### Contents

1. General Oligopolistic Market Equilibrium Problem via Tensor Variational Inequalities 2
2. Near-optimal Robust Bilevel Optimization 4
3. Approximation and exact penalization in hierarchical optimization 5
4. Solving equilibrium problems using extended mathematical programming 6
5. Portfolio optimization via a returns over risk ratio 7
6. Dynamic Marketing Policies with Online-Review-Sensitive Consumers: A Mean-Field Games Approach 8
7. The Replicator Dynamics of Generalized Nash Games 9
8. Nonsmooth dynamics of generalized Nash games 10
9. An Operator Theoretic Approach to Distributed GNE Computation in Games 11
10. A pollution control model with incomplete information 12

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1 General Oligopolistic Market Equilibrium Problem via Tensor Variational Inequalities
GENERAL OLIGOLOPISTIC MARKET EQUILIBRIUM PROBLEM VIA TENSOR VARIATIONAL INEQUALITIES: THEORETICAL RESULTS AND NUMERICAL METHODS

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The aim of the talk is to consider a general oligopolistic market equilibrium problem in which each firm produces several commodities. The equilibrium condition is expressed by a generalization of the Cournot-Nash principle and is characterized by a tensor variational inequality in which every operator is defined in a tensor Hilbert space. As a consequence, tensor variational inequalities, recently introduced in [1], have a fundamental role to study the model. For this reason, we show existence and regularity results for solutions to such inequalities. Moreover, we investigate on the convergence of solutions to regularized tensor variational inequalities to a solution to a tensor variational inequality making use of the set convergence in Kuratowski’s sense (see [2]). Then, we focus our attention on some numerical schemes based on the projection operator to compute the solutions (see [3]). Finally, we apply the theoretical results to our model.

References
2 Near-optimal Robust Bilevel Optimization

Author: Mathieu Besançon

Abstract: Bilevel optimization studies problems where the optimal response to a second mathematical optimization problem is integrated in the constraints. Such structure arises in a variety of decision-making problems in areas such as market equilibria, policy design, product pricing or network protection. We introduce the concept of near-optimal robustness for bilevel problems, protecting the upper-level decision-maker from limited deviations of the lower level, related to a bounded rationality assumption. We show it is a restriction of the corresponding so-called pessimistic bilevel problem. Essential properties are derived in generic and specific problem settings. This model finds an intuitive interpretation in various situations cast as bilevel optimization problems. We develop a duality-based solution method for cases where the lower level is convex, leveraging the methodology from robust and bilevel literature. In the case of linear-linear bilevel problems, an extended formulation can be derived, replacing the non-convex quadratic terms with disjunctions over linear constraints. The models obtained are tested numerically using different solvers and formulations, showing the successful implementation of the near-optimal robust bilevel problem.
3 Approximation and exact penalization in hierarchical optimization

Author: Giancarlo Bigi

joint work with Lorenzo Lampariello and Simone Sagratella

Abstract: Hierarchical programs are optimization problems whose feasible set is implicitly defined as the solution set of another, lower-level, problem. As a major departure from the more general bilevel structures, this talk focuses only on lower-level problems that are non-parametric with respect to the upper level variables. In particular, the minimization of an objective function over the solution set of a lower-level variational inequality is considered, which is a special instance of semi-infinite programs and encompasses simple bilevel problems and selection of Nash equilibria as particular cases. To tackle this hierarchical problem, a suitable approximated version is introduced. On the one hand, this does not perturb the original (exact) program too much, on the other hand it allows relying on suitable exact penalty approaches whose convergence properties are established.
4 Solving equilibrium problems using extended mathematical programming

Author: Michael C. Ferris

Joint work with Youngdae Kim and Andy Philpott

Abstract: We introduce an extended mathematical programming framework for specifying equilibrium problems and their variational representations, such as generalized Nash equilibrium, multiple optimization problems with equilibrium constraints, or quasi-variational inequalities, and computing solutions of them from modeling languages. We define a new set of constructs with which users annotate variables and equations of the model to describe the equilibrium and variational problems. Our constructs enable a natural translation of the model from one formulation to another more computationally tractable form without requiring the modeler to supply derivatives. In the context of many independent agents in the equilibrium, we facilitate expression of sophisticated structures such as shared constraints and additional constraints on their solutions. We define a new concept, shared variables, and demonstrate its uses for sparse reformulation, equilibrium problems with equilibrium constraints, a mixed pricing behavior of agents, and so on. The shared variables also enable back-end solvers to exploit the problem structure for improved performance. We give some equilibrium and variational examples from the literature and describe how to formulate them using our framework. Experimental results comparing performance of various complementarity formulations for shared variables combined with structure exploitation of the back-end solver are given. Our framework has been implemented and is available within GAMS/EMP. Agents problems will be formulated to allow risk preferences and some computational results related to energy systems and other applications will be given.
5 Portfolio optimization via a returns over risk ratio

Title: Warren Hare

Based on joint work with J Braun, J Brar, and D Wang.

Abstract: Portfolio optimization is the process of choosing the best investment decision across a set of financial instruments or assets. Investors are faced with a trade-off between risk and expected returns. Most researchers have considered this problem from one of two perspectives: maximize a portfolio’s expected return for a given risk or minimize a portfolio’s risk for a given expected return. In this talk, we consider a fractional model with a returns over risk ratio that keeps both expected returns and risk flexible simultaneously. The considered model provides us with the optimal investment portfolio for which the expected returns per unit of risk will be maximized. We demonstrate how to reformulate the basic model as a linear program. Numerical tests demonstrate that the model is no harder to solve and provides a more balanced portfolio when compared to previous models.
Dynamic Marketing Policies with Online-Review-Sensitive Consumers: A Mean-Field Games Approach

Author: Jerome Le Ny

Joint work with Rabih Salhab, Roland Malham and Georges Zaccour

Abstract: We consider a large group of consumers who can choose between two products at each purchasing occasion. Their choice is influenced by the marketing strategies of the firms, e.g., price, advertising, the reputation of the brands, and by the product reviews. The problem is modeled as a Stackelberg mean-field game, with one firm acting as leader and the consumers as followers. We determine the conditions under which an equilibrium exists and provide a numerical scheme to compute it. We give some examples to illustrate the type of insight that can be obtained with our model.
7 The Replicator Dynamics of Generalized Nash Games

Author: Jason Lequyer

Abstract: Generalized Nash Games are a powerful modelling tool that have seen some significant developments in the past two decades. Evolutionary Games have been around a bit longer and seek to describe how natural selection can drive phenotypic changes in interacting populations. We show how these two independently formulated models can be linked under a common framework and how this framework can be used to expand each model. At the center of this unified model is the Replicator Equation and the relationship we establish between it and the lesser-known Projected Dynamical System.
8 Nonsmooth dynamics of generalized Nash games

Author: Tangi Migot

Abstract: The generalized Nash equilibrium problem (GNEP) is an N-player noncooperative game, where each player has to solve a nonlinear optimization problem whose objective function and constraints depend on the choices of the other players. As in the case of classic Nash games, where other players' choices only impact a player's objective function, a natural question arises as to how players might evolve their strategies over time, and whether or not this evolution would allow them to reach a Nash equilibrium strategy. The approach in classical Nash games is that of introducing some form of differential equations/systems whose stable points are exactly the Nash strategies of the game. This approach leads to considering projected dynamical systems and sweeping processes. In this paper, we show that these dynamical system approaches can be extended to the case of the GNEP. We present dynamical systems that are useful in this context and discuss the new difficulties introduced by this more complex game. Finally, we show how to exploit the existence proof to build numerical methods and solve GNEP problems from the literature.
9 An Operator Theoretic Approach to Distributed GNE Computation in Games

Author: Lacra Pavel

Abstract: We consider continuous-kernel games with shared coupled constraints and the problem of how to compute an equilibrium solution, namely a variational generalized Nash equilibrium (GNE). Based on a variational inequality characterization and the KKT conditions, we show that the problem can be reformulated as that of finding zeros of a sum of monotone operators. Based on this, GNE seeking algorithms can be developed via operator-splitting methods, guaranteed to globally converge with fixed step-sizes under perfect information on the other players. We consider how to distribute such algorithms in partial-information settings, when players can only communicate with their neighbours over an arbitrary undirected graph. To distribute the problem, we augment variables, so that each player has local decision estimates and local copies of Lagrangian multipliers. We then show how the problem can be reframed as one of finding the zeros for a sum of augmented monotone operators, with a special preconditioning matrix. Proper selection of parameters can ensure that these augmented operators have desired monotonicity/cocoercivity properties, thus guaranteeing convergence to a variational GNE.
A pollution control model with incomplete information

Author: Laura Scrimali

Abstract: This paper investigates a pollution control model with incomplete information, in which different countries simultaneously aim to determine the optimal investment allocation in environmental projects and the tolerable pollutant emissions so as to maximize their welfare. Damage deriving from pollution of each country could be considered as a continuous random variable due to the influence of atmospheric and geologic factors that countries cannot really control. The equilibrium concept governing the model is that of Bayesian Nash equilibrium, which is given as a solution of a variational inequality in a Hilbert space. The existence of solutions is investigated.