} \right) \left( \frac{\phi_1 + \phi_2}{\phi_1} \right) \right)$$

(III) If $\mu < 0$ and $\mu/\sigma^2 > -\frac{1}{2}\gamma_2$ and $|\mu|/\sigma^2 \geq \frac{1}{2}\frac{\phi_1}{\phi_2}\gamma_1$, the agent stops everywhere at and above the break-even point $y_R$, but waits below the break-even point. Thus if the agent sells, she exactly breaks even

(IV) If $\mu/\sigma^2 \leq -\frac{1}{2}\gamma_2$, the agent sells immediately at all price levels
Figure 2: (II). \( \mu = -0.03, s(y_R) = 1.455 \). The agent stops for \( \theta > 1.54 \); equivalently, for prices \( y > 1.15 \). Parameters are: \( \sigma = 0.4, \phi_1 = 0.2, \phi_2 = 1, \gamma_1 = 3, \gamma_2 = 1 \) and reference level, \( y_R = 1 \).
Remarks

• Kyle et al (2006) study this eg. using variational techniques - non-differentiability implies cannot use smooth-pasting
• ...but agent *never* chooses to sell at a loss ... so ”strong” disposition effect!
Model 2: Piecewise Power $S$-shaped utility and Exponential BM

Proposition 3 The solution to problem (3) with $h(y) = y$, $dY = Y(\mu dt + \sigma dW)$, and $U(z)$ is given by piecewise power $S$-shape, consists of three cases. Define $\beta = 1 - \frac{2\mu}{\sigma^2}$.

(I): If $\beta \leq 0$; or if $0 < \beta < \alpha_1 < 1$, the agent waits indefinitely and never liquidates

(II) If $0 < \alpha_1 < \beta \leq 1$ or $\alpha_1 = \beta < 1$, the agent stops at a level higher than the break-even point. If the agent liquidates, she does so at a gain

(III) If $\beta > 1$, the agent stops when the price reaches either of two levels. These two levels are on either side of the break-even point - liquidates either at a gain or at a loss
Figure 3: (III). $\beta = 1.5$, $\alpha_1 = 0.7$, $s(y_R) = 1$. The agent waits for $\theta \in (0.1723, 1.0105)$ and stops otherwise. Equivalently, the agent waits for $y \in (0.31, 1.007)$. Parameters are: $\lambda = 2.2$, $\alpha_2 = \alpha_1$ and reference level $y_R = 1$. 
Remarks

• Conclusions (and findings of Kyle et al) not robust to changing the $S$-shaped function
• Piecewise power functions lead to situation where if odds are bad enough (price transient to zero, a.s), agent “gives up” and sells at a loss - consistent with eg. of Shefrin and Statman (1985)
• Is it consistent with the disposition effect? Is selling at a gain more likely than at a loss?
Figure 4: Probability of liquidating at a gain in Case (III), as a function of $\beta$ and $\alpha_1$. The reference level is $y_R = 1$ and take $y = 1; \lambda = 2.2$. 
Extension to Divisible Claims

• In both piecewise exponential and piecewise power models, agent follows “all-or-nothing” sales strategy
• ...in contrast to an agent with standard concave utility (over wealth) where units are sold-off over time (cf. Grasselli and Henderson (2006), Rogers and Scheinkman (2007), or Henderson and Hobson (2008))
Concluding Remarks

- In contrast to existing literature, we provide prospect theory optimal stopping model (with Tversky and Kahneman (1992) piecewise power functions) under which the agent will liquidate at a loss, enter the position ex-ante, and will be more likely to sell at a (small) gain than a (large) loss, consistent with disposition effect.
- Agent’s strategy not robust to change in $S$-shaped function
- Extend to divisible positions and show prospect agent prefers to liquidate on an “all-or-nothing” basis