
Fields Quantitative Finance Seminar
Toronto, 27th November 2013

Kartik Anand, Financial Stability Department, Bank of Canada

The views expressed here do not necessarily reflect those of the Bank of Canada’s Governing Council
Agenda

1. Macro Stress Tests (MSTs) at the Bank of Canada (BoC)
2. MFRAF Overview
   a. Solvency risk
   b. Funding liquidity risk
   c. Information contagion / Contagious runs
   d. Network effects
3. Calibrating MFRAF
4. Some hypothetical results
5. Conclusion
1. Overview of MSTs

- Annual exercise conducted jointly by the BoC and OSFI

- **Objective:** To assess the resilience of the financial system to extreme but plausible shocks

- Involves the “big six” Canadian banks

1. MST scenario

2. Bottom-up stress test exercise:
   a. Bank’s apply MST scenario to their balance sheet
   b. Focuses on solvency risk only

3. Top-down stress test exercise:
   a. MFRAF
1. Example of a MST scenario

- Materialization of key risks identified in the FSR, e.g.,
  - Euro area crisis
  - Canadian household finance and housing price shock

- **Trigger**: Disorderly default of a peripheral eurozone country

- **Transmission mechanisms**: Disruption in funding markets; financial contagion; adverse confidence and wealth effects

- **Outcome**: Severe and persistent economic recession and slow recovery over a 3-year horizon
1. Bottom-up stress testing

- Banks apply the MST scenario to their balance sheets using internal models and report the results back to OSFI.

- The exercises yield detailed information on the resilience of banks.

- Drawbacks:
  - Does not offer a systemic perspective, as it ignores liquidity risks and network effects.
1. Top-down stress testing: MFRAF

- MST scenario is consistently applied to a suite of internally developed and calibrated models (MFRAF) that accounts for different risks:
  - Solvency risk
  - Funding liquidity risk and contagious runs
  - Interbank network spillovers

- MFRAF provides a systemic perspective on risks to the banking sector, and also serves as a consistency check for the bottom-up stress test exercise

- Disadvantage: “A model is only as good as its assumptions”
1. Objectives of MFRAF

- Provides a consistency check for the bottom-up exercise

- Quantitative tool for assessing the systemic impact of key risks to the financial system

- Framework to look at policy options, e.g.
  - Capital vs. liquidity requirements
  - Measure of systemic risk contribution of an individual bank
2. Structure of MFRAF
2. MFRAF: sequential framework

- **Solvency risk**
  - Core credit model

- **Stress Scenario**
  - Default (losses) due to (non-banks) borrowers defaults

- **Funding Liquidity Risk**
  - Default (losses) due to interactions between funding strategies and solvency concerns

- **Network Effects**
  - Default (losses) due to interbank counterparty defaults

- **Aggregate loss distribution**
2. MFRAF: sequential framework (continued)

Stress scenario

Corporate and household defaults

Credit losses for banks

Banks’ capital falls

Banks default

Banks’ creditors review losses

Creditors rollover

Creditors withdraw

Interbank network

Creditors of solvent banks update their beliefs on fire-sales

Bank defaults due to illiquidity

Aggregate loss distribution
2. Timelines

Bottom-up stress test

Top-down stress test (MFRAF)

Funding liquidity risk

Network effects

CET1 ratio with systemic effects

Year 1

2012Q2

X months

2013Q1

12 - X months

2014Q1

2015Q1

Year 2

Year 3
2. Bank’s $t_0$ (initial) balance sheet

- **Illiquid assets** $I_0$
- **Liquid assets** $M_0$
- **Long term liabilities**
- **Short term liabilities** (coming due in $X$ months) $S_0$
- **Capital** $E_0$
2.a Solvency risk

- Banks’ loan portfolios subject to credit risk across different sector, e.g., business, government, consumer

Expected Losses
\[ \text{Expected Losses} = \text{Probability of Default} \times \text{Loss Given Default} \times \text{Exposure at Default} \]

- PDs (distribution) – function of macro-variables.
- LGDs – judgement based, e.g., from bottom-up exercises
- EADs – banks’ regulatory reported values

- Derive annual loss distributions for each sector and for each bank
2. a Solvency risk

- Each realization of the expected annual losses, $P^{(E)}$, must be translated into the time structure of MFRAF

- Losses $P^{(1)}$ realized at date $t_1$ (interim period)

- Losses $P^{(2)}$ realized at date $t_2$ (final period)

\[
P^{(1)} = \frac{P^{(E)}}{12/X}, \text{ and } P^{(2)} = P^{(E)} \times \left(1 - \frac{1}{12/X}\right)
\]
2.a Bank’s ex-post \((t_2)\) balance sheet

Illiquid assets

\[ I_0 - P^{(1)} - P^{(2)} \]

Liquid assets

\[ M_0 \]

Long term liabilities

Ex-post solvency condition

\[ E_0 - P^{(1)} - P^{(2)} > 0. \]
2.b Liquidity risk

- At the interim date, $t_1$, following the realization of the $P^{(1)}$ losses, a bank’s creditors may decide to run

- Runs may occur due to:
  - Concerns over the bank’s future solvency;
  - Low liquidity, relative to it’s wholesale funding
2.b Liquidity risk

- **Illiquidity condition:** a bank fails if the fraction of creditors who foreclose ($\ell$) is greater than the banks’ recourse to liquidity, i.e.,

\[
\ell \times S_0 > M_0 + \bar{\psi} \times (I_0 - P^{(1)}),
\]

where $\bar{\psi}$ is the expected fire-sale price for the bank’s illiquid assets.

**Balance Sheet Liquidity:**

\[
\lambda \equiv \frac{M_0 + \bar{\psi} \times (I_0 - P^{(1)})}{S_0}
\]
2.b Liquidity risk – the rollover game

- Decisions of creditors modeled as a simultaneous move coordination game

- Binary choice model – each creditor must decide whether to
  (2) withdraw deposits, or
  (1) rollover deposits

- Payoffs for an individual creditor:
  - Withdraw – $r^F$, irrespective of whether the bank survives, or not
  - Rollover – $r^S > r^F$, if the bank survives, and zero otherwise
2.b Liquidity risk – the rollover game

<table>
<thead>
<tr>
<th></th>
<th>$\ell \leq \lambda$</th>
<th>$\ell &gt; \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rollover</strong></td>
<td>$r^S$</td>
<td>0</td>
</tr>
<tr>
<td><strong>Withdraw</strong></td>
<td>$r^F$</td>
<td>$r^F$</td>
</tr>
</tbody>
</table>
2.b Liquidity risk – the rollover game

- Solve using the global games paradigm

- Creditors use threshold strategies
  - rollover if $P^{(1)} < P^*$
  - foreclose otherwise

- Bayes-Nash Equilibrium – $P^*$ solved from FPE

\[
\lambda(P^*) \times \text{Prob}(E_0 - P^* - P^{(2)} > 0) \times r^S = r^F
\]
2.c Contagious runs

Secondary market $\psi_H$ or $\psi_L$?
2.c Contagious runs

Credit shock

Secondary market
\( \psi_H \) or \( \psi_L \)?

Safe Asset
2. c Contagious runs

Credit shock

Secondary market \( \psi_H \) or \( \psi_L \)?

Bank Defaults

Safe Asset

\( \bar{\psi} \)
2.c Contagious runs

Secondary market $\psi_H$ or $\psi_L$?

Bank Defaults

Credit shock

Safe Asset

Creditor's observe default and update their beliefs

$\bar{\psi}^{(1)}$
2.c Contagious runs

Credit shock

Secondary market $\psi_H$ or $\psi_L$?

Bank Defaults

Safe Asset

Creditors observe default and update their beliefs

$\bar{\psi}$

$\bar{\psi}^{(1)}$
2.c Contagious runs

Secondary market $\psi_H$ or $\psi_L$?

Bank Defaults

Credit shock

Safe Asset

$\bar{\psi}$

$\bar{\psi}^{(1)}$
2.c Contagious runs– Bayesian updating

- Define \( w_j \) to be the subjective belief held by the creditors of bank \( j \) that \( \psi = \psi_H \), and \( \eta_k \in \{0,1\} \) as an indictor for whether bank \( k \) has defaulted (1), or not (0), and \( i \) as the iteration-step.

\[
\begin{align*}
w_j^{(i+1)} & = \text{Prob}\left(\psi = \psi_H \mid \{\eta_k^{(i)}\}_{k \neq j}\right) \\
& = \frac{\text{Prob}\left(\eta_1^{(i)} \mid \{\eta_k^{(i)}\}_{k \neq j, 1}, \psi_H\right) \times \cdots \times \text{Prob}\left(\eta_{N-1}^{(i)} \mid \eta_N^{(i)}, \psi_H\right) \times \text{Prob}\left(\eta_N^{(i)} \mid \psi_H\right) \times \text{Prob}(\psi = \psi_H)}{\text{Prob}\left(\{\eta_k^{(i)}\}_{k \neq j}\right)} \\
& = \text{Prob}(\psi = \psi_H) \times \prod_{k \neq j} \frac{\text{Prob}(\eta_k^{(i)} \mid \psi_H)}{\text{Prob}(\eta_k^{(i)})}
\end{align*}
\]
2.d Network effects

A owes 1 to B and 1 to C
B owes 1 to C
C owes to nothing to A and B

<table>
<thead>
<tr>
<th>Bank</th>
<th>IA</th>
<th>IL</th>
<th>Net IA</th>
<th>Net non-IA</th>
<th>Net worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>1</td>
<td>(-2)+1 = -1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0 + 0 = 0</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2 + 0 = 2</td>
</tr>
</tbody>
</table>

IA: interbank assets
IL: interbank liabilities
Net non-IA: net non-interbank assets after credit losses
2.d Network effects

A in solvency default
A promised to pay 1 to B and 1 to C but is only willing to pay 1
How to allocate 1 between B and C?
B holds 50% of A’s interbank liabilities → ½ to B
C holds 50% of A’s interbank liabilities → ½ to C

<table>
<thead>
<tr>
<th>Bank</th>
<th>IA</th>
<th>IL</th>
<th>Net IA</th>
<th>Net non-IA</th>
<th>Net worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>1</td>
<td>(-2)+1 = -1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0 + 0 = 0</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2 + 0 = 2</td>
</tr>
</tbody>
</table>
2.d Network effects

<table>
<thead>
<tr>
<th>Bank</th>
<th>IA</th>
<th>IL</th>
<th>Net IL</th>
<th>Net non-IA</th>
<th>Net worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>2</td>
<td>-2</td>
<td>1</td>
<td>(-2)+1 = -1</td>
</tr>
<tr>
<td>B</td>
<td>-1 ½</td>
<td>1</td>
<td>½ -1 = - ½</td>
<td>0</td>
<td>- ½ + 0 = - ½</td>
</tr>
<tr>
<td>C</td>
<td>2 1 ½</td>
<td>0</td>
<td>1 ½</td>
<td>0</td>
<td>1 ½ + 0 = 1 ½</td>
</tr>
</tbody>
</table>

B gets ½ instead of 1 => B defaults because A has not made full payment: spillover default
2.d Network effects

B promised to pay 1 to C but is willing to pay $\frac{1}{2}$ only. C remains solvent.
3. Calibrating MFRAF
## 3. Calibrating MFRAF

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Source for calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_0$</td>
<td>Dollar value of illiquid assets</td>
<td>NCCF report</td>
</tr>
<tr>
<td>$\psi_H$</td>
<td>Liquidation value of assets in the “high” state</td>
<td>Judgement on haircuts</td>
</tr>
<tr>
<td>$\psi_L$</td>
<td>Liquidation value of assets in the “low” state</td>
<td>Judgement on haircuts</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Dollar value of liquid assets</td>
<td>NCCF report</td>
</tr>
<tr>
<td>$S_0$</td>
<td>Cumulative short term liabilities that come to maturity in $t_1$</td>
<td>NCCF report</td>
</tr>
<tr>
<td>$RWA$</td>
<td>Risk weighted assets (CET1 Basel III)</td>
<td>Provided by the banks</td>
</tr>
<tr>
<td>$\text{INCOME}$</td>
<td>Operating income (internally generated capital)</td>
<td>Satellite models</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Bank’s starting capital levels (CET1 Basel III)</td>
<td>Provided by the banks</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Minimum threshold level for bank’s capital ratio (7% or 4.5%)</td>
<td></td>
</tr>
<tr>
<td>$X$</td>
<td>Interbank network</td>
<td>Regulatory filings</td>
</tr>
</tbody>
</table>
3 Calibrating MFRAF

- Banks reported their holdings of liquid and illiquid assets using the Net Cumulative Cash Flow (NCCF) definitions.

- Liquid assets have to be unencumbered and eligible for central bank open market operations:
  - Cash and deposit accounts at the BoC
  - Government securities (Canada, U.S., and Euro Area)
  - Other eligible securities (e.g. BAs and NHA-MBS)
3. Assumptions on recovery rates (1 – haircuts)

<table>
<thead>
<tr>
<th>Instrument</th>
<th>State H</th>
<th>State L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposits with banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Other Securities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other government</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortgage Backed Securities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset Backed Securities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate CP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corporate bonds</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precious Metals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other commodities</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Assumptions on recovery rates

<table>
<thead>
<tr>
<th>Instrument</th>
<th>State H</th>
<th>State L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residential mortgages - insured</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residential mortgages - uninsured</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Personal loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit cards</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Business and government loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Customers' liabilities under BAs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Swapped Intra-bank Loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Call Loans</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reverse Repurchase Agreements</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Securities borrowed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Derivatives related amounts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Assets</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Starting capital level (CET1 Basel III)

- “Front-load” income generated over the 1-year MFRAF horizon onto the starting capital level, i.e.,

\[ E_0 = \kappa \times RWA_0 + Income \]

- To determine the default threshold, we look at the level of capital in excess of the regulatory minimum,

\[ E_0 = \kappa \times RWA_0 + Income - \tau \times RWA_0 \]
3. Accounting for losses

- Credit risk losses
  
  \[ P^{(1)} + P^{(2)} \]

- Losses following a bank run
  
  \[ z \text{ percent of } \tau \times RWA_0 \]

- Losses after default due to network contagion
  
  endogenous clearing
4. Some *hypothetical* results
4. Results – loss distribution (solvency & liquidity)
4. Results – loss distribution (all effects)
4. Results – sensitivity to beliefs and prices
5. Conclusions
5. Conclusions

- MFRAF is a top-down stress testing tool that investigates the interactions between solvency and liquidity risk.

- Results depend starting capital ratios and balance sheet liquidities.

- Calibrating prices is very much an art form, and ideas for a more robust modeling would be very welcome.
5. Conclusions: Model Improvements – Key priorities

- Feedback effects to the real economy
  - TVAR with endogenous Financial Stress Index (FSI) to generate stress scenarios
  - Link FSI to outputs from MFRAF (e.g., via losses).

- RWA model to account for impact of liquidity risk and network effects.

- Link market liquidity ($\psi$ parameters) with funding liquidity risk, i.e., endogenous relationship.
Thank you!