EDWARD BIERSTONE Appointed Next Director of the Fields Institute

Edward Bierstone, Professor of Mathematics at the University of Toronto, has been appointed Director of the Fields Institute for Research in Mathematical Sciences, beginning July 2009.

Ed Bierstone has made pathbreaking contributions to mathematics, in the areas of singularity theory, analytic geometry and differential analysis. Ed earned his B.Sc. from the University of Toronto and his Ph.D. from Brandeis University.

Ed’s honours include Fellowship in the Royal Society of Canada (1992), the Jeffery-Williams Prize of the Canadian Mathematical Society (2005), and an invited address at an American Mathematical Society Annual Meeting. He has served as Editor-in-Chief of the Canadian Journal of Mathematics, and is currently Editor-in-Chief of the Mathematical Reports of the Royal Society. This year he was awarded the Excellence in Teaching Award by the Canadian Mathematical Society (CMS). He is currently the Chair of the Research Committee of the CMS.

“The wide scope of Ed’s interests in mathematical sciences places him in a superb position to direct Fields, one of the world’s leading research and training institutes,” says Board of Directors Chair John Gardner. “We look forward to working more with him.”

Ed’s recent research is in analysis and algebraic geometry, concerning singularities. Singularities describe irregularities of spaces and functions in many branches of mathematics and applications. Ed’s work (in collaboration with Pierre Milman) has involved the discovery of links among algebraic, combinatorial and analytic aspects of singularities, leading to the solution of old problems of Whitney and Thom on composition and extension of differentiable functions, and of Hironaka on stratification of subanalytic sets. His work on resolution of singularities has played a major part in a revival of interest in the area; the constructive techniques developed by Bierstone and Milman have led to applications in areas as diverse as logic and analysis.

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THEMATIC PROGRAM ON O-MINIMAL STRUCTURES AND REAL ANALYTIC GEOMETRY

JANUARY-JUNE 2009

Modern real analytic geometry was started in the 1960’s by Lojasiewicz and Hironaka with the introduction of the notions of semianalytic and subanalytic sets. A subset of real Euclidean $n$-space $\mathbb{R}^n$ is semianalytic if it is described locally around every point of its closure by finitely many equations and inequalities among real analytic functions. Subanalytic sets are proper projections of semianalytic sets. Using techniques such as resolution of singularities, it was shown that subanalytic sets admit locally finite stratifications by connected analytic manifolds that are themselves subanalytic sets. Moreover, the family of all subanalytic sets is easily seen to be stable under the basic set-theoretic operations of taking finite unions, finite intersections, projections and cartesian products. Inspired by Tarski’s theorem for real algebraic sets, Gabrielov proved in 1968 the much more difficult fact that this family is also stable under taking complements.

One way to view real analytic geometry is as an instance of what Grothendieck calls tame topology in his Esquisse d’un programme. Our thematic program is focussed on generalizations of subanalytic geometry

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Organized by David Ellwood (CMI), Andrew Granville (Montreal), Bryna Kra (Northwestern), Izabella Laba (UBC; committee chair) and Trevor Wooley (Bristol).

This conference, co-sponsored by the Clay Mathematics Institute, was the largest event of the Winter-Spring 2008 Harmonic Analysis thematic program, and was a major event in the field, probably the largest in the last two years. It featured over forty speakers, including most of the leaders in the area, junior researchers, and leading scientists in related fields whose work has connections to additive combinatorics. We were particularly honoured to host three Fields medalists: Jean Bourgain (IAS), Timothy Gowers (Cambridge) and Terence Tao (UCLA), as well as Endre Szemerédi, who had just been awarded the Steele prize for his 1974 proof of what is now known as Szemerédi’s theorem. The Distinguished Lecture Series given by Tim Gowers (April 7-9) was a major highlight of the conference.

The meeting was very well attended, both by senior mathematicians and by students and junior scientists. For the beginners, it was an excellent way to be introduced to the subject; the experts had an opportunity to catch up on recent developments, and engage in discussions and collaboration.

Additive combinatorics is an emerging area of mathematics that combines elements of number theory, combinatorics, harmonic analysis and ergodic theory. Its best known results include Szemerédi’s theorem on arithmetic progressions in dense sets of integers, the Green-Tao theorem on arithmetic progressions in the primes, sum-product estimates, the Freiman-Ruzsa theorem on the structure of sets with small sumsets, and much more. The problems under consideration can often be stated in terms of grade-school arithmetic, while solutions can range from surprisingly simple to deep and highly sophisticated.

Szemerédi’s theorem states that if a set $A \subseteq \mathbb{Z}$ contains a positive proportion of the integers, it must contain a $k$-term arithmetic progression for any $k$. At this point, it has several substantially different proofs: combinatorial via regularity lemma (due to Szemerédi), ergodic-theoretic (Furstenberg), harmonic-analytic (Gowers), combinatorial via hypergraph theory (Gowers and, independently, Nagle-Rödl-Schacht-Skokan). Each of them was a major milestone in combinatorics, introducing new methods and opening new directions of research: for example, Furstenberg’s ergodic-theoretic approach led to several far-reaching generalizations such as the multidimensional Szemerédi theorem and the density Hales-Jewett theorem (both due to Furstenberg-Katznelson), or the polynomial Szemerédi theorem (Bergelson-Leibman). We are still quite far, however, from a complete understanding of the subject, and many questions remain open.

The conference included a number of talks related to Szemerédi’s theorem and its extensions. Vitaly Bergelson and Nikos Frantzikinakis talked about their recent work (joint with Alexander Leibman and Randall McCutcheon, and with Maté Wierdl and Emmanuel Lesigne, respectively) on extensions of the Bergelson-Leibman theorem to new classes of functions that are not polynomial, but still have good enough equidistribution properties. Bernard Host lectured on his joint work with Bryna Kra on nilsequences and their applications in ergodic theory and additive combinatorics. The combinatorial approach was represented by Balázs Szegedy, who presented a “symmetric” variant of the regularity lemma and an application to additive number theory. Tim Austin discussed a quantitative version of the Furstenberg-Katznelson proof of the Hales-Jewett theorem.

One of the best-known results in additive combinatorics is the proof by Ben Green and Terence Tao that the primes contain arbitrarily long arithmetic progressions. Subsequently, Green and Tao went on to develop a program aimed at a very general conjecture (due to Hardy-Littlewood and Dickson) on the asymptotic number of solutions to linear equations in the primes. The conjecture includes the Green-Tao theorem as a

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AGM LECTURE: Arithmetic Geometry and Hyperbolic Geometry

PIT-MANN WONG  
(University of Notre Dame)  
JUNE 26, 2008

To many mathematicians, the two subjects of Pit-Mann Wong’s talk appear to have little relation to each other although they are both “geometry.” But it has become increasingly clear in recent years, as Wong convincingly argued in his lecture, that there is in fact a deep connection between them which is not yet completely understood. One of the main goals of this fall’s program is to investigate and further clarify the connections between these two geometries by bringing together many of the experts in each field.

In some ways, the arithmetic geometry in question goes back to the 19th century when Liouville proved in 1844 that transcendental numbers exist, followed not long after (1873) by Cantor’s startling discovery that the set of real numbers is uncountable. The “theory” of transcendental numbers was given even more prominence through Hilbert’s 7th problem – if an algebraic number differs from 0 and 1 and is algebraic and irrational, is α0 transcendental? This was solved affirmatively by A.O. Gelfond and Th. Schneider independently in 1934, and is now known as the Gelfond-Schneider theorem.

Shortly thereafter, C.L. Siegel initiated the use of function theory in the subject by considering a transcendental function f: C → C ∪ {∞} and investigating the cardinality of the set f⁻¹(K) where K is an algebraic number field. He showed that for many transcendental functions, f⁻¹(K) is finite – in fact he gave an explicit upper bound for its cardinality. Siegel’s work was reformulated and extended by Serge Lang, and the most general form at this time is E. Bombieri’s theorem (1970), proved using deep results from analysis – viz. the method of L^2-estimates of Hörmänder:

Let f = (f₁, f₂, ..., fₙ): C^n → C^n be a map with the fᵢ meromorphic such that the field K(f₁, f₂, ..., fₙ) has transcendence degree at least m + 1 and the partial derivatives ∂/∂zᵢ take K(f₁, f₂, ..., fₙ) into itself, for all i. Then f⁻¹(K) is contained in an algebraic set P(z₁, z₂, ..., zₙ) = 0 (where P is a nonzero polynomial in C[z₁, z₂, ..., zₙ], of explicitly bounded degree).

Lang had proved this for m = 1 – in which case f⁻¹(K) is of course finite.

Wong continued at this point with an example showing the relationship between Diophantine equations and Diophantine approximation, using the example x³ = 2y³ = 1. The Diophantine approximation problem relevant to this Diophantine equation is

| x/y - 2 | ≤ C/|y|^3

where x and y are integers and C is a positive constant. “Roth’s Theorem”,

Let r be an irrational algebraic number. Then for any ε > 0, there exists a constant C > 0 such that there are only finitely many rational numbers x/y satisfying

| x/y - r | ≤ C/|y|^{2+ε},

when applied in the foregoing case (when r = 21/3) shows that x³ = 2y³ = 1 has only finitely many integral solutions. A. Baker later found an effective form of Roth’s Theorem, providing an upper bound for |y| in solutions (x,y) and enabling all integral solutions to be found.

The following “logarithmic” form of Roth’s Theorem

Then for any ε > 0,

log 1/|x/y - r| ≤ (2 + ε)h(x/y) + O(1)

holds for all but finitely many rational numbers x/y where h(x/y) = log max |x|, |y| is the “height” of x/y.

is useful for comparison to the Second Main Theorem in Nevanlinna Theory which is stated later.

A crucial step, as far as the theme of this fall’s program is concerned, was the extension of Roth’s Theorem to higher dimension (multiple variables, simulta-

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The 2008 Society for Mathematical Biology Conference held July 30 – August 2 brought together 330 mathematicians, scientists, statisticians and computer scientists (and of course an entire spectrum of hybrids of these) in Toronto, a city as diverse as the backgrounds of the meeting’s participants. Those able to arrive early on July 29th enjoyed a pre-conference reception at Fields. Early the next morning was the official Conference commencement at the Medical Sciences building at the University of Toronto.

Following introductory remarks, the first plenary talk (Mechanochemistry and motility) was delivered by L. Mahadevan of Harvard University. This inspiring lecture served as a springboard into the various mini-symposia and contributed talk sessions that made up the bulk of the three-and-a-half day conference. The sessions focused on a broad range of topics in mathematical biology. Other plenary speakers included Timothy Secomb, Herb Levine, Natalia Komarova, Yicang Zhou, Mark Lewis and Martin Golubitsky. These speakers discussed varied and distinct topics but were consistently excellent. Between talks, guests were able to socialize and share ideas over coffee, tea and pastries.

On Thursday, a new SMB tradition began as the first Lee Segel Prizes were awarded. These prizes, to be awarded biennially, recognize what are determined to be the best articles appearing in the Bulletin of Mathematical Biology over the previous two years. The inaugural Lee Segel Prizes went to Emma Y. Jin in the student category for her work Combinatorics of RNA structures with pseudoknots (with Jing Qin and Christian M. Reidys) and to Tomas de-Camino-Beck for A new method for calculating net reproductive rate from graph reduction with applications to the control of invasive species. Since Tomas was unable to attend, his work was presented by collaborator Mark Lewis. Emma Jin also gave an exceptional presentation on the research that earned her the prize. Following these talks, conference attendees retreated to a lovely reception at Massey College, a lush oasis in the heart of the U of T campus. The setting was further enhanced by the ideal weather, which shocked locals by continuing to cooperate for (almost) the entire conference.

With Friday came the annual poster viewing, as over 70 students and post-docs competed for glory (and more tangible prizes) in an event that truly shadowed the Olympic Games. While all posters were interesting and appreciated, the top honours went to Luay Almassalha of the University of Michigan in the graduate student category (Understanding the formation of the Arabidopsis root epidermis through an intimate collaboration between modeling and experiment) and Richard Brown of the University of Canterbury (NZ) (A meta-population model for the growth of nassella tussock) for best poster by a postdoctoral fellow. To recognize the significant contribution of undergraduate researchers to this year’s event, the organizers also awarded a top undergraduate poster prize to Jacob Barker and Naomi Pollica of the University of Vermont (A statistical approach to population studies of Chagas Disease). Congratulations to the winners, as well as to all who participated!

Friday was made even more memorable by the presentation of the first annual Torcom Chorbajian Lecture, given by Melissa Knothe Tate (Engineering an ecosystem: taking cues from Nature’s paradigm to build tissue in the lab). This lecture series was established to pay tribute to the continuing service of Torcom Chorbajian, who was a founding member of the SMB and has served as its treasurer for over 30 years. His generosity of time and spirit cannot be overstated. The Torcom Reception followed the lecture and the time went quickly, but not as quickly as the free liquor, as the conference attendees prepared for the banquet. The banquet was a lively affair that culminated with the after-dinner speech, wherein Vito Quaranta humorously pondered the sometimes-obscure relationship between mathematicians and biologists.

Saturday morning contained the last of the talks of the 2008 SMB Conference. Some lucky guests were able to experience the (relatively) nearby Niagara Falls following the conclusion of the meeting, before once again dispersing until next year’s conference in Vancouver. Many people deserve thanks for helping to facilitate this year’s meeting, but perhaps none more so than the chair of the local organizing committee, Siv Sivaloganathan. So, thanks to Siv and the rest of the organizing committee, the executives of the SMB, the staff of the Fields Institute, and of course to all of the attendees for the success of this year’s meeting in Toronto. We look forward to next year’s meeting in Vancouver!
The Canadian IMO team members - including four from Ontario and two from Alberta - were selected from among more than 200,000 students in grades 7-12 who participated in local, provincial and national mathematics contests.

Attending the Send-off Reception were representatives from the Ontario Ministry of Education, the Spanish Consulate, sponsors from both the public and private sectors, the Fields Institute, Wilfrid Laurier University and the Canadian Mathematical Society. All were on hand to offer best wishes and words of encouragement, along with some special gifts to take to Spain.

“These six students have demonstrated the exceptional problem solving skills and creativity that is essential to compete against the very best at the international level,” said Dr. Tom Salisbury, President of the Canadian Mathematical Society, the organization responsible for the selection and training of Canada’s IMO team. “They represent the best of our students and will be excellent ambassadors for Canada.”

The team visited Fields before attending a special one-week IMO Training Seminar at Wilfrid Laurier University. While in Madrid from July 7-22, they pitted their skills against more than 540 of the world’s best math students from nearly 100 countries. At the closing ceremony on July 21st, Silver Medals were awarded to Yan (Cynthia) Li and Jonathan Schneider, and Bronze Medals to Alexander Remorov, XiaoLin (Danny) Shi, Chen Sun and Chengyue (Jarno) Sun. “The Canadian team performed spectacularly with everyone bringing home a medal. The leaders are very proud of their achievements,” said Dr. Felix Recio, Team Leader.

“The IMO is the world championship high school mathematics competition. The problems were difficult and the talent and creativity displayed by the Canadian team has been outstanding. We are very pleased to see their hard work has been rewarded,” said Dr. Graham Wright, Executive Director of the Canadian Mathematical Society (CMS), the organization responsible for the selection and training of Canada’s IMO team.

The team was accompanied by the Team Leader Dr. Felix Recio, Senior Lecturer, University of Toronto; the Deputy Team Leader, Mr. Yufei Zhao, a Canadian IMO gold medalist in 2005 and an undergraduate student at Massachusetts Institute of Technology; and the Deputy Leader Observer, Ms. Lindsey Shorser, a graduate student and lecturer, University of Toronto.

Although students compete individually, country rankings are obtained by adding the team scores. The maximum score for each student is 42 and for a team of six students the maximum is 252. The Canadian team placed 22nd out of 97 competing countries with a score of 135.

The 2008 IMO contest was set by an international jury of mathematicians, one from each country, and was written on Wednesday July 16th and Thursday July 17th. On each day of the contest, three questions had to be solved within a time limit of four and a half hours. Team members must be less than 20 years old when they write the IMO.

The top 10 teams and their scores are: China (217); Russia (199); USA (190); South Korea (188); Iran (181); Thailand (175); North Korea (173); Turkey (170); Taiwan (168); and Hungary (165).

Since 1981, Canadian students have received a total of 16 gold, 35 silver, and 61 bronze medals at the IMO. Last year’s team won one silver medal and three bronze in Hanoi, Vietnam.

The 50th International Mathematical Olympiad will take place in Bremen, Germany, in July 2009.
NONCOMMUTATIVE GEOMETRY WORKSHOP

MAY 27-31, 2008
Held at the Fields Institute

Organizers: Masoud Khalkhali (Western), Matilde Marcolli (Max Planck Institute, Bonn), and Guoliang Yu (Vanderbilt)

This was the second International Conference on Noncommutative Geometry (NCG) at the Fields Institute – the first took place 13 years ago in June 1995 at the Fields Institute’s earlier site in Waterloo. The proceedings of that meeting were published and are a testament to the breadth and scope of the subject at that point. Several of the speakers in this meeting also spoke at the 1995 meeting, but one could not fail to notice a much younger generation of mathematicians in attendance this year.

A highlight of the meeting was the Distinguished Lecture Series by Alain Connes that took place during this meeting. Connes gave three lectures with titles: The spectral characterization of manifolds, A CKM invariant in Riemannian geometry, About the field with one element. In his introduction to Connes’ lectures, Juris Steprans, the deputy director of Fields, mentioned that Connes is the first mathematician who has delivered a Distinguished Lecture series for a second time. In fact his first lecture series also took place at the previous meeting in 1995. Since there will be a separate review of Connes’ lectures, in the following we shall merely report on the conference talks.

All areas of noncommutative geometry were present in the talks. Roughly speaking one can divide the talks into three groups. This is not an artificial division, since in fact it coincides with the three phases of the development of Noncommutative Geometry by Alain Connes and his school. The first group of talks touched on aspects of noncommutative geometry related to index theory and its generalizations, the Baum-Connes conjecture and applications to algebraic topology (the Novikov conjecture), and the relevant tools like cyclic cohomology, K-theory and KK-theory. The second group was related to metric aspects of noncommutative geometry, spectral triples, applications to the standard model of elementary particles and quantum field theory. Finally there were several talks that highlighted recent interactions between number theory, algebraic geometry and noncommutative geometry.

The talks by Higson, Guentner, Baum, Emerson, Nica and Pourkia covered aspects of NCG related to the Baum-Connes conjecture, KK-theory and cyclic cohomology, and its applications to noncommutative geometry. Those by Tang, Fathizadeh, Phillips, Tsygan and Gorokhovsky were related to index theory in the noncommutative setting. Talks by Landi, Rieffel, Li, Ponge, Hajac, and Wulkenharr centered around the notion of spectral triples, Dirac operators on noncommutative spaces and quantum groups. Finally the talks by Consani, Ha, Mahanta, Plazas, and Yao dealt with interactions between noncommutative geometry, number theory, algebraic geometry, and the theory of motives.

**SPEAKERS:**

Nigel Higson, (Penn State)

Mackey’s analogy and admissible representations of complex semisimple groups

Jorge Plazas (IHES)

Heisenberg modules and arithmetic properties of noncommutative tori

Yi-Jun Yao (Vanderbilt)

Some results on Rankin-Cohen deformations

Marc Rieffel (Berkeley)

Dirac operators for coadjoint orbits

Xiang Tang (Washington)

Algebraic higher index theorems

Arash Pourkia (UWO)

Hopf-cyclic cohomology in braided monoidal categories

Giovanni Landi (Trieste)

Monopoles and Laplacians on quantum Hopf bundles

Alexander Gorokhovsky (Colorado)

Algebraic index theorem for Fourier integral operators

Bogdan Nica (Vanderbilt)

Relatively spectral morphisms and applications to K-theory

Raphael Ponge (Toronto)

Noncommutative geometry and lower dimensional volumes in Riemannian geometry

Erik Guentner (Hawaii at Manoa)

Decomposition complexity of metric spaces

John Phillips (Victoria)

An index theory for certain gauge invariant KMS states on C*-algebras

Farzad Fathizadeh (UWO)

Towards a local index formula for twisted spectral triples

Boris Tsygan (Northwestern)

Deformation quantization of Lagrangian submanifolds

Raimar Wulkenhaar (Münster)

A spectral triple for harmonic oscillator Moyal space

Piotr M. Hajac (Polish Academy of Sciences)

Equivariant pullbacks and finite free distributive lattices

Eugene Ha (Johns Hopkins)

On Z^∞ structures and the Bost-Connes system

Paul Baum (Penn State)

Geometric structure in the representation theory of reductive P-adic groups

Katia Consani (Johns Hopkins)

The integral BC-endomotive and its reduction mod p

Heath Emerson (Victoria)

Duality in equivariant Kasparov theory

Snigdhaan Mahanta (Toronto)

Noncommutative correspondence categories and homotopy groups of separable C*-algebras

Hanfeng Li (SUNY Buffalo)

Metric aspects of noncommutative Heisenberg manifolds.

Masoud Khalkhali
On October 24, 1882, John Charles Fields was proposed as a member of the University of Toronto Mathematical and Physical Society. Founded just a few months before, the purpose of the undergraduate society was “the encouragement of study and original research in mathematics and physics, and the preservation of the results of such work.” In its early years, the solution of problems was a feature of each program.

At its meeting of January 23, 1883, Mr. J.C. Fields presented a solution to one of the problems posed at the previous meeting. According to the undergraduate newspaper, the Varsity, at the following meeting in February, “Mr. Fields occupied considerable time in the discussion of a problem paper and offered some elegant solutions to a difficult problem.”

At his graduation in 1884 with a BA, Fields won the Gold Medal in Mathematics. The Varsity noted on the occasion: “a graduate of Hamilton Collegiate Institute, from the time of his matriculation,” John Charles Fields “has always stood at or near the top of his department.” He then disappears from the pages of the Varsity for a number of years while he completed his PhD and two years of post-doctoral studies at Johns Hopkins. He then taught at Allegheny College in Meadville, Pennsylvania, and spent a decade studying and doing research in Paris, Berlin, and Göttingen. In 1908, however, he reappeared in the Varsity and in the affairs of the undergraduate Mathematical and Physical Society of University of Toronto, now as its faculty advisor.

The Society had changed during his absence, and no longer focused upon posing and solving mathematical problems. Meetings feature lectures by faculty and students, and the topics ranged widely. A.T. DeLury addressed the society on “Mathematical Allusions in Literature,” followed by violin solos. A presentation was given on “The History of Sun Spots and Their Relation to Magnetic Disturbances” (illustrated with lantern slides), and one on “The Dynamics of the Golf Ball.” Fields spoke on “The Thread of Life or Story of a Physicist” (which physicist the minutes of the meeting do not say).

Before long, the February meetings of the Society were taking place at a nearby skating rink, followed by refreshments; it is reported that attendance was high. There were musical soirées with vocal recitals, piano duets, and recitations. There were debates over mostly frivolous topics. “Resolved that pleasure should not be secondary to work in a college course,” was one topic Fields chaired (he decided in the negative). Another was “Resolved that in these days of high-cost living it is better to have loved and lost than to have loved and won.” What can Fields have thought?

In 1919, Mathematical and Physical Society members heard a lecture on “The Science of Gunnery,” which addressed problems of mines and bombs, and was illustrated with slides. They heard a lecture on the life of Galois, and one on “Mathematics and the Poet.” There were skating parties, and a dance with card tables for those who preferred not to dance.

But Fields was absent. He had turned the full force of his energy towards a wider audience – the university, leading manufacturers of Toronto, the Royal Canadian Institute, local newspapers, and politicians. His work from then until his death was unvarying: the importance of scientific research.

Elaine Riehm
Summer School in Analytic Number Theory and Diophantine Approximation

JUNE 30 – JULY 11, 2008
Held at the University of Ottawa

Organizing committee: Nathan Ng (Lethbridge), Damien Roy (Ottawa)
Scientific advisory committee: Nathan Ng, Damien Roy, Kannan Soundararajan (Stanford)

This two week summer school attracted 85 participants from 46 different post-secondary institutions all over the world. Thanks to financial support from the Fields Institute and the National Science Foundation, we were able to partially fund the living and travel expenses of 19 Canadian students, 15 students from the United States, and 18 students from 10 other countries. The main objective of the school was to give young mathematicians an opportunity to learn the latest methods and results in analytic number theory and Diophantine approximation. These closely related fields have made considerable progress in the last ten years. Some examples of important results that have recently been proven are Goldston, Pintz, and Yildirim’s theorem on small gaps between primes and Ball and Rivoal’s theorem on infinitely many irrational odd zeta values. The summer school exposed the participants to key ideas and tools in these respective fields. Moreover, it was affiliated to the tenth conference of the Canadian Number Theory Association (CNTA X) which took place July 13 - July 18, 2008 in Waterloo.

The first week of the school consisted of introductory lectures by Michel Waldschmidt (Paris VI), Damien Roy, and Nathan Ng. Waldschmidt spoke on irrationality and transcendence methods with an emphasis on the historical development of the subject and the construction of auxiliary functions. This was a very clear and motivated introduction to the theory. The lectures of Roy concerned zero estimates, Philippon’s criterion for algebraic independence, and their application to Gelfond’s problem, showing how the two tools can be applied in a specific problem. Ng spoke on the analytic theory of Dirichlet L-functions, primes in arithmetic progressions, and zeros of the Riemann zeta function, covering some background material in preparation for the more advanced courses of the second week. Each course consisted of five lectures of 80 minutes each.

The second week of the summer school had the same format, consisting of three series of five lectures by Kannan Soundararajan, Francesco Amoroso (Université de Caen), and Andrew Granville (Université de Montréal). Soundararajan gave a series of lectures on L-functions. He discussed probabilistic models, the distribution of values, and moments of L-functions. In his final lecture Soundararajan presented exciting new work on weak sub-convexity for L-functions. In a complementary series of lectures Granville gave a comprehensive overview of the theory of sums of multiplicative functions. He discussed many advances obtained jointly with Soundararajan, including their recently introduced notion of pretentiousness. Amoroso’s lectures first dealt with Lehmer’s problem which asks for a lower bound for the Mahler measure of non-zero algebraic numbers which are not roots of unity. Then, going to higher dimensions, he defined the height of an algebraic sub-variety of an algebraic torus and proved several fascinating results, mostly due to him and Sinnou David, concerning generalizations of Lehmer’s problem in this setting.

The conference took place in the pleasant setting of Ottawa. The student accommodations were well-situated next to the lecture rooms and library. In addition, they were near many famous landmarks of Ottawa including Parliament Hill and the Museum of Civilization. All of this created a motivating and congenial environment. As the courses were finished each day at 3:20 pm, the students had the opportunity to review the lectures of the day, ask questions, and hold discussions with the lecturers. During the two weeks of school the interactions among participants increased both academically and socially. In the evenings and weekends there was a number of organized and spontaneous activities. Some of these included the Canada Day festivities on Parliament Hill, biking in Gatineau park, and organized soccer matches. There were also a luncheon and two suppers organized for the whole group.

The organizers of this conference were very happy with the large number of participants and by their enthusiasm and eagerness to learn. In addition, we were pleasantly surprised by the diverse representation from 12 countries. We believe that this summer school was inspiring and helpful for the participants’ research and will lead to further advances in the fields of analytic number theory and Diophantine approximation.

Sponsors: The Fields Institute, University of Ottawa, and the National Science Foundation

Damien Roy
GENERAL SCIENTIFIC ACTIVITIES: A COMMUNITY RESOURCE

ALL THE CANADIAN MATHEMATICS RESEARCH INSTITUTES support activities initiated by members of the mathematical sciences community. The GSA program at Fields supports activities not related to our thematic programs, both onsite (at Fields) and distributed (among PSU’s, across the province, across the country and occasionally internationally). While some events are initiated and managed by the Fields directorate, the majority are community-initiated activities. Our proposal mechanism was devised to invite these activities. The directorate, consulting with referees and members of the SAP, makes decisions about support. The selection is fairly rigorous. While we reject few submissions, we discourage applications for events that seem of only marginal mathematical interest, that seem overpriced or impractical, or that do not strive for high quality. (A few years ago, we had to reject some excellent proposals because of budget constraints; however, following recent increases in our funding, we are now inviting applications.) Fields adds value both by encouraging people to propose activities that create, teach and communicate new mathematics, and by suggesting changes that improve the chance of success. We provide resources not only to fund travel and accommodation for attendees but to handle all the logistics of organization, from setting up credit-card registrations to catering. Fields frees the organizers and participants to do what they are good at and what they want to do: bring together members of the mathematics community, and bring mathematicians together with other scientists and educators to talk about their research and to start new collaborations. Besides the benefits of having us do the organization, organizers can take advantage of our advertising and our reputation.

Fields solicits a wide range of community input into its scientific programs. For the next three years, we plan to spend $450,000 annually on activities in this category. Most of the support is given out in small amounts. In 2007-08, for example, Fields funded 49 GSA events, at an average cost of $7,800 each.

Fields makes the case, by its very existence, for the importance of communication and collaboration in mathematics research. In addition, many of the events we sponsor contribute substantially to graduate and postdoctoral training. We support at least one summer school each year, and all workshops make provision for students to attend. Discussions with our stakeholders indicate that this increased activity benefits the research level in their departments. While thematic programs remain our signature events, they affect only a small fraction of the mathematical research community in Ontario in a given year, while GSA reaches many more people.

There are other important reasons for GSA. It allows us representation in areas of mathematics in which we have not, and may not be likely to, run a thematic program. GSA brings the institute into contact with topics and individuals who might be recruited to organize future thematic programs. GSA is also an important way that Fields stays connected to PSUs and Affiliates.

Fields supports GSA events in a number of categories: small and medium-sized workshops, birthday conferences, student-oriented events, seed funding for international conferences, summer schools, exploratory workshops and ongoing seminar series at Fields.

OPERATION OF GSA PROGRAM:
Any event with a contribution of $20,000 or more from Fields must be approved by the SAP, which meets twice a year (November and May). We support events irrespective of the affiliation of the proposers, based only on the quality of the event and on its relevance to the Fields community. However, we expect our Affiliate Universities, and especially our PSUs, to be the major sources of proposals for GSA events. Each PSU has a “SUAC”, or “Sponsoring Universities Activities Committee”, which solicits activity from all the mathematical sciences departments at that university, and screens proposals before they are sent to us. We send every proposal to a referee (an objective expert from the community), and often ask the proposers to make changes – in the composition or number of speakers, or in some organizational aspect of the event. Once an event is approved, a formal funding letter is written, and the event is passed along to the program team.

Our reputation as an efficient conference facility has resulted in frequent appeals to contribute logistical support for meetings (particularly on-line registration and reimbursement of attendee expenses), even if direct funding is not needed. But every event that uses any of our resources – money, staff or facilities – is a “Fields event”. And we cannot lend our name to activities that have not undergone our approval process, including vetting by a SUAC if applicable, no matter how small the contribution from Fields appears to be.

We welcome ideas and initiatives that come from the community. In fact, the community is us. The people who request support for their events become part of Fields, and take responsibility for partial management of these events, including communicating with us about proposals, event logistics and reports in a timely way.

THE FIELDS VISION FOR GSA:
To keep the program viable and to manage its projected growth, it must be:
• regulated so that all events are of high scientific quality and are well-run
• seen to be open to the entire mathematical sciences community
• efficiently managed without overextending the institute’s human resources
• connected to our mandate, with reporting lines that respect our accountability.

This ideal is made possible by a number of things.

1. Active and consistent SUAC involvement. Each PSU should have an active, rotating set of representatives that include, over time, all departments that might propose events. All proposals from a PSU should be recommended by its committee. Its members need to be willing to assess and to actively solicit a
good stream of proposals, and to pass them along to Fields promptly. Affiliates are welcome to form SUACs as well. In particular, any affiliate with more than one mathematical sciences department participating in Fields activities should ensure that all departments are aware of how to take advantage of the opportunity to propose GSA.

2. Observance of deadlines for proposal submission and funding announcements. We are turning to a more flexible, but also more enforceable, schedule of three deadlines per year. Furthermore, since what the proposers need to know is when they can expect an answer, we will try to maintain reporting deadlines. The submission deadlines, beginning in Fall 2008, are

i. **October 15.** Decisions announced November 30.
ii. **February 15.** Decisions announced March 31.
iii. **June 15.** Decisions announced July 31.

People planning large events (over $20,000) should be aware that the only deadline which fits conveniently into the SAP meeting schedule is the first. Proposals for a major event, such as a summer school, that require SAP approval, will not receive final decisions until the end of May (if submitted in February) or the end of November (if submitted in June).

3. Consistency in making resources of Fields available. Every event that uses Fields resources to any degree – space in our building, time of our staff, or funding from our grants – is an activity of Fields, and becomes an entry in our list of activities. Approval of the event is marked by issuing a “funding letter” (or “no-funding letter” if no direct funding is awarded). And the event is not completed until a report has been written for the Annual Report. (This is the document of record for our activities.) The web page for GSA proposals contains enough detail to guide proposal-writing, though conversations with the director or deputy director are encouraged. In particular, any request for support, after a preliminary conversation to determine that this is something we could support, will be answered with a request that the proposer send us a proposal. The proposal must be electronic and must be complete, as indicated on our “proposals” page.

4. Simplification of event management and reporting. Here we plan to

- Communicate with first-time organizers about running GSA events
- Make clear to organizers what can be paid from Fields funds
- Be clear about deadlines: for announcements, registration pages, workshop schedules
- Obtain prompt reports on activities. Particularly interesting reports will be rewarded by presentation in *Fields Notes*. All reports will be part of the Annual Report.

*Barbara Lee Keyfitz (Fields Institute)*

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**CANADIAN UNDERGRADUATE MATHEMATICS CONFERENCE**

**JULY 9-12 2008**

Held at the University of Toronto

**This was the 15th instance of this annual nation-wide event, organized by students at a different Canadian university every year, bringing together undergraduates with an interest in mathematics.**

Close to 150 students attended CUMC 2008. The variety of backgrounds and experience of the participants this year made for an interesting blend of beginning and graduating students, French and English speakers, coming from universities from the West to the East coast who had the chance to meet and share thoughts and ideas on mathematics and their academic goals.

The central part of the conference was the student talks – over 80 students gave a 25 or 40 minute presentation on a topic which they are researching or are simply interested in. The talks ran for all of the second and third days, with a short break for lunch, as well as for the first half of the final day of the conference. The range of student talks was enormous – from stochastic processes and “Transient results for M/M/1/c queues via path counting” to Arrow’s Impossibility Theorem and “The mathematics of voting” to “A foray into algebraic graph theory”. A highlight of the conference was the lectures by the five keynote speakers – distinguished professors who are well established in their areas of research. The conference was opened by Dror Baron (Toronto) who gave a talk on using Gaussian elimination to solve Rubik’s cube and various other puzzles, as well as hinting at further applications and generalizations to which this method could be applied. The following day, Nantel Bergeron (York) introduced some surprising and interesting ways of looking at permutation groups and Professor Edward Barbeau (Toronto) spoke of his experience going from undergraduate student to professor in mathematics and challenged the audience with a few math puzzles. Barbara Keyfitz (Fields) gave an insight into how partial differential equations feature in the workings of traffic lights and in the closing lecture of the conference Alexander Holroyd (UBC) explained and showed some beautiful examples of cellular automata.

Its continuity being ensured by the CMS Student Committee, the CUMC is one of the main events of undergraduate mathematics in Canada and this year was no different as students got to learn some new mathematics and talk to their peers as well as experienced professors from all over Canada. The conference will be held at Carleton University next year.

CUMC 2008 was supported by CMS, the Fields Institute, CAIMS, AARMS, PIMS, SSC, the MITACS Student Advisory Committee, the Departments of Mathematics and Statistics at the University of Toronto, UTSC and the Faculty of Arts & Science.

*Iva Halacheva*
Meeting of the Canadian Number Theory Association

JULY 13-18, 2008
Held at the University of Waterloo,
Organizers: Kevin Hare (Waterloo),
Wentang Kuo (Waterloo), Yu-Ru Liu
(Waterloo), David McKinnon (Waterloo),
Michael Rubinstein (Waterloo), Cameron
Stewart (Waterloo)

The biennial meeting of the CNTA has become one of the largest conferences in number theory in the world, and is regularly attended from all over the globe. The tenth meeting attracted over 200 attendees from outside Waterloo, and was well attended by local mathematicians, graduate students, and even several undergraduates.

The purpose of CNTA meetings is to bring together a wide variety of leading number theorists, junior researchers, and graduate students to discuss all areas of number theory. The tenth meeting had

“In virtually every branch of number theory is represented at CNTA meetings, including applications such as cryptography”

In fact, CNTA X featured eight plenary lectures, forty invited lectures, and over sixty contributed talks in the space of a week. The talks covered the breadth of number theory from computing moments of $L$-functions to rational points on algebraic surfaces to Iwasawa theory to transcendence theory and beyond – virtually every branch of number theory is represented at CNTA meetings, including applications such as cryptography. In 2008, there was a school for graduate students on analytic number theory and diophantine approximation in Ottawa that took place during the two weeks immediately preceding the CNTA meeting, and there were a significant number of participants who took part in both events.

In addition, the Ribenboim Prize is awarded at CNTA meetings for distinguished research in number theory by a mathematician who has strong ties to Canada. In 2008, Paulo Ribenboim himself presented the Ribenboim Prize to Adrian Iovita of Concordia University, who gave his prize lecture on $p$-adic Hodge theory.

David McKinnon
June 25-27, 2008
Held at the University of Ottawa
Organizers: Ian Blake (University of Toronto), Ali Miri and Monica Nevins (University of Ottawa)

This hugely successful event drew over 90 participants and speakers from all over North America, and from as far away as England, Australia and Jordan.

The first day of the workshop was devoted to mini-courses on some of the major themes in modern cryptography. In the morning, Kenny Paterson (Royal Holloway, London) gave a wonderful overview of the history and necessity of Identity-Based Encryption, and then brought the audience to the current cutting edge of research in that area. The afternoon’s course, given by Ali Miri (Ottawa), gave an overview of the techniques and ideas of accelerating scalar multiplication on Elliptic Curve Cryptosystems (ECC), with particular emphasis on recent open problems.

The second day featured invited lectures by world-renowned researchers working in Canadian universities, in mathematics, computer science and engineering. Many of the speakers specifically highlighted current open problems, both theoretical and practical in nature, for graduate students to explore. The opening talk by Kumar Murty (Toronto) explored the mysteries of hash functions. New variants of elliptic curves for use in cryptography were the subject of Renate Scheidler’s (Calgary) talk, while Francesco Sica (Mount Allison) explored new ideas in scalar multiplication on algebraic curves. Doug Stinson (Waterloo) focused on new results on authentication protocols, and Amr Youssef (Concordia) presented analysis of Boolean functions for use in cryptography. Evangelos Kranakis (Carleton) closed the day with an intriguing and thought-provoking talk about security models.

The final day of the workshop featured speakers from government and industry. The first talk, by representatives of the CSEC, gave an overview of the mandate and interests of this governmental body and was of particular interest to the many graduate students in attendance. The final three talks, by Rene Struik (Certicom), Tom St Denis and Dana Neustader (Elliptic Semiconductor) and Phil Eisen (Cloakware), introduced a variety of problems and issues that are subjects of intense industrial research. These range from speeding up implementations of ECC to ensuring security in a world where even cryptographic icon “Bob” can’t be trusted.

Cloakware and Certicom generously offered additional sponsorship for this workshop. Consequently, all students and junior researchers who applied for funding received some support for their travel and accommodation expenses. Moreover, the workshop was able to provide all coffee breaks, as well as lunches on the three days, at no charge to the participants. These breaks and meal times invited and encouraged much additional interaction among the participants and speakers, and were enjoyed by all.

The talks were held in the beautiful SITE building of the University of Ottawa, which overlooks the Rideau Canal. After delightful sunny weather the first two days, some rain arrived on the last day. Nevertheless, some participants braved the soggy weather to enjoy the boat cruise on the Rideau Canal after the close of the workshop. Three intense days and a wealth of problems to pursue: we look forward to the next time!

Monica Nevins
ORAHS PHD WORKSHOP

JULY 24 - 25, 2008
Held at the Fields Institute

Organizers: Dionne Aleman, Ali Vahit Esensoy and Daphne Sniekers (Toronto)

This two-day workshop was organized as a precursor to the Operational Research Applied to Health Services (ORAHS) 34th annual conference held at the University of Toronto. ORAHS is a special interest group of the Association of European Operational Research Societies (EURO), and aims to provide a network for researchers involved in the application of systematic and quantitative analysis in support of planning and management in the health services sector. Operational research methods can help the health services sector, which is in need of quantitative tools to help guide decisions in an environment of the growing demands on the healthcare system, emerging technologies and shifting policy positions.

The goal of this event was to provide an intimate forum for current and recently graduated PhD students to showcase and discuss their research. The workshop was composed of tutorials and seminars focusing on operational research and healthcare topics run by prominent researchers in these fields. There were 38 researchers from North America and Europe who participated in the event.

The first day of the event started with a tutorial by Dr. Steven Schechter (University of British Columbia) on Medical Decision Making using mathematical models. The tutorial introduced various modeling techniques that help physicians and patients determine the best course of treatment, including decision trees, Markov models, and Monte Carlo simulations with examples on applying these techniques to decisions regarding liver transplantation and HIV treatment. This was followed by interactive seminars run by PhD students on the topics of cancer and chronic care, high-level healthcare applications and hybrid simulation models. Hybrid simulation models based on system dynamics and discrete event approaches drew a lot of questions and discussion. These models seem to be promising in merging operational and strategic optimization decisions in one model package. The participants had the opportunity to continue the discussions on the day’s presentations and to network with colleagues at a dinner hosted by Dr. Michael Carter (Toronto).

The second day of the workshop started with an inspiring presentation by Dr. Murray Côté (University of Colorado) on feasibility models for healthcare resource planning. Dr. Côté’s tutorial was made possible by the videoconferencing facilities of FIELDS, as Dr. Côté was forced to join from the safety of his home in Denver following an emergency aircraft landing on his way to Toronto. This presentation was followed by another set of PhD student led interactive sessions on the applications of operations research in disease planning, patient and wait list management. The presentation on developing quantitative measurement approaches to prioritize surgical wait lists aptly demonstrated the difficulty of quantitatively assessing the urgency of patients by involving the audience in a mock wait list decision process.

At the end of the two-day workshop, the participants relaxed at a Blue Jay’s game. Due to the success of this event ORAHS is already planning the next workshop to be held in Leuven, Belgium in the summer of 2009.

SPONSORS:
Centre for Research in Healthcare Engineering
Fields Institute
MITACS
European Operational Research Societies (EURO)
SAS Institute
Cancer Care Ontario
Courtyard Group
Hay Group
Siemens
Canadian Operational Research Society

PRESENTATIONS:
Kirandeep Chahal (Brunel)
A symbiosis between system dynamics and discrete event simulation in healthcare

Murray Côté (Colorado)
A feasibility model for healthcare resource planning: Applications and potential future research opportunities

David Hutton (Stanford)
Cost effectiveness of Hepatitis-B catch-up vaccination among children and adolescents in China

Beste Kucukyazici (McGill)
The role of operations research through the journey of chronic care

Mariel Lavieri (British Columbia)
Applying operations research in cancer care

Jessica McCoy (Stanford)
Planning for a response to a bio-terrorism attack: An example of an anthrax scenario planning model

Brijesh Patel (Southampton) (Presented by Joe Viana and Shivam Desai)
Taxonomy of modeling and simulation models in healthcare and a toolkit method selection for healthcare requirements

Steven Schecter (British Columbia)
Introduction to medical decisionmaking

Sherry Weaver (Toronto)
Managing the waiting experience for patients

Peter VanBerkel (Twente)
Tutorial and case study in outpatient planning

Joe Viana, Shivam Desai (Southampton)
Hybrid models at the University of Southampton

Mike Carter
special case, corresponding to a system of $k \cdot 1$ equations $x_{j+2} + x_j = 2x_{j+1}$ in $k$ variables $x_1, \ldots, x_k$, but it also makes the stronger claim that the number of such progressions in the primes less than $N$ follows certain prescribed asymptotics. It also implies formally the Goldbach conjecture (every even number is a sum of two primes) and the twin primes conjecture (there are infinitely many pairs of primes $p, p+2$). The ultimate goal of the Green-Tao program is to prove the “non-degenerate” case of the Dickson conjecture. “Non-degenerate” means that the system of equations does not include or encode implicitly any equation in two variables; thus the success of the Green-Tao program would not resolve “binary” questions such as the Goldbach and twin primes conjectures, but it would yield precise asymptotics on the number of $k$-term arithmetic progressions in the primes for every $k$.

About two years ago, Green and Tao reduced the problem to proving two conjectural statements: the inverse Gowers norm conjecture and the Möbius-nilsequences conjecture. They also proved both statements to the extent needed to resolve the case of 2 equations in the primes. Independently and in a different context, Alex Samorodnitsky proved an analogous case of the inverse Gowers norm conjecture in fields of characteristic 2.

In his two lectures at the conference, Terence Tao reported that he and Green have now proved the Möbius-nilsequences conjecture in its full generality; their proof combines number-theoretic methods (the “circle method”) and a new Ratner-type theorem for nilmanifolds. Ben Green gave an update on the inverse Gowers norm conjecture. Last year, Lovett-Meshulam-Samorodnitsky and (independently and about the same time) Green-Tao found finite field counterexamples to the most general form of the conjecture. However, Green and Tao have recently proved that this particular type of counterexample cannot occur in fields of sufficiently large characteristic; hence they continue to be optimistic that the conjecture will be true in $\mathbb{Z}/N\mathbb{Z}$.

Tamar Ziegler lectured on another major development inspired by the Green-Tao theorem: the proof, by Tao and Ziegler, of the polynomial Szemerédi theorem in the primes. Their work relies on the ergodic-theoretic methods of Furstenberg and Bergelson-Leibman, as well as a new variant of the “transference principle” of Green and Tao. The latter states, roughly speaking, that sets such as primes—which have asymptotic density zero, but are sufficiently randomly distributed—can be modelled by sets of positive asymptotic density. This allows us to transfer results known for sets of positive density, such as the Szemerédi and Bergelson-Leibman theorems, to the primes setting.

Another very active area of research concerns the sum-product estimates. Let $A$ be a set of real numbers. We will write $A+A=\{a+a': a,a' \in A\}$ and $A\cdot A=\{aa': a,a' \in A\}$. What can we say about the minimum size of $A+A$ and $A \cdot A$? It is easy to see that each set has cardinality at least $2|A| - 1$ and that this lower bound is attained if $A$ is an arithmetic or geometric progression, respectively. Nonetheless, Erdős and Szemerédi conjectured that at least one of $A+A$ and $A \cdot A$ must be large. Just before this article was written, Jozsef Solymosi proved that $\min(|A+A|, |A \cdot A|) \geq \frac{1}{2} |A|^{2/3}(\log |A|)^{1/3}$, which is the best lower bound known so far.

A few years ago, Bourgain-Katz-Tao first extended this type of estimate to the finite field setting. This was followed by a rush of activity, motivated by the applications that sum-product estimates have found in number theory, combinatorics and computer science. Mei-Chu Chang, Alexei Glibichuk, Alex Iosevich and Chun-Yen Shen all spoke on sum-product estimates in finite fields, including variants for multiple sumsets and product sets and applications to exponential sum estimates and distance set problems. Harald Helfgott lectured on his “growth theorem” in $SL_3$: if a set $A \subset SL_3(\mathbb{Z}/p\mathbb{Z})$ is not “too large”, then $|A^* A^* A^*| \geq |A|^{1+\epsilon}$.

There were many other fascinating talks on combinatorics, additive and analytic number theory, given by Antal Balog, Ron Graham, Alex Gamburd, Andrew Granville, Vsevolod Lev, Maté Matolcsi, Jaroslav Nesetřil, Tom Sanders, Jozsef Solymosi, Endre Szemerédi, Trevor Wooley, and others.

I would like to close by mentioning a few talks that featured connections between additive combinatorics and other areas of research. Jean Bourgain talked about a question in ergodic theory concerning the rigidity of invariant measures on 2-dimensional tori; the solution (joint work with Furman, Lindenstrauss and Mozes) features an application of sum-product estimates. Alex Samorodnitsky spoke on pseudorandomness and structure in “property testing” in theoretical computer science: this is the line of work that led him, independently of the Green-Tao work, to the Gowers inverse norm conjecture mentioned earlier in connection with Green’s talk. Akshay Venkatesh gave a “speculative” lecture about his joint work with Jordan Ellenberg on nothing-theoretic phenomena by the statistics of seemingly unrelated random objects such as matrix groups.

Izabella Laba

Distinguished Lecture Series:

TIMOTHY GOWERS

TIMOTHY GOWERS, ROUSE BALL PROFESSOR of Mathematics at Cambridge and a 1998 Fields Medalist, is one of the founders of – and unquestioned leaders in – additive combinatorics. His work on Szemerédi’s theorem has transformed the field and opened the way to many further developments, including the Green-Tao theorem and quantitative versions of combinatorial results previously only available in a qualitative form. His ideas have found their way into several other areas of research, from harmonic analysis to theoretical computer science.
Gowers spoke on "quadratic Fourier analysis", a subject that goes back to his 1998 proof of Szemerédi’s theorem and continues to be developed by many authors, with major contributions by Green, Tao, Samorodnitsky, and Gowers himself. In his first lecture, addressed to a general audience, Tim gave an introduction to $U^d$ uniformity and inverse theorems. Suppose that we want to prove Szemerédi’s theorem: if a set $A \subset \mathbb{Z}$ contains a positive proportion of integers, then it must contain a $k$-term arithmetic progression for each $k$. The general strategy is well known by now. If $A$ is randomly distributed, in the sense that it does not exhibit any noticeable "special patterns", then there are many $k$-term progressions in $A$. If on the other hand $A$ is not random, we take advantage of that by choosing structured subsets of integers on which $A$ has higher density. Iterating the argument, we eventually prove the theorem.

Exactly what it means for a set to be random, or to exhibit a special pattern, depends in a crucial way on the length $k$ of the progression that we are looking for. It also depends, to a somewhat lesser extent, on the choice of approach to Szemerédi’s theorem: graph-theoretic, ergodic, or harmonic-analytic. We will focus on the harmonic-analytic proof, due to Roth for $k = 3$ and to Gowers for general $k$. Here, uniformity is defined in terms of the so-called $U^d$ Gowers norms: if a set is uniform with respect to the $U^k$ norm, it contains the statistically correct number of $k$-term arithmetic progressions.

A $U^2$-uniformity, also known as linear uniformity, has a Fourier-analytic interpretation. If $A$ is not $U^2$-uniform, its characteristic function has a large Fourier coefficient; in other words, $A$ correlates with a phase function, where $\varphi(x)$ is linear. (This is where the terminology comes from: a set is linearly uniform if and only if it has no linear patterns.)

What about $U^3$-uniformity? It has been known for some time (due perhaps to Furstenberg and Weiss) that a set of integers can have a statistically disproportionional number of 4-term arithmetic progressions if it exhibits "quadratic patterns", in the sense that its characteristic function correlates with $e^{2\pi i \phi(x)}$, where $\phi$ is a quadratic homomorphism. (Quadratic homomorphisms are somewhat more general than quadratic polynomials in $x$: they also include linear projections of quadratic polynomials in several variables.) Similarly, higher degree polynomial patterns contribute to $U^k$ nonuniformity for higher $d$.

What we need in the proof of Szemerédi’s theorem is an inverse theorem: if a set is not $U^d$-uniform, it must exhibit a polynomial pattern. This turns out to be very difficult to prove. The first such result for $d \geq 3$ was proved by Gowers; his result is sometimes known as a weak inverse theorem, in the sense that the polynomial correlations occur only on very small parts of the set. Other applications require stronger inverse theorems: in particular, Tim mentioned the Green-Tao inverse theorem for the $U^3$ norm and its application to counting 4-term arithmetic progressions in the primes. That’s where the first lecture ended.

In the second lecture (based on Gowers’s joint work with Julia Wolf) we were introduced to decomposition theorems. A decomposition theorem for the $U^3$ norm can be stated as follows: if $f$ is a function on either $\mathbb{Z}_N$ or $\mathbb{F}_p^n$ with $\|f\|_{U^3} \leq 1$, there is a decomposition $f = \sum Q_i$ where the $Q_i$ are "generalized quadratic phase functions" and $g$ and $h$ are error terms with $\|g\|_{U^2}$ and $\|h\|_{U^2}$ small. This can be deduced from the $U^8$ inverse theorem of Green-Tao; in fact, a similar statement was already implicit in their work. Tim presented a new approach to deducing decomposition theorems from inverse theorems, based on functional-analytic arguments involving the geometry of normed spaces and, specifically, a variant of the Hahn-Banach theorem.

This can be applied to the question of counting solutions to systems of linear equations in sets. Suppose that we are interested in finding sensible conditions under which a set $A \subset \mathbb{F}_p^n$ will have the statistically correct number of solutions to a system of linear equations. For instance, it is well known that $U^3$-uniformity guarantees the right number of 4-term arithmetic progressions $x, x+r, x+2r, x+3r$. Green and Tao prove a more general result of this type: they introduce the notion of complexity of a system of linear forms and prove that systems of complexity $k$ are controlled by $U^{k+1}$ norms.

Gowers and Wolf, however, do not stop there. Suppose that we are interested in counting configurations of the form, say, $x, y, z, x+y+z, x-y+2z, x+y-2z$. The complexity of this system in the sense of Green-Tao is 2, hence a set $A$ uniform with respect to the $U^4$ norm will contain the “right” number of such configurations. Gowers and Wolf, however, can prove that $U^4$-uniformity already guarantees the same conclusion! Why is this example different from 4-term progressions? The squares $x^2, (x+r)^2, (x+2r)^2, (x+3r)^2$ are linearly dependent, whereas $x^2, y^2, z^2, (x+y+z)^2, (x-y+2z)^2, (x+y-2z)^2$ are not. Gowers and Wolf prove that such “square independence” is in fact both sufficient and necessary for a system of complexity 2 to be controlled by the $U^3$ norm. The proof is based on the decomposition theorem described earlier.

The last lecture focused specifically on the Hahn-Banach type functional-analytic arguments. Tim started out by explaining parts of the proof of the decomposition theorem from the second lecture, then went on to present further applications of the Hahn-Banach approach. One concerns a new proof of the transference principle of Green-Tao and Tao-Ziegler, based on functional-analytic arguments instead of the more involved energy-increment iteration. (I learned later that a similar approach had also been developed, independently and around the same time, by Omer Reingold, Luca Trevisan, Madhur Tulsiani and Salil Vadhan, who were motivated by certain questions in computer science.) The second application is a refinement of Tao’s “structure theorem” from his quantitative version of the ergodic-theoretic proof of Szemerédi’s theorem.

Izabella Łaba
The SMT can be generalized to higher dimension:

Let \( L_1, \ldots, L_q \) be \( q \) hyperplanes in general position in \( \mathbb{P}^n \). Then for any linearly non-degenerate holomorphic map \( f: \mathbb{C} \to \mathbb{P}^n \),

\[
\sum \log \frac{1}{|f(re^{it}) - a_j|^{2e}} \leq (2 + \varepsilon) T^*(r) + O(\log r)
\]

where the estimate holds for all \( r \) outside an exceptional set of finite Lebesgue measure.

The left side is known as the proximity function and \( T^*(r) \) is called the characteristic function. It is the analogue of the height function of number theory.

The SMT can be translated to Diophantine geometry and yields the higher dimensional analogue of Siegel’s theorem in dimension 1. Namely the complement of \( 2n+1 \) hyperplanes, defined over \( \mathbb{Q} \), in \( \mathbb{P}^n \) admits only finitely many integral points.

2. P. Corvaja and U. Zannier extended Schmidt’s Subspace Theorem for hyperplanes to the case of hypersurfaces of degree \( d \). M. Ru translated the proof to prove the SMT for hypersurfaces.

There are also other results of hyperbolic geometry such as Bloch’s Theorem on subvarieties of Abelian varieties and Lang’s conjecture on an ample divisor of an Abelian variety, proved by Y.-T. Siu and S.-K Yeung, which have been proved “directly” (in this case properties of elliptic curves by Faltings) but have not yet been proved by “translation”.

There are many open and intriguing problems in this marriage between Diophantine geometry and hyperbolic geometry. For example there is a well-developed \( p \)-adic Nevanlinna theory and \( p \)-adic hyperbolic geometry but their relationship to \( p \)-adic Diophantine geometry remains unexplored. Similarly there is a theory of \( p \)-adic dynamics but again its relationship to complex dynamics has not yet received much attention.

The organizers of the Fields thematic program on arithmetic and hyperbolic geometry hope that progress will be made on these and many other problems of this kind.

Carl Riehm
secure funding, and the partnership of more universities in Ontario and elsewhere than ever before, Fields is just beginning to realize its potential to stimulate research,

THETMIC PROGRAM ON O-MINIMAL
continued from page 1

in the spirit of this tame topology. In the 1980’s model theorists observed that many of Groethendieck’s suggestions could be realized for collections of subsets of real Euclidean space that satisfy the seemingly simple assumptions that: (a) the collection is closed under the basic set-theoretic operations mentioned above and (b) that every subset of R that belongs to this collection is a finite union of points and intervals. Such a collection is called an o-minimal structure. The first known example of such a structure is the collection of all semialgebraic sets. The o-minimality is a consequence of Tarski’s quantifier-elimination theorem for real closed fields. While the collection of all subanalytic sets is not an o-minimal structure, it can be associated to (and recovered from) one in a unique way. A subanalytic subset of \( \mathbb{R}^n \) is said to be globally subanalytic if its canonical embedding in projective space \( \mathbb{P}^n \) is a subanalytic subset of \( \mathbb{P}^n \). It follows from the theorems mentioned above that the collection of all globally subanalytic subsets of Euclidean space forms an o-minimal structure.

Many o-minimal structures have been found over the last fifteen years that strictly contain the globally subanalytic sets. These structures are typically generated by functions arising in various problems in the theory of dynamical systems: pfaffian functions (solutions of triangular partial differential equations with polynomial coefficients), various non-oscillatory trajectories of analytic vector fields, and even Poincaré return maps for certain planar analytic vector fields. In each of these examples, the method of proving o-minimality has contributed as much new insight into the problems as the o-minimality result, as it combines asymptotic analysis, resolution of singularities and differential geometry in new ways.

One of the aims of our program is to extend these methods to other classes of functions arising from dynamical systems. Of particular interest are such classes within which each function is uniquely characterized by a generalized series, that is, a series whose monomials are not restricted to real powers of the identity function, but may be logarithmic-exponential terms or similar. Such functions are prominent in both Ecalle’s and Ilyashenko’s solution of Dulac’s problem, and they arise as solutions of Dirichlet’s problem on certain subanalytic domains.

This focus of our program is reflected by our planned activities. Our three graduate courses are split into three month-long modules each. In addition to covering many of the currently known methods of establishing o-minimality, we were able to enlist several lecturers working at the forefront of resolution of singularities and generalized asymptotics. These classes are preceded by a week-long introductory winter school in o-minimal geometry, intended to familiarize graduate students with the basic model theory and geometry needed to follow the courses. The Fields Institute Distinguished Lecture series takes place in May and will be given by Fields medalist Jean-Christophe Yoccoz. Throughout the second half of our program, various participants will organize mini-workshops, intended to discuss new developments in o-minimal geometry. We finish our program with a workshop on finiteness problems in dynamical systems.

“the method of proving o-minimality has contributed as much new insight into the problems as the o-minimality result, as it combines asymptotic analysis, resolution of singularities and differential geometry in new ways”

message from the director
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no longer relevant to the world, it can be associated with the ideas of structure. Such a collection is called an o-minimal structure. The first known example of such a structure is the collection of all semialgebraic sets. The o-minimality is a consequence of Tarski’s quantifier-elimination theorem for real closed fields. While the collection of all subanalytic sets is not an o-minimal structure, it can be associated to (and recovered from) one in a unique way. A subanalytic subset of \( \mathbb{R}^n \) is said to be globally subanalytic if its canonical embedding in projective space \( \mathbb{P}^n \) is a subanalytic subset of \( \mathbb{P}^n \). It follows from the theorems mentioned above that the collection of all globally subanalytic subsets of Euclidean space forms an o-minimal structure.

Many o-minimal structures have been found over the last fifteen years that strictly contain the globally subanalytic sets. These structures are typically generated by functions arising in various problems in the theory of dynamical systems: pfaffian functions (solutions of triangular partial differential equations with polynomial coefficients), various non-oscillatory trajectories of analytic vector fields, and even Poincaré return maps for certain planar analytic vector fields. In each of these examples, the method of proving o-minimality has contributed as much new insight into the problems as the o-minimality result, as it combines asymptotic analysis, resolution of singularities and differential geometry in new ways.

One of the aims of our program is to extend these methods to other classes of functions arising from dynamical systems. Of particular interest are such classes within which each function is uniquely characterized by a generalized series, that is, a series whose monomials are not restricted to real powers of the identity function, but may be logarithmic-exponential terms or similar. Such functions are prominent in both Ecalle’s and Ilyashenko’s solution of Dulac’s problem, and they arise as solutions of Dirichlet’s problem on certain subanalytic domains.

This focus of our program is reflected by our planned activities. Our three graduate courses are split into three month-long modules each. In addition to covering many of the currently known methods of establishing o-minimality, we were able to enlist several lecturers working at the

NOTE

RICHARD BOND, former Fields Board of Director Member, has received the Gruber Cosmology Prize for major theoretical insights into the origin of the universe. Bond, who is now the director of the Canadian Institute for Advanced Research Cosmology and Gravity Program, is being honoured for groundbreaking theoretical work on structure formation and evolution of the universe. Bond received the prize in September at the Harvard Smithsonian Center for Astrophysics in Cambridge, Mass.

DAVID BRYDGES, Fields Scientific Advisory Panel Member, has been appointed as Deputy Director of the Pacific Institute for the Mathematical Sciences. His term commenced on August 1 2008. He is also serving as the PIMS-UBC Site Director. Brydges is a Professor of Mathematics and Canada Research Chair at the University of British Columbia.

NOTED

Patrick Speissegger
Call for Proposals, Nominations, and Applications

For detailed information on making proposals or nominations, please see the website: www.fields.utoronto.ca/proposals

General Scientific Activities*
Proposals for short scientific events in the mathematical sciences should be submitted by October 15, February 15 or June 15 of each year, with a lead time of at least one year recommended. Activities supported include workshops, conferences, seminars, and summer schools. If you are considering a proposal, we recommend that you contact the Director, Barbara Keyfitz, or Deputy Director, Juris Steprans (proposals@fields.utoronto.ca). Also see www.fieldsinstitute.ca/proposals/other_activity.html

Thematic Programs *
Letters of intent and proposals for semester long programs at the Fields Institute are considered in the spring and fall each year, and should be submitted by March 15 or August 31. Organizers are advised that a lead time of several years is required, and are encouraged to submit a letter of intent prior to preparing a complete proposal. The Fields Institute has started a new series of two-month-long summer thematic programs focusing on interdisciplinary themes. Proposals for the summer of 2010 are now being considered. Organizers should consult the directorate about their projects in advance to help structure their proposal.

Postdoctoral Opportunities
Applications are invited for postdoctoral fellowship positions for the 2009-2010 academic year. The thematic program on Foundations of Computational Mathematics is running in the fall of 2009, and the program on Quantitative Finance Foundations and Applications will run in winter/spring 2010. Qualified candidates who have recently completed a PhD in a related area of the mathematical sciences are encouraged to apply. The fellowships provide for a period of engagement in research and participation in the activities of the Institute. They may be offered in conjunction with partner universities, through which a further period of support may be possible. One recipient will be awarded the Institute’s prestigious Jerrold E. Marsden Postdoctoral Fellowship. Applicants seeking postdoctoral fellowships funded by other agencies (such as NSERC or international fellowships) are encouraged to request the Fields Institute as their proposed location of tenure, and should apply to the address below for a letter of invitation. Additional support is available from NSF to support junior US visitors to some programs. Applications are encouraged from all qualified candidates, particularly aboriginal peoples, persons with disabilities, members of visible minorities and women.

The deadline for postdoctoral applications for the 2009-2010 programs is December 7, 2008, although late applications may be considered. Postdoctoral opportunities also exist at some of the Fields Institute’s sponsoring universities. Consult www.fields.utoronto.ca/proposals/#pdf for details.

CRM–Fields–PIMS Prize
Nominations are invited for this joint prize in recognition of exceptional achievement in the mathematical sciences. The candidate’s research should have been conducted primarily in Canada or in affiliation with a Canadian university.

Please send nominations to: directeur@CRM.UMontreal.ca

Nominations for the CRM–Fields–PIMS Prize should reach the CRM by November 1, 2008.

Distinguished Lecture Series in Statistical Science (DLSS)
Nominations are being solicited for the eighth Fields Institute Distinguished Lecture Series in Statistical Science, to be given in 2009. The awardee will be an internationally prominent statistical scientist, who will give two lectures (one general, one specialized) at the Fields Institute.

Nominations for the DLSS should reach the Institute by October 1, 2008, although late applications may be considered. DLSS nominations should be sent to Nancy Reid c/o Fields Institute or directly to reid@utstat.utoronto.ca

*A note on diversity. In proposing any activity, applicants are requested to consider the mandate of the Institute to broaden and enlarge the community. Applicants should explain how they plan to include women and members of visible minority groups in the proposed activity. As well, they should ensure that the proposed participant lists include scientists representing a range of career levels, types of institutions and geographical locations in Canada and abroad.
Fields Activities
Bridging Research, Education and Industry

SEPTMBER 2008 – JUNE 2009
at Fields unless otherwise indicated
Detailed information: www.fields.utoronto.ca/programs

Thematic Programs

THEMATIC PROGRAM ON ARITHMETIC GEOMETRY, HYPERBOLIC GEOMETRY AND RELATED TOPICS
JULY-DECEMBER, 2008
Organizers: John Bland (Toronto), Caterina Consani (Johns Hopkins), Stephen Kudla (Toronto), Min Ru (Houston), Paul Vojta (UC Berkeley), Pit-Mann Wong (Notre Dame)

WEEK OF SEPTEMBER 29-OCTOBER 3, 2008 (TENTATIVE)
COXETER LECTURE SERIES: SHOU-WU ZHANG, COLUMBIA UNIVERSITY

OCTOBER 20-24, 2008
Workshop on Arithmetic Geometry: Diophantine approximation and Arakelov theory

OCTOBER 27-28, 2008
Mini-workshop on p-adic dynamics

NOVEMBER 10-14, 2008
Mini-workshop on complex dynamics

WEEK OF NOVEMBER 10-14, 2008 (TENTATIVE)
DISTINGUISHED LECTURE SERIES: YUM-TONG SIU, MATHEMATICS DEPARTMENT, HARVARD

NOVEMBER 17-21, 2008
Workshop on Complex Hyperbolic Geometry and Related Topics

THEMATIC PROGRAM ON O-MINIMAL STRUCTURES AND REAL ANALYTIC GEOMETRY
JANUARY-JUNE 2009
Organizers: David Marker (Chicago), Chris Miller (Ohio State), Jean-Philippe Rolin (Bourgogne), Patrick Speissegger (McMaster), Carol Wood (Wesleyan)

JANUARY 12-16, 2009
Winter School in o-minimal geometry

MAY 25-29, 2009 (TENTATIVE DATES)
Distinguished Lecture Series: Jean-Christophe Yoccoz, Collège de France

JUNE 22-26, 2009
Workshop on Finiteness problems in dynamical systems

General Scientific Activities

SEPTMBER 12-21, 2008
Ottawa-Carleton Algebra Week
Carleton and Ottawa Universities

SEPTMBER 18-20, 2008
Conference on Non-linear Phenomena in Mathematical Physics:
Dedicated to Cathleen Synge Morawetz
on her 85th birthday

SEPT. 24-28, 2008
International Conference on Infinite Dimensional Dynamical Systems
York University, Toronto

SEPTEMBER 25-27, 2008
Quantum Critical Phenomena Statics and Dynamics Meeting

OCTOBER 24-26, 2008
2nd Annual Pure and Applied Mathematics Graduate Student Conference
McMaster University

NOVEMBER 1-2, 2008
Workshop on Algebraic Varieties
Fields Institute

NOVEMBER 2-7, 2008
Workshop on Women in Numbers (WIMS)
Banff International Research Station for Mathematical Innovation and Discovery, Banff, Alberta

NOVEMBER 9-10, 2008
IFID/MITACS Conference on Financial Engineering for Actuarial Mathematics Fields Institute

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MESSAGE FROM THE DIRECTOR

Getting the Most out of Fields

In this issue of Fields Notes, as in every issue of the newsletter, you can read about many activities that take place at Fields, or are sponsored by Fields elsewhere. If you have been wondering what this has to do with you, then the purpose of this fall’s column is to give an answer. Some people know already the extent to which Fields welcomes initiatives from the mathematical community. Those people have already been taking advantage of the Institute to organize programs and workshops. This message is for everyone else.

The most obvious track for community involvement is to propose to organize some event under the heading of General Scientific Activities, and an invitation to do so is issued elsewhere in this newsletter. Here I want to talk about additional opportunities, some old and some new.

First, and simplest: Participate. No Fields event is “by invitation only”, and everyone is welcome to attend everything. For formally organized activities like workshops, we ask people to register, and registration closes if it appears that the room will be full to capacity. (For this reason, we try to make sure that only people who really intend to come to events actually register. That is one purpose of charging a small registration fee.)

Second: Organize. Any “member of the Canadian mathematical sciences community” is invited to propose a Fields event, on any scale from a thematic program to a special lecture. Not a member of the Canadian mathematical sciences community? That provision is inserted into our operating procedures to ensure that everyone who tries to make a difference to mathematics research and training in Ontario. But a typical organizing committee is truly international. (Fields is happy to broker partnerships, too – sometimes organizers get to know each other by working to arrange a thematic program.)

Third: Innovate. If you have ideas for activities that would benefit the mathematical sciences research community, broadly defined, then bring them to Fields. This could be done through a formal proposal, through a letter or e-mail to the Director or Deputy Director, or by communicating with a member of the Scientific Advisory Panel or the Board of Directors. Not all ideas are practical, but many of the best things done at Fields started with suggestions that came from you. In the past year, we have formed an Institute-to-Institute partnership with the Liu Bie Ju Center of the City University of Hong Kong, which has resulted so far in cooperation on Industrial Problems Workshops, and in research visits. With support from Fields, George Gadanidis at Western has begun a competition for elementary school students, the Mathematics Performance Festival. Recently, Carleton University started a public lecture series, the Fields-Carleton Distinguished Lectures.

The best programs are yet to be designed. With a magnificent building,