

JOHN BAEZ
University of California

The Homotopy Hypothesis

Crudely speaking, the Homotopy Hypothesis says that n -groupoids are the same as homotopy n -types - nice spaces whose homotopy groups above the n th vanish for every basepoint. We summarize the evidence for this hypothesis. Naively, one might imagine this hypothesis allows us to reduce the problem of computing homotopy groups to a purely algebraic problem. While true in principle, in practice information flows the other way: established techniques of homotopy theory can be used to study coherence laws for n -groupoids, and a bit more speculatively, n -categories in general.

IGOR BAKOVIC
Rudjer Boskovic Institute

Bigroupoid principal 2-bundles

We introduce the concept of an action of a bicategory and a bigroupoid. Namely, any bicategory naturally defines a pseudomonad on the category Cat of small categories and the actions are obtained as pseudoalgebras over that pseudomonad. Then, we describe principal 2-bundles for a given bigroupoid B , and we give the complete classification of these objects in terms of the second nonabelian cohomology with coefficients in B . Finally we show that such bigroupoid principal 2-bundles gives Glenn's simplicial 2-torsors after the application of the Duskin nerve functor for bicategories.

JULIE BERGNER
Kansas State University

Model categories Quillen equivalent to the quasi-category model structure

The model category structure for quasi-categories can be regarded as providing a model for homotopy theories or a model for $(\infty, 1)$ -categories, depending on one's point of view. In this talk we will describe various Quillen equivalences between connecting this model structure with several others: the model structure on the category of simplicial categories, two Segal category model structures on the category of Segal precategories, and the complete Segal space model structure on the category of simplicial spaces.

EUGENIA CHENG
The University of Chicago

Batanin omega-groupoids and the homotopy hypothesis

I will discuss work of Batanin, Berger and Cisinski towards proving a version of the Homotopy Hypothesis for Batanin's theory of weak omega-groupoids. Building on the work of Berger, Cisinski has proved part of Batanin's hypothesis that Batanin weak omega-groupoids model homotopy types; specifically, he proves that the homotopy category of CW-complexes can be embedded in the homotopy category of Batanin's weak omega-groupoids. This talk will be expository. In particular we will not go into technical model category theoretical details. A general idea of model categories and homotopy categories will be sufficient, as covered in Mike Shulman's talk on Tuesday.

ALISSA CRANS
Loyola Marymount University

A Survey of Higher Lie Theory

In preparation for the day's talks, we focus on categorifying Lie theory, concentrating on Lie 2-groups and Lie 2-algebras. These are categorified versions of Lie groups and Lie algebras, where we have replaced the associative law and Jacobi identity, respectively, by natural isomorphisms called the "associator" and the "Jacobiator". We will consider alternative proposals for what categorified Lie theory should look like and discuss the advantages and limitations of these different choices.

NICK GURSKI
Yale University

Tricategories

I will discuss tricategories from the ground up, assuming no prior knowledge of their technical aspects. I will show how the definition can be obtained as a naive categorification of the definition of bicategory, and I will discuss the coherence theory for tricategories with the focus being on how to use this theory to make tricategories more manageable.

ANDRE HENRIQUES**Westfaelische Wilhelms-Universitaet***Integrating L_∞ -algebras*

An *L_∞finity* algebra is a chain complex equipped with the structure of a Lie algebra, where the usual laws hold up to coherent homotopy. Examples include the canonical one-parameter family of "Lie 2-algebras" associated to any simple Lie algebra, or more generally the "Lie n-algebras" associated to cohomology classes on Lie algebras. Given any Lie n-algebra, we describe how to construct a corresponding "Lie n-group": that is, a Kan simplicial manifold.

ANDRE JOYAL**UQAM***From quasi-categories to higher categories*

Quasi-categories were introduced by Boardman and Vogt in their work on homotopy invariant algebraic structures. They belong as much to the world of homotopy theorists than to the world of category theorists. It turns out that category can be wholly extended to quasi-categories. The extension is non-trivial and it opens new avenues in every domain where category theory plays an important role: algebra, logic, computer science, homological algebra, homotopical algebra, algebraic geometry, etc. We call quasi-algebra the resulting extension of algebra. Quasi-algebra is a new methodology for handling homotopy invariant algebraic structures and coherence problems in homotopy theory. It can be used for handling coherence problems in higher category theory as well. As an example, we propose a notion of n-quasi-category by using quasi-algebra. For this we use the notion of reduced category, a set theoretic version of the notion of complete Segal space of Rezk. We conjecture that the notion of n-quasi-category is equivalent to the notion of Segal n-category introduced by Hirschowitz and Simpson.

STEVE LACK**University of Western Sydney***A survey of the theory of Gray-categories*

I'll discuss the theory of Gray-categories, including the connections with braided monoidal categories and with homotopy 3-types, the relationship between Gray-tensor products and other tensor products of 2-categories, Gray-enriched limits, and pseudomonads in Gray-categories.

AARON LAUDA
University of Cambridge

Frobenius algebras, quantum topology and higher categories

After recalling the well-known construction of 2d TQFTs from commutative Frobenius algebras, we show how this fits into a bigger picture involving open-closed TQFTs and the Fukuma-Hosano-Kawai state sum model. Some of these ideas can already be categorified to study higher dimensions. If time permits, we will also sketch how this machinery can be used in a construction of Khovanov homology, not just for links but also for tangles.

TOM LEINSTER
University of Glasgow

A survey of the theory of bicategories

A mature version of the coherence theorem for bicategories should not be limited to ‘every weak 2-category is equivalent to a strict one’: it should also say something about the functors etc. between 2-categories. With this in mind, I will discuss the various ways to collect together (strict or weak) 2-categories to form a single structure, and what coherence does and does not say about how these structures are related. For instance, if **Str-2-Cat** denotes the 3-category consisting of strict 2-categories, strict 2-functors etc., and similarly **Wk-2-Cat** (everything weak), then the inclusion **Str-2-Cat** \rightarrow **Wk-2-Cat** is *not* an equivalence. This is a precise expression of the view (long advocated by Bénabou) that the most important aspect of the theory of bicategories is not that they themselves are weak, but that the maps between them are weak.

Although this talk will consist of elementary observations, I will assume knowledge of basic bicategory theory: see for instance the references below.

- Francis Borceux, *Handbook of Categorical Algebra 1: Basic Category Theory*, Encyclopedia of Mathematics and its Applications 50, Cambridge University Press, 1994.
- Tom Leinster, Basic bicategories, math.CT/9810017, 1998.
- Ross Street, Categorical structures, in M. Hazewinkel (ed.), *Handbook of Algebra Vol. 1*, North-Holland, 1996.

PETER MAY

University of Chicago

Applications of bicategories to algebraic topology

How does duality theory extend from symmetric monoidal categories to bicategories? What does that have to do with parametrized homotopy theory? It turns out that there are two very different kinds of duality theory in parametrized homotopy, homology, and cohomology theory, and it is virtually impossible to understand them without first understanding duality in bicategories. The theory here (joint with Johann Sigurdsson) sheds new light even on classical Poincaré duality.

Duality theory in symmetric monoidal categories leads to traces of maps. What are traces in bicategories? What do they have to do with fixed point theory? It turns out (work of Kate Ponto) that to understand the converse of the Lefschetz fixed point theory, one should understand Reidemeister traces as traces in a bicategory.

I'll give an informal introduction to these ideas.

JOSHUA NICHOLS-BARRER

Massachusetts Institute of Technology

Fibred Quasi-Categories and Stacks

We discuss three distinct models for the quasi-categories of left/right/op-Cartesian/Cartesian fibrations over a simplicial set S : space-valued functors, the simplicial nerve construction of Joyal and Lurie and the opFib/S and Fib/S constructions (“fibred quasi-categories”) of the speaker’s thesis, all of which give equivalent quasi-categories. We examine at least one form of the Yoneda lemma and look at descent in these different pictures, arriving at a few appropriate quasi-categories of stacks on a site. We’ll talk a little about how moduli problems arise in geometry and how they fit naturally into the fibred quasi-category picture. If there is time, we’ll talk a bit about Lurie’s Giraud-type characterization theorem for quasi-categories of stacks in Kan complexes (what he calls ∞ -topoi, and what might more precisely be termed $(\infty, 1)$ -topoi).

SIMONA PAOLI

Macquarie University

Semistrict Tamsamani’s n -groupoids and connected n -types

The modelling of homotopy types provides an important link between higher category theory and homotopy theory. In this talk we compare two models of connected n -types: cat^{n-1} groups and Tamsamani’s weak n -groupoids (with one object). The first model arose in homotopy theory, generalizing earlier work of Whitehead on crossed modules. The second arose in higher category theory. As a result of this comparison we identify

a subcategory of Tamsamani's weak n -groupoids whose objects are 'less weak' than the ones used by Tamsamani to model connected n -types. Thus objects of this subcategory are semistrict. We show that every Tamsamani weak n -groupoid representing a connected n -type is in a suitable sense equivalent to a semistrict one. A large part of the talk will be devoted to the case $n = 3$, where we will also make a connection with Gray groupoids. We will then illustrate the main ideas involved in the argument for higher n .

URS SCHREIBER
Universitt Hamburg

Parallel transport in low dimensions

A vector bundle with connection can be conceived as a suitable parallel transport 1-functor from paths in base space to vector spaces. We categorify this and discuss various examples of locally trivializable 2-transport, some of them relevant for the description of charged 2-particles in formal high energy physics. We also point out how some surface transport really has to be conceived as 3-transport and we discuss the Chern-Simons 3-transport.

URS SCHREIBER
Universitt Hamburg

On 2-dimensional QFT: from arrows to disks

The quantization of the charged point particle relates two 1-functors into vector spaces: the parallel transport on target space is turned by quantization into propagation on parameter space. We categorify this and discuss how the 2-dimensional quantum field theory describing the charged 2-particle relates two 2-functors with values in 2-vector spaces. Our goal is to explain and illuminate this way aspects of the categorical description of rational conformal field theory by the FRS theorem; like the relation of boundary fields (living on "D-branes") to internal modules and of bulk fields to internal bimodules.

MICHAEL SHULMAN
The University of Chicago

Model categories for beginners

This talk will begin with some fuzzy thoughts about 'sameness' and 'homotopy', what a 'homotopy category' is and what it's good for (like comparing different models for homotopy types, or for higher categories), and the tools people use to get a handle on it. Then we'll work up to model categories, which are a neat compact package that incorporates all the information about equivalences, homotopy, and useful tools for reasoning about them. We'll also briefly discuss Quillen adjunctions and Quillen equivalences, which are two of the ways model categories make it easier to compare different models of the 'same'

objects. The goal is to convey enough of the language of model categories that everyone can follow the later talks in which it is used.

MICHAEL SHULMAN
The University of Chicago

Introduction to quasicategories

Quasicategories are one way to define infinity-categories in which all cells of dimension above 1 are invertible, also known as ‘(infinity,1)-categories’. Following the homotopy-hypothesis intuition, we can also describe them as ‘categories up to coherent homotopy’; this is how they originally arose in the work of Boardman and Vogt. Of the known definitions for (infinity,1)-categories, quasicategories are one of the simplest and most tractable. For instance, they support good theories of limits and colimits, functor categories, fibrations, and adjunctions. This talk will be a basic introduction to the theory of quasicategories, with an emphasis on intuition and why you should care. We’ll do some things explicitly, to get a feel for how things work, and then say a bit about the model structure for quasicategories and related tools that help manage the unavoidable combinatorial complexity. Later talks will expand on various aspects of the theory of quasicategories.

DANNY STEVENSON
University of California, Riverside

Lie 2-algebras and Higher Gauge Theory

There has been recent interest in generalisations of principal bundles, in which the structure group is replaced by a 2-group, which is a certain kind of groupoid. Understanding the geometry of these generalised principal bundles, known as “2-bundles” or “nonabelian gerbes”, forms part of the subject of higher gauge theory. In the case of ordinary principal bundles, Atiyah gave an elegant formulation of the notion of a connection in terms of a splitting of an extension of Lie algebras. Just as the usual cohomology of groups allows one to classify central extensions, Schreier theory allows one to classify arbitrary extensions of groups. Atiyah’s approach to connections and curvature for bundles can be nicely understood using a Lie algebra version of Schreier theory. After reviewing the basic concepts of Lie 2-algebras, we show that Schreier theory for Lie 2-algebras clarifies the theory of connections and curvature for 2-bundles.