Universal Approach to Dynamical Control of Decay and Decoherence

G. Gordon, D. Petrosyan, S. Pellegrin, A. G. Kofman and G. Kurizki,

The Weizmann Institute of Science,
Rehovot 76100, Israel
Dynamical control of decay and decoherence: Universal formula


Weak coupling to environment: \( \hat{V}_s = \sum_j \mu_j \langle e \rangle \langle j \rangle + \text{h.c.} \)
Amplitude/phase modulation/perturbation: \( \hat{V}(t) = \epsilon(t)\hat{V}_s \).

Exact (reversible) evolution:

\[
\dot{\alpha} \equiv \frac{d}{dt} \langle e | \Psi(t) \rangle = - \int_0^t dt' \epsilon^*(t') \epsilon(t) \Phi(t-t') e^{i\omega_a(t-t')} \alpha(t'),
\]

\( \Phi(t-t') = \sum_j |\mu_{ej}|^2 e^{-i\omega_j(t-t')} \) (reservoir memory function),
\( \alpha(t) \) decays slower than \( \Phi(t) \) \( \Rightarrow \) \( \alpha(t') \approx \alpha(t) \).

\( \Rightarrow \) Coherent or random \( \epsilon(t) \) obeys universal modified decay rate:

\[
R(t) = 2\pi \int_{-\infty}^{\infty} d\omega G(\omega + \omega_a) F_t(\omega).
\]

Overlap of reservoir coupling spectrum

\[
G(\omega) = \pi^{-1} \text{Re} \int_0^{\infty} dt e^{i\omega t} \Phi(t) \rightarrow \rho(\omega) |\mu(\omega)|^2
\]

and the spectral intensity of modulation

\[
F_t(\omega) = |\epsilon_t(\omega)|^2.
\]

Overlap of \( G(\omega) \) and \( F_t(\omega) \) determines either suppressed or enhanced coupling to environment: Quantum Zeno effect (QZE) or anti-zeno effect (AZE).
Tunneling - barrier modulation: “α-decay” control
Fischer, Gutierrez and Raizen, PRL 87, 040402 ('01):
Optical potential

“Washboard” potential on – $\tau_1$, off – $\tau_0 \gg \tau_1$.

\[ G(\omega) \text{ does not change over } 2\pi/\tau_0 \implies F_t(\omega) \sim \text{measurement-induced broadening.} \; \nu \sim 1/\tau_1: \text{MHz.} \]

**QZE conditions:** $\nu \gg \Gamma_R \gtrsim 1/\tau_c$.  
**AZE conditions:** $1/\tau_c \ll \nu \ll \Gamma_R$

1, 4 – no modulation. 2 – QZE (compared to curve 1) $\tau_1 = 0.8 \mu$s. 3 – AZE (compared to curve 4) $\tau_1 = 2 \mu$s, $\tau_0 \simeq 50 \mu$s.
Lattice tilt (acceleration) – 15 km/s$^2$ ,
Na atoms barrier $\omega_g \sim 100$ kHz.
Josephson junction with bias-current (GHz) modulation: "Washboard" potential control.


(a) $G_{n=12}(\omega)$ and $F_{t=4\tau_0}(\omega)$ with $\tau_1 = 1/\omega_0 \sim 0.1$ ns, $\tau_0 = 5\tau_1 \, (\omega_0 -$ fundamental frequency in the well).

(b) $G_{n=15}(\omega)$, $\tau_1 = 0.3/\omega_0$, and $F_{t=4\tau_0}(\omega)$.

$$R_n \approx \frac{2\pi \tau_1}{\tau_0} \sum_{k=-\infty}^{\infty} \text{sinc}^2 \left( \frac{k\pi \tau_1}{\tau_0} \right) G \left( \omega_n + \frac{2k\pi}{\tau_0} \right).$$

QZE: $\frac{1}{\tau_1} \gg \frac{\omega_0}{\text{F}_t \, \text{width}} \rightarrow R(\tau_1) \ll R_0 = 2\pi G(\omega).$

(a) $R$ (in units of Golden-Rule rate $R_{GR}$) for $n = 12$, as a function of interruption time $\tau_1$ (in units of $1/\omega_0$) for $\tau_0 = 5\tau_1$ (curve 1) and $\tau_0 = 50\tau_1$ (curve 2).

(b) for $n = 15$, $R$ exhibits QZE behavior. Upper inset—$P$ vs. total time $t$, showing impulsive jumps: $I_b = 0.9928 \pm 2 \times 10^{-4} I_c$. 

\[ \begin{array}{c}
\text{(a) } G_{n=12}(\omega) \text{ and } F_{t=4\tau_0}(\omega) \\
\text{(b) } G_{n=15}(\omega), \tau_1 = 0.3/\omega_0, \text{ and } F_{t=4\tau_0}(\omega).
\end{array} \]
Dynamic (coherent) control of qubit decoherence

A. G. Kofman and G. Kurizki, PRL 87, 270405 (2001)

a) **Resonant field** can dynamically reduce proper dephasing ($\mathcal{E}(t)$ fluctuations).

$$\omega_e = \bar{\omega}_e + \epsilon(t), \quad T_2^{-1} = \langle \epsilon^2 \rangle > \tau_c.$$ 

$\tau_c$ - correlation time

$$\Omega \gg 1/\tau_c \rightarrow T'_2 \gg T_2(\Omega\tau_c)^2.$$ 

CW dynamical decoupling simpler than an echo ("bang-bang") pulse sequence. For spectrally-biased fluctuations: usual ”bang-bang” fails, tailor $\Omega(t)$.

b) **Phase modulation (PM)**: control of vibrational decay.

AC Stark modulation: $\delta(t) \sim \Omega_s^2(t)/\Delta$.

$$F_t(\omega) \sim \sum_k |\epsilon_k|^2 \delta(\omega - \omega_k).$$

$$R \approx 2\pi \sum_k |\epsilon_k|^2 G(\omega_a + \omega_k).$$

Phase jumps by $\phi$

at $\tau, 2\tau, \ldots$

Periodic PM with $\phi \ll 1$ – most effective near band edge.

Random PM ($QZE$) – ineffective.

Periodic PM with $\phi = \pi$ (Agarwal, Scully, Walther, 2001)– most effective for lorentzian bands.
Dynamical control of qubit decoherence at finite $T$


Zwanzig’s method used to write most general Master Eq. for driven/modulated systems, coupled to bath B, without RWA:

$$\dot{\rho} = -\frac{i}{\hbar}[H_S(t), \rho] + \int_0^t dt' \{ \Phi_T(t, t') [\tilde{S}(t', t) \rho \tilde{S}(t)] - S(t) \tilde{S}(t', t) \rho \} + \text{H.c}.$$  

Quasiperiodic modulation of $S(t) \propto \epsilon(t) = \sum_k \epsilon_k e^{i\omega_k t}$ ($k = 0, \pm 1, \ldots$),

$$R_{\epsilon(g)}(t \to \infty) = 2\pi \int_{-\infty}^{\infty} d\omega F(\omega) G_T(\pm \omega) = 2\pi \sum_k |\epsilon_k|^2 G_T(\pm (\omega_a + \omega_k)) \quad (1)$$

$$G_T(-\omega) = e^{-\beta \omega} G_T(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} \Phi_T(t) e^{-i\omega t} dt \quad (2)$$

Fast modulation, high $\omega_k$: Non-RWA $g \rightarrow e$ transitions even at $T = 0$!

![Bath spectrum](image1.png)

Solid: $G_0(\omega)$; dashed: $G_S(\omega) = [G_T(\omega) + G_T(-\omega)]/2$, $\beta = 10/\omega_D$; dark: $F(\omega)$. $\omega_a = 0.94\omega_D$. 3: $\phi = -0.15$; 4: $\phi = \pi$. Curve 3 is optimal. Dotted: RWA.
Qubit decoherence control: Conclusions

A. How to control state decay into energy continuum/reservoir? Perturb system on quasi-reversible memory time scale.

B. Our simple universal formula results in general criteria for dynamical control of decay, decoherence, and quantum information (QI)/fidelity loss.

C. We considered in detail various systems: tunneling in optical lattices, Josephson junctions, entangled photon states.

D. Coherent (unitary) modulation of the coupling to the reservoir (continuum) can be designed for much more effective suppression of decoherence/QI loss than QZE.

E. We account for thermal and antiresonant (non-RWA) effects: reservoir-induced excitation of the system at $T = 0(!)$ in the presence of phase modulation.

F. Radiative decay requires different control: Subradiant two-atom interference or sudden phase jumps near continuum edge.
Sudden Change Dynamics

\[
\alpha_{\text{dyn}}(t) = \alpha_A^{\text{stat}}(\tau) \alpha_B^{\text{stat}}(t-\tau) + \int_0^\infty \beta_{\omega,A}^{\text{stat}}(\tau) \beta_{\omega,B}^{\text{stat}}(t-\tau) \rho(\omega) \, d\omega, \quad t \geq \tau
\]

\[
\alpha_{A/B}^{\text{stat}} = \text{excitation amplitude}
\]

\[
\beta_{\omega,A/B}^{\text{stat}} = \text{mode } \omega \text{ amplitude}
\]

\(\omega_A \rightarrow \omega_B \) increased detuning

\(\omega_B \rightarrow \omega_A \) reduced detuning
Nonadiabatic Dynamical Protection from Decoherence in PBGs: Periodic Frequency Jumps
Sophie Pellegrin & Gershon Kurizki

Fidelity and phase gates

Periodic “counterintuitive” sudden changes

$\pi/2$ phase shift

Static case

Gate
Dipole-dipole interacting diatom qubit

Eigenstates of the system:
\[ |G\rangle = |g_1g_2\rangle, \quad |E\rangle = |e_1e_2\rangle, \quad |\pm\rangle = \frac{1}{\sqrt{2}} (|e_1g_2\rangle \pm |g_1e_2\rangle) \]

For \( \zeta \ll 1 \) \( \Rightarrow \) \( \Delta \approx \frac{3\gamma}{4\zeta^3} \gg \gamma, \quad \Gamma_+ \approx \gamma, \quad \Gamma_-=\Gamma_E \approx 2\gamma \)

\( |G\rangle \) and \( |\pm\rangle \) are the qubit states.

\[ \Omega_- \approx \frac{\Omega(\vec{r}_{12})}{\sqrt{2}} = \frac{\Omega\zeta}{\sqrt{2}}, \quad \Omega_+ = \sqrt{2}\Omega \quad (\zeta \ll 1) \]

Single-qubit gate operation—rotation:
\[ T_{\text{flip}} = \frac{\pi}{2\Omega_-} \Rightarrow P_{\text{decay}} = \Gamma_- T_{\text{flip}} = \frac{\pi\gamma\zeta}{5\sqrt{2}\Omega} \]

Take: \( \zeta \approx 0.02, \Omega/\gamma \approx 30 \Rightarrow P_{\text{decay}} \approx 3 \times 10^{-4}, \Delta \approx 10^5\gamma, \gamma T_{\text{flip}} \approx 3.7 \)
Dynamical Control of Multiparticle/Multilevel Systems

G. Gordon, G. Kurizki, A. Kofman

- Common ground state and n excited states, energies $\omega_n$.
- Collection of reservoirs: partial or no-cross correlations.
- Modulating AC Stark shifts via $\varepsilon_n(t)$.

Matrix equation for the excited-states vector $\alpha = \{\alpha_n\}$

$$\frac{\partial \alpha(t)}{\partial t} = -i\Omega \alpha(t) - \int_0^t dt' K(t, t') \Phi(t-t') e^{i\omega_n t - i\omega_n t} \alpha(t')$$

Rabi matrix: $\Omega_{nn'} = \mu_{nn'} \cdot \vec{E}_n(t)$ computing/entanglement

Modulation matrix: $K_{nn'}(t, t') = \varepsilon^*_n(t) \varepsilon_{n'}(t')$, $F_t(\omega)$: Spectral Density of $K(t, t')$

Relaxation matrix: $G_{nn}(\omega) = \sum_{\lambda} g_{n\lambda}^* g_{n'\lambda} \delta(\omega - \omega_{\lambda})$, $g_{n\lambda} = \mu_ng \cos^2 \theta_{n\lambda}$

cross-correlations: $\cos \theta_{n\lambda} \cos \theta_{n'\lambda}$

$$\alpha(t) = e^{-R(t)} \alpha(0)$$

$$R_{nn'}(t) = 2\pi e^{i(\omega_n - \omega_{n'}) t} \int_{-\infty}^{\infty} d\omega G_{nn'}(\omega + \omega_{n'}) F_t, nn'(\omega)$$

Minimize $|R_{nn'}(t)|$, by choosing appropriate $\varepsilon_n(t)$
Create quasi "decoherence-free subspaces" although $\{G_{nn'}\} \neq 0$

Compare:
Zanardi&Rasetti
Lidar&Whaley

$$\frac{N_{control}}{\text{no. of control parameters}} > \frac{n(n+1)}{2 \text{ no. of eqs.}}$$
Coherent quasi-periodic modulation

\[ \mathcal{E}_n(t) = \sum_k \kappa_{n,k} e^{-i \nu_{n,k} t} \]

\[ \sum_k |\kappa_{n,k}|^2 = 1 \]

2nk degrees of freedom with n constraints. For a given set of \( \nu_{n,k} \)
Search for \( \kappa_{n,k} \) such that \( \sum_{nn'} |R_{nn'}(t)| \to 0 \)

**Long time modulation:** QZE or AZE

\[ R_{nn'}(t) = 2\pi t \delta_{n,n'} \sum_k |\kappa_{k,n}|^2 G_{nn'}(\omega_n + \nu_{nk}) \]

**Ultrashort time modulation:** Full reversibility

\[ R_{nn'}(t) = t^2 e^{i(\omega_n - \omega_{n'}) t} \int d\omega G_{nn'}(\omega) \sum_{kl} \kappa_{n,k} \kappa_{n',l} \]

**No modulation:** Golden rule

\[ R_{nn'}^{ref} = 2\pi t e^{i(\omega_n - \omega_{n'}) t} G_{nn'}(\omega_{n'}) \]
Numerical examples

Relaxation matrix:
\[ G_{nn'}(\omega) = \cos \theta_n \cos \theta_{n'} e^{-\omega^2/2\Gamma^2} \]

Modulation frequencies:
- \( \nu_{1,1} = -p\Gamma \) \( \Gamma \) - reservoir width
- \( \nu_{1,2} = p\Gamma \)
- \( \nu_{2,1} = -p\Gamma + \Delta \)
- \( \nu_{2,2} = p\Gamma + \Delta \)

\( n=2 \)

Long time modulations \( t \gg 1/\Delta, 1/\Gamma \) QZE or AZE

Short time modulations \( t \sim 1/\Delta, 1/\Gamma \) Channels interference

Ultrashort time modulations \( t \ll 1/\Delta, 1/\Gamma \) Full reversibility & \( |R_{nn'}| \) suppression

\( n=4 \)

Preliminary results

\[ \sum_{nn'} |R_{nn'}(t)| / \sum_{nn'} |R_{nn'}^{ref}| \text{ [dB]} \]

\[ \sum_{nn'} |R_{nn'}(t)| / \sum_{nn'} |R_{nn'}^{ref}| \text{ [dB]} \]

\[ \log_{10} p \]
Conclusions

• Efficient control of multi-qubit decoherence (also chaos) by multiple pulsed AC Stark shifts. Pulse engineering replaces ancilla.
• Works for both local and correlated reservoirs.
• Decoherence suppressed for AC Stark shifts within bath spectrum due to short-time multichannel interference.
• Unified dynamical theory of driven multipartite coupling to arbitrary environments & unitary control of their irreversibility & classicality.