EFFICIENT PARAMETRIC AMPLIFICATION IN DOUBLE $\Lambda$ SYSTEMS IN THE ABSENCE OF TWO-PHOTON MAXIMAL COHERENCE

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Two-level system: pump and probe

- If self-focusing and diffraction balanced, Gaussian pump propagates as a spatial soliton
- If pump-induced cross focusing of probe balanced by diffraction, the weak probe propagates as if it is a spatial soliton
- If pump is sufficiently intense, radiation generated at FWM frequency
- Parametric amplification between probe and FWM via the pump
- Process occurs over many diffraction lengths: EIPM important
Self-focusing

Nonlinear refractive index

\[ n = n_0 + n_2 I \]

Thus

\[ n_2 \Rightarrow \frac{dn}{dI} \]

Focusing obtained when

\[ \frac{dn}{dI} > 0 \]

Self-focusing obtained when laser is detuned to the blue!
Coherent Population Trapping (CPT)

- Fields are equally intense
- Population trapped in lower levels
- Two-photon coherence is maximal: $|\rho_{21}|^2 = \rho_{11}\rho_{22} = \frac{1}{4}$
- Zero absorption
Electromagnetically Induced Transparency (EIT)

- Pump and probe fields
- Population optically pumped into state $|2\rangle$
- Two-photon coherence is small
Double $\Lambda$ System

- Several possible configurations
- Highly efficient FWM when CPT occurs (Harris)
Model

- Three or four beams with Gaussian transverse intensity profile (GTIP)
- Beams copropagate
- Assume steady-state
- Compare results with plane-wave beams
- Cases studied:
  - CPT with maximal coherence, either initially or on propagation
  - Four identical beams with 0 and $\pi$ phase
  - Three strong fields
  - Two strong fields
  - Incoherent pumping from state 2 to 4 (not shown here)
  - Raman detuning (not shown here)
Maxwell-Bloch Equations

\[
\frac{\partial}{\partial z} V'_{ij} = \frac{i}{4L_D} \nabla^2 V'_{ij} + \frac{i}{L_{ij}} \rho'_{ij}
\]

\[
\nabla_T^2 = \frac{\partial^2}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial}{\partial \xi} + \frac{1}{\xi^2} \frac{\partial^2}{\partial \theta^2}
\]

| Rabi frequency for $|j\rangle \rightarrow |i\rangle$ transition | $V'_{ij}$ |
|---------------------------------------------------------------|----------|
| Density matrix element                                        | $\rho'_{ij}$ |
| Diffraction length                                            | $L_D$    |
| Interaction length                                            | $L_{ij}$ |
| Direction of propagation                                      | $z$      |
| Transverse radial coordinate                                  | $\xi$    |
Bloch Equations

\[ \dot{\rho}_{11} = i(V_{13} \rho'_{31} + V_{14} \rho'_{41} - V_{31} \rho'_{13} - V_{41} \rho'_{14}) - \gamma_{12} \rho_{11} + \gamma_{21} \rho_{22} + \gamma_{31} \rho_{33} + \gamma_{41} \rho_{44}, \]
\[ \dot{\rho}_{22} = i(V_{23} \rho'_{32} + V_{24} \rho'_{42} - V_{32} \rho'_{23} - V_{42} \rho'_{24}) + \gamma_{12} \rho_{22} - \gamma_{21} \rho_{11} + \gamma_{32} \rho_{33} + \gamma_{42} \rho_{44}, \]
\[ \dot{\rho}_{33} = i(V_{31} \rho'_{13} + V_{32} \rho'_{23} - V_{13} \rho'_{31} - V_{23} \rho'_{32}) - (\gamma_{13} + \gamma_{23}) \rho_{33} + \gamma_{43} \rho_{44}, \]
\[ \dot{\rho}_{44} = i(V_{41} \rho'_{14} + V_{42} \rho'_{24} - V_{14} \rho'_{41} - V_{24} \rho'_{42}) - (\gamma_{41} + \gamma_{42} + \gamma_{43}) \rho_{44} - r_{24} (\rho_{44} - \rho_{22}), \]
\[ \dot{\rho}'_{21} = i(V_{23} \rho'_{31} + a V_{24} \rho'_{41} - V_{31} \rho'_{23} - a V_{41} \rho'_{24}) - (\Gamma_{21} + i\Delta_{21}) \rho'_{21}, \]
\[ \dot{\rho}'_{31} = i(V_{31} \rho'_{11} + V_{32} \rho'_{21} - V_{13} \rho'_{33} - V_{41} \rho'_{34}) - (\Gamma_{31} + i\Delta_{31}) \rho'_{31}, \]
\[ \dot{\rho}'_{32} = i(V_{32} \rho'_{22} + V_{31} \rho'_{12} - V_{23} \rho'_{33} - a * V_{42} \rho'_{34}) - (\Gamma_{32} + i\Delta_{32}) \rho'_{32}, \]
\[ \dot{\rho}'_{41} = i(V_{41} \rho'_{11} + a * V_{42} \rho'_{21} - V_{31} \rho'_{43} - V_{41} \rho'_{44}) - (\Gamma_{41} + i\Delta_{41}) \rho'_{41}, \]
\[ \dot{\rho}'_{42} = i(V_{42} \rho'_{22} + a V_{41} \rho'_{12} - a V_{32} \rho'_{43} - V_{42} \rho'_{44}) - (\Gamma_{42} + i\Delta_{42}) \rho'_{42}, \]
\[ \dot{\rho}'_{43} = i(V_{41} \rho'_{13} + a * V_{42} \rho'_{23} - V_{13} \rho'_{41} - a * V_{23} \rho'_{42}) - (\Gamma_{43} + i\Delta_{43}) \rho'_{43}. \]
Notation

- $a = \exp(i\Phi)$, $\Phi = \varphi_{31} - \varphi_{32} + \varphi_{42} - \varphi_{41}$ is initial relative phase,
- $\gamma_{kl}$ is longitudinal decay rate from state $|k\rangle \to |l\rangle$,
- $\gamma_i$ is total decay rate from state $|i\rangle$,
- $\Gamma_{kl} = 0.5(\gamma_k + \gamma_l) + \Gamma_{kl}^* + r_{24}\delta_{k4}\delta_{l2}$ is transverse decay rate,
- $\Gamma_{kl}^*$ is rate of phase-changing collisions,
- $r_{24}$ is rate of incoherent pumping from state $|2\rangle \to |4\rangle$,
- $\Delta_{ij} = \omega'_{ij} - \omega_{ij}$ is one-photon detuning from resonance; $i = (3, 4)$, $j = (1, 2)$,
- $\rho'_{ij} = \rho_{ij} \exp[-i(\Delta_{ij} t + k_{ij} z - \varphi_{ij})]$,
- $\rho'_{21} = \rho_{21} \exp\{-i[(\Delta_{31} - \Delta_{32}) t + (k_{31} - k_{32}) z - (\varphi_{31} - \varphi_{32})]\}$,
- $\rho'_{43} = \rho_{43} \exp\{-i[(\Delta_{41} - \Delta_{31}) t + (k_{41} - k_{31}) z - (\varphi_{41} - \varphi_{31})]\}$. 
Multiphoton Resonance Condition

\[ \omega_{31} - \omega_{32} + \omega_{42} - \omega_{41} = 0 \]

\[ \Rightarrow \Delta_{31} - \Delta_{32} = \Delta_{41} - \Delta_{42} = \Delta_{21}, \quad \text{two-photon detuning} \]

or \[ \Delta_{41} - \Delta_{31} = \Delta_{42} - \Delta_{32} = \Delta_{43}, \]

\[ \Rightarrow \Delta k_0 = k_{31} - k_{32} + k_{42} - k_{41} = 0, \quad \text{initial phase mismatch} \]
Analytical Solution

$$\rho'_{ij} = \rho^{(1)}_{ij} + \rho^{(3)}_{ij},$$

$$\rho'_{31} = \chi^{(1)}_{31} V_{31} + a \chi^{(3)}_{31} V_{32} V_{24} V_{41},$$

$$\rho'_{32} = \chi^{(1)}_{32} V_{32} + a^* \chi^{(3)}_{32} V_{31} V_{14} V_{42},$$

$$\rho'_{41} = \chi^{(1)}_{41} V_{41} + a^* \chi^{(3)}_{41} V_{42} V_{23} V_{31},$$

$$\rho'_{42} = \chi^{(1)}_{42} V_{42} + a \chi^{(3)}_{42} V_{41} V_{13} V_{32}.$$
CPT and Maximal Two-Photon Coherence

- When CPT exists, no absorption or focusing or defocusing occurs since $\chi^{(1)}=0$
- Phase-matching unimportant
- Maximum FWM occurs within a propagation distance less than diffraction length
- This is completely different from a two-level system
CPT and Maximal Two-Photon Coherence

$V_{31} = 8; \quad V_{32} = 8; \quad V_{42} = 1; \quad V_{41} = 0.001;$

$\Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = 4;$

$L_{NL}/L_D = 1.66 \times 10^{-3};$
Transverse Intensity Profile
Comparison: GTIP’s and PW’s
CPT and Maximal Two-Photon Coherence

\[ V_{31} = 8; \quad V_{32} = 8; \quad V_{42} = 8; \quad V_{41} = 0.001; \]

\[ \Delta_{31} = \Delta_{32} = \pm 4; \quad \Delta_{41} = \Delta_{42} = \pm 100; \]

\[ \frac{L_{NL}}{L_D} = 1.66 \times 10^{-4}; \]
Transverse Intensity Profile
Comparison: GTIP’s and PW’s
Onset of CPT vs. Maximum Conversion

• For detuning $\Delta_{41} = \Delta_{42} = \pm 100$, CPT exists at the outset. Maximum conversion of 87% occurs at 0.047.
• For detuning $\Delta_{41} = \Delta_{42} = \pm 10$, CPT occurs at 0.1, whereas maximum conversion of 73% occurs at 0.009.
• Thus, it is possible to get efficient conversion before CPT, without focusing, defocusing or ring formation.
CPT and Maximal Two-Photon Coherence

$V_{31}=8; \quad V_{32}=8; \quad V_{42}=8; \quad V_{41}=0.001;\quad \Delta_{31}=\Delta_{32}=\pm 4; \quad \Delta_{41}=\Delta_{42}=\pm 10;\quad L_{NL}/L_D=1.66\times 10^{-4};
Focusing

- Focusing can occur before CPT established
- Beams blue-detuned
- Nonlinear length sufficiently long
- Maximum FWM can still occur within a short propagation distance
- Phase-matching still unimportant
Three strong lasers

$V_{31} = 8; \quad V_{42} = 8; \quad V_{32} = 8; \quad V_{41} = 0.001;\\
\Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = -4;\\
\frac{L_L}{L_D} = 1.52 \times 10^{-3};$
Initial gain at FWM frequency

\( \text{Gain} \)

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<th>Amplitude</th>
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\( z/L_d \)
Focusing

\[ |V_{31}| \text{ Amplitude} \]

\[ |V_{32}| \text{ Amplitude} \]

\[ |V_{41}| \text{ Amplitude} \]

\[ |V_{42}| \text{ Amplitude} \]
Maximum focusing and conversion

\begin{align*}
|V_{31}| & \quad \text{Amplitude} \\
|V_{32}| & \quad \text{Amplitude} \\
|V_{41}| & \quad \text{Amplitude} \\
|V_{42}| & \quad \text{Amplitude}
\end{align*}

\begin{align*}
\text{Focusing} \\
\text{Focusing} \\
\text{Focusing} \\
\text{Focusing}
\end{align*}
Comparison: GTIP’s and PW’s

![Graphs showing comparison between GTIP's and PW's](image-url)
Two strong lasers: strong-weak-strong – weak configuration

\[ V_{31} = 4; \quad V_{42} = 4; \quad V_{32} = 0.1; \quad V_{41} = 0.001; \]

\[ \Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = -4; \]

\[ L_L / L_D = 1.66 \times 10^{-3}; \]
Two strong lasers: strong-weak-strong – weak configuration

\[ V_{31} = 4; \quad V_{42} = 4; \quad V_{32} = 0.1; \quad V_{41} = 0.001; \]

\[ \Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = -4; \]

\[ \frac{L_L}{L_D} = 1.66 \times 10^{-3}; \]
Two strong lasers:

**strong-weak-strong – weak configuration**

\[ V_{31} = 4; \quad V_{42} = 4; \quad V_{32} = 0.1; \quad V_{41} = 0.001; \]

\[ \Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = -4; \]

\[ \frac{L_{NL}}{L_D} = 1.66 \times 10^{-3}; \]
Focusing on propagation
Phase dependence

• When field at FWM frequency absent or small at outset, phase is unimportant
• When field at FWM frequency present at outset, dramatic phase effects can be obtained
Four Strong Fields: phase 0 vs. $\pi$

$V_{31} = 4; \quad V_{42} = 4; \quad V_{32} = 4; \quad V_{41} = 4; \quad \Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = -4; \quad L_{NL}/L_D = 1.11 \times 10^{-3};$
Zero Phase
No Change on Propagation: CPT
Four strong fields: phase $\pi$

$V_{31} = 4; \quad V_{42} = 4; \quad V_{32} = 4; \quad V_{41} = 4;$

$\Delta_{31} = \Delta_{32} = \Delta_{41} = \Delta_{42} = -4;$

$L_{NL}/L_D = 1.11 \times 10^{-3}; \quad \Phi = \pi;$

![Graph showing GTIP on-axis population and coherence](image)
Phase $\pi$

Focusing on propagation: no CPT
Conclusions

- Efficient FWM in double lambda systems can be obtained even before CPT occurs.
- It is obtained at short propagation distances.
- Focusing can be obtained by blue one-photon detuning.
- Often accompanied by ring formation.