Distinguished Lecture Series: Gregory Margulis

The Distinguished Lecture Series for 2005-2006 was delivered by Gregory Margulis of Yale University from January 9-11, 2006. The series of three lectures exposed the interplay of ideas between ergodic theory, the theory of Lie groups and homogeneous spaces, and Diophantine approximation.

Margulis is one of a handful of mathematicians who have been awarded both the Fields Medal and the Wolf Prize. Since the beginning of his career in late 1960s, his work has been characterized by the establishment of connections between seemingly unrelated areas of mathematics. His proof of the arithmeticity of lattices in higher rank Lie groups, for which he received the Fields Medal, used a variety of methods from ergodic theory, geometry and representation theory. Applications of his work can also be found in graph theory and combinatorics. Terms such as Margulis measure, the Margulis Lemma, the Margulis invariant, have become commonplace in modern mathematics.

Shortly after the lectures Professor Margulis was honored at a conference

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“Lie Groups: Dynamics, Geometry, Arithmetic” at Yale University on the occasion of his 60th birthday.

The title for two of Margulis’s three lectures was “Irrational Quadratic Forms”. Their subject was the Oppenheim conjecture, proved by Margulis in the mid-1980s, which was concerned with the distribution of values of indefinite irrational quadratic forms at integral points.

He started his first lecture by stating the conjecture (now his theorem) and its various extensions, such as estimates and asymptotics for the number of solutions of inequalities \( a < Q(x) < b, \) \(|x| \leq T \) as \( T \to \infty \), where \( Q \) is an indefinite quadratic form, \( a,b \in \mathbb{R} \) and \( x \in \mathbb{Z}^d \) – this is joint work of Margulis with Dani and also with Eskin and Mozes. He spent some time describing the history and various approaches to the problem, in particular Raghunathan’s reduction of the Oppenheim conjecture to studying orbits of the \( SO(2,1) \)-action on \( \text{SL}_3(\mathbb{R})/\text{SL}_3(\mathbb{Z}) \).

The latter reduction motivated deep conjectures of Raghunathan, Dani and Margulis on orbit closures and invariant measures of unipotent actions on homogeneous spaces, proved in full generality by Ratner.

At the end of the talk Margulis stated new results involving positive definite quadratic forms \( Q \). Recently Friedrich Götze established

“terms such as … the Margulis invariant, have become commonplace”

new bounds for the error term in the “circle problem”, that is, the difference between the volume of the ellipsoid \( |x| \leq T \) and \( (x,x) \leq s \) and the number of integer points in it. Previously such estimates were obtained by Bentkus and Götze when \( d \geq 9 \); new methods allowed the reduction of the number of variables to 5. A corollary of the new estimate is the validity of the Davenport-Lewis conjecture for forms of five or more variables.

The goal of the second lecture was to shed light on the methods of proofs of some of the results mentioned in the first talk. Margulis described a reduction of the counting problem in the Oppenheim conjecture to computing certain integrals over the space of lattices. The audience was introduced to the construction mentioned in the title to the third lecture, namely to “one interesting class of functions on the space of lattices”. These functions \( \alpha_j \) and various combinatorial inequalities between them, played the crucial role in the work of Eskin, Margulis and Mozes.

The last lecture was entirely devoted to positive definite forms, that is, to the work of Götze, the main technical tools of which are Fourier transforms and estimates of theta-sums. However Margulis chose to describe his modification of Götze’s proof, showing how his favorite functions \( \alpha_j \) enter the picture. At the end of the talk the proof of the error term bound was reduced to estimating integrals over certain orbits on the space of lattices.

Dmitry Kleinbock (Brandeis University)

Although he regrets not having as much time to read as he would like, he mentioned his lasting pleasure in Cervantes’ Don Quixote, and, recently, in the novels of Anthony Trollope, particularly the quartet of novels that comprise the Barchester Chronicles, that amusing, sad, and intimate portrait of an unworldly Church of England vicar in Victorian times.

Stepans agrees with his colleagues who say that outside of his family, in-line skating, cross-country skiing and a bit of pool-playing, he has “no life” beyond his work. But his interest in mathematics and in mathematics education extends well beyond. He has served his profession through activity in the Canadian Mathematical Society, and on the Natural Sciences and Engineering Council as Chair of one of the grants selection committees. He has also organized numerous research programs at the Fields Institute and elsewhere. We are fortunate that his energy and single mindedness will now be turned to the benefit of the mathematics research and education programs of the Fields Institute.

Elaine Riehm
NICOLE TOMCZAK-JAEGERMANN, of the University of Alberta, has been awarded the 2006 CRM-Fields-PIMS prize. According to the citation, “She has made outstanding contributions to infinite dimensional Banach space theory, asymptotic geometric analysis, and the interaction between these two streams of modern functional analysis. She is one of the few mathematicians who have contributed important results to both areas. In particular, her work constitutes an essential ingredient in a solution by the 1998 Fields medalist W. T. Gowers of the homogeneous space problem raised by Banach in 1932.”

Tomczak-Jaegermann received her Master’s (1968) and Ph.D. (1974) degrees from Warsaw University, where she held a position until moving to the University of Alberta in 1983. There she holds a Canada Research Chair in Geometric Analysis. She is a Fellow of the Royal Society of Canada, lectured at the 1998 ICM, and has won the CMS’s Krieger-Nelson Prize Lectureship. She has served the Canadian research community in many ways including the Scientific Board of BIRS and as a Site Director of PIMS.

What is this area of mathematics which produced two recent Fields medals (of J. Bourgain and T. Gowers)? The story starts with the 1932 book of Banach where he laid the foundation of Banach space theory. It was to be a foundational theory for many applied problems in differential equations and other applied fields, but the intellectual curiosity of the customers of the “Scottish cafe” in Lwoff took over, and the quest for a classification theory for infinite dimensional Banach spaces started soon after. Most problems turned out to be deep and hard and way beyond the reach of the mathematicians of the 30’s and 40’s. All these questions have now been answered but many had to wait till the end of the century. While the questions look like mere mathematical curiosities, the techniques developed to answer them turned out to be rich and far reaching: from convex analysis, to infinite dimensional Ramsey theory, and from the refined asymptotics of various parameters connected to convex bodies, to the theory of random matrices and Gaussian processes.

Banach’s book posed several intriguing problems about the structure of infinite dimensional spaces. Are they isomorphic to their own hyperplanes? To their squares or their cubes? Among the best known is the homogeneous space problem. Because of space constraints I will focus on that rather than Nicole’s other profound contributions on topics such as metric entropy, Banach-Mazur distances, the best projection constants problem, complex convexity in infinite dimensions, or Sudakov’s minoration theorem in the theory of Gaussian processes.

Problem: Is Hilbert space the only homogeneous Banach space? i.e., is it the only one that can be isomorphic to all of its infinite dimensional subspaces?

Now we know that the answer to this question of Banach is affirmative, thanks to independent and remarkably complementary contributions by Gowers on one hand, and by N. Tomczak-Jaegermann and Komorowski on the other. The first breakthrough came when Nicole and Komorowski proved that much can be said if the space has an unconditional basis.

Theorem A: A Banach space with an unconditional basis contains either a Hilbertian subspace or a subspace without an unconditional basis.

Next Gowers and Maurey devised a novel construction that opened a whole new understanding of infinite-dimensional phenomena. In fact, the Gowers-Maurey space has a stronger property: no subspace is a topological direct sum of two infinite-dimensional spaces. Such a space is called hereditarily indecomposable (an HI space) and they went on to prove the following:

Theorem B: An HI Banach space is not isomorphic to any proper subspace of itself.

Many interesting examples of spaces having this and related properties were eventually constructed by Gowers-Maurey, but the precise connection between these concepts was finally clarified by the spectacular structural dichotomy proved by Gowers in 1993:

Theorem C: Every infinite-dimensional Banach space either has an infinite dimensional subspace with an unconditional basis or has an HI subspace.

Once all these results were proved, the solution to the homogeneous space problem is simple. By Theorem A, a homogeneous space $X$ not isomorphic to Hilbert space cannot have an infinite-dimensional subspace with an unconditional basis. By Theorem C it must have an HI subspace, and hence $X$ itself is HI since it is homogeneous. But then, Theorem B says that it cannot then be isomorphic to any proper subspace of itself, which means that $X$ is not homogeneous after all.

The CRM-Fields-PIMS prize is Canada’s leading award recognizing exceptional research in the mathematical sciences, and was established in 1994 as the CRM-Fields prize. In 2005 PIMS became an equal partner and the name was changed to the CRM-Fields-PIMS prize. Previous recipients of the prize are H.S.M. Coxeter, G. A. Elliott, J. Arthur, R. V. Moody, S. A. Cook, S. M. Sigal, W. T. Tutte, J. B. Friedlander, J. McKay, E. Perkins, D. A. Dawson, and D. Boyd. An expanded version of this article will be published elsewhere and made available through the Fields website.

Nassif Ghoussoub (UBC)
Three generations of complex dynamicists gathered at the Institute on March 7–11 to celebrate John Milnor’s 75th birthday. This festive occasion was an opportunity to take stock of the explosive development the subject has experienced since the early 1980’s, and to chart new directions for research. Many presentations were made with impressive high-tech support and accompanied by beautiful computer-generated images (see the facing article in this issue for more on this topic).

In an inspired opening lecture B. Branner described some of Milnor’s contributions to the development of complex dynamics.

The scientific highlight of the workshop was a mini-course given by X. Buff, A. Chéritat, A. Douady and M. Shishikura on a construction of polynomial Julia sets of positive measure. The question of the existence of such sets had been one of the central open problems in the field. In part, the motivation is the Rigidity Conjecture postulating the absence of non-trivial deformations of non-hyperbolic quadratic polynomials. If the Julia set has zero area, the map cannot be deformed. Yet, even though the Rigidity Conjecture is generally believed to be true, we now know that Julia sets of positive measure do exist.

About a decade ago Adrien Douady laid down a program for constructing such pathological examples. In his thesis, Chéritat, developing earlier work of Jellouli, made a major advance in the program by showing, in particular, that a Siegel Julia set can be perturbed to a parabolic one without losing much area. The converse statement is based on very recent results of Inou and Shishikura on renormalization of parabolic maps. Building on it (and on results of McMullen on the geometry of Siegel disks), Buff and Chéritat completed Douady’s program. (Another conceivable completion of the program by means of renormalization of Siegel disks was outlined by Gaidashev and Yampolsky at the Workshop on Renormalization in Dynamical Systems in November.)

An important step towards resolving the Rigidity Conjecture in the quadratic family was announced by J. Kahn, who established a priori bounds for infinitely renormalizable quadratics of bounded primitive type. Combined with the existing theory, this implies that these maps are rigid. (Further rigidity results and the underlying new analytic techniques developed by Kahn and Lyubich have been discussed in Lyubich’s graduate course at the Fields Institute.)

The field of complex dynamics in several variables offers further exciting challenges. Polynomial diffeomorphisms in \( \mathbb{C}^2 \), such as the celebrated Hénon maps first studied in this context by Friedland & Milnor and Hubbard & Oberste-Vorth, are a natural bridge between the one- and two-dimensional worlds. In one complex dimension, the action of a quadratic polynomial at the circle of directions at infinity is simply the angle-doubling map. This observation is of profound importance to the field, leading to a natural combinatorial model of the Julia set. For a Hénon map, the corresponding object is a hyperbolic map of the dyadic solenoid obtained by an infinite sequence of blow-ups to resolve the indeterminacy of the action at infinity.

J. Smillie discussed perspectives of this approach, such as the possibility of constructing an analogue of the Yoccoz puzzle, and J. Hubbard presented the blow-up techniques in the study of Newton’s method for finding the intersection of two conics. N. Sibony’s talk complemented these lectures by describing the pluripotential theory approach to the construction of the measures of maximal entropy. These were but a few of the many excellent talks at the workshop, which explored the relations of one and several dimensional complex dynamics with hyperbolic geometry, combinatorial group theory, dynamics in several variables, holomorphic foliations and Riemann surface laminations.

The scientific program was complemented by a number of enjoyable social events. A game show “Who wants to be a Milnor-aire” by D. Schleicher, J. Rückert and R. Perez was hilarious; somewhat more serious stories (and a song) were delivered at the banquet in honour of Milnor.

Many participants stayed in Toronto for a few weeks longer (and many came earlier) creating a vibrant research atmosphere at the Institute. The conference was partly supported by the NSF.

Mikhail Lyubich and Michael Yampolsky (Toronto)
The March Workshop on Holomorphic Dynamics included a competition. Twenty-eight computer-generated images were entered and displayed around the Institute. The winning entry was submitted by Lasse Rempe of the University of Liverpool. The following note describes the mathematics it represents.

This figure shows a detail of the parameter space of exponential maps $E_\kappa: z \to \exp(z) + \kappa$. The fan-like regions are hyperbolic components and represent particularly simple dynamical behavior: for parameters in these components, almost every orbit converges to a stable periodic cycle. The grey background region, on the other hand, represents the bifurcation locus: the region of parameter space where dynamical behavior changes significantly under a small perturbation of the parameter.

By a theorem of Schleicher, each hyperbolic component has a distinguished boundary point, which is the landing point of two parameter rays (certain dark curves in the bifurcation locus). The figure demonstrates this fact for a hyperbolic component (in the middle of the picture), and three other hyperbolic components which bifurcate from it. It was created to illustrate recent results (to appear in Proc. AMS), which exploit this combinatorial structure of parameter space to obtain information in the dynamical plane.

The boundaries of the four hyperbolic components are drawn by repeatedly using a Newton’s method in two variables to find parameters with a neutral periodic cycle. A similar method is used to draw parameter rays. The background picture is a combination of a heuristic which decides whether a given pixel intersects the bifurcation locus, and a colour scheme on hyperbolic components.

A colour image can be found at www.fields.utoronto.ca/programs/scientific/05-06/holodynamics/holodynamics_workshop/images/index.html

Lasse Rempe (Liverpool)
Partially hyperbolic dynamics, laminations, and Teichmüller flow

This workshop, organized by Giovanni Forni, Mikhail Lyubich, Charles Pugh and Michael Shub of the University of Toronto and held January 5–9, 2005, brought together several leading experts in two very active fields of contemporary dynamical systems theory: partially hyperbolic dynamics and Teichmüller flow. They are unified by ideas coming from the theory of laminations and foliations, dynamical hyperbolicity, and ergodic theory, which made interaction between experts from different fields natural and productive. The large group of students and young researchers attending the workshop had a chance to learn about the most recent events in the field. Two mini-courses given during the workshop offered them a friendly introduction to the main themes.

Roughly speaking, the main goal of dynamical systems theory is to describe the statistical behavior of trajectories of a typical system of some natural class. Tremendous progress in this direction was made in the 1960–1970’s by Smale, Anosov, Sinai, Ruelle, and others, who achieved this goal for uniformly hyperbolic systems. These are systems for which all orbits diverge (uniformly) exponentially rapidly, either in forward or backward time. These systems exhibit sensitive dependence on initial conditions, and generate horseshoes, strange attractors, and a variety of complex behaviours. Paradoxically, this extreme instability makes them both treatable and statistically robust.

A natural extension of this class relaxes the condition of exponential divergence of all orbits leading to the class of partially hyperbolic systems. In the mid 1990’s C.Pugh and M.Shub formulated a program of exploring the Boltzmann Ergodicity Conjecture for this class. This led to an explosive development of the field in the past decade that was reflected at the workshop.

The main conjecture by Pugh and Shub is that a generic partially hyperbolic system is robustly ergodic. The impressive progress that has been achieved recently in this direction was surveyed in a mini-course by Keith Burns and Federico Rodriguez Hertz.

The main result is that the conjecture is true in the case of one-dimensional central direction.

“Paradoxically, this extreme instability makes them both treatable and statistically robust.”

The Teichmüller flow is a conservative flow on the moduli space of quadratic or abelian differentials on Riemann surfaces which plays a crucial role in the study of several basic classes of dynamical systems, such as interval exchange transformations and billiards in polygons with rational angles. It can be viewed as the renorm-group for these dynamical systems, which links it closely to the theme of the Fields Institute Fall 2005 program on Renormalization and Universality. It is also linked to holomorphic dynamics (which is a major theme of the current program) through ideas coming from the theory of deformations of conformal structures.

The foundation of the field of Teichmüller dynamics was laid down about twenty years ago in the pioneering work of H.Masur and W.Veech who proved by its means the (unique) ergodicity of the typical interval exchange transformation. More recently, A. Zorich and M. Kontsevich formulated a series of conjectures on the Lyapunov spectrum of the flow and on the related deviations for the ergodic averages of interval exchange transformations. The Kontsevich-Zorich conjectures have been completely proved by G. Forni and A. Avila, and M. Viana.

An introduction to this field was given in the first two lectures of a mini-course by Anton Zorich. In the third lecture, he went on to describe the connected components of the strata of the moduli space in topological and combinatorial terms. This discussion was further elaborated upon in the talk by Howard Masur.

These fields, together with others such as foliations, formed the basis for many of the interesting and memorable talks at the workshop, which was in turn followed by Gregory Margulis’ Distinguished Lecture Series. For more details about the workshop, please consult the program website.

The workshop was supported by the Fields Institute and the National Science Foundation (U.S.).

Mikhail Lyubich (Toronto)
The 2006 Distinguished Lecture Series in Statistical Science was delivered on April 3 and 4 by Elizabeth Thompson, Professor of Statistics at the Department of Statistics and at the new Genome Centre of the University of Washington. This annual series was established in 2000, with previous lectures given by Peter Hall, D.A.S. Fraser, Donald Dawson, Sir David Cox, and Brad Efron.

Since 1999, Elizabeth Thompson has developed both research and education in statistical genetics at the University of Washington. She is well known for her contributions to statistical methods for inference from genetic data, and particularly from data observed on large and complex pedigree structures. In September 2000, based on lectures presented at a 1999 CBMS Summer Research Conference, she published an IMS monograph on modern methods of Pedigree Analysis, where she describes some of the foundations of her work. She is a pioneer in the development of Markov chain Monte Carlo (MCMC) and Monte Carlo likelihood methods for the analysis of data on individuals within a known pedigree structure.

The two lectures given by Thompson relate to her most recent work on fuzzy p-values and confidence intervals for composite null hypotheses. While the uncertainty in Geyer and Meeden’s paper relates to the discreteness of data random variables, Thompson addressed the problem of uncertainty associated with latent variables.

The first lecture developed the basic concepts of the approach, showing its interest for a test of association in a 2 by 2 table. In latent variable problems, the natural definition of a fuzzy p-value is the distribution, given observed data, of that function of latent variables that would be the p-value were the latent variables observed. This notion puts our uncertainty directly onto the p-value scale, and permits simultaneous expression of the strength of the evidence and our uncertainty.

The second lecture addressed more specifically the application to genetics. Latent variables are a common problem in genetic analyses since the pattern of DNA inheritance in pedigrees is often not measured but inferred from observed data. Thompson showed how fuzzy p-values can summarize both the strength of evidence for linkage and the uncertainty about that evidence. She explained how realizations from the fuzzy p-value distribution may be obtained efficiently with only two sets of Monte Carlo realizations. No simulation of marker data is required, and the procedure, being conditional on the observed marker data, shares with permutation-based tests a partial robustness to the genetic map and assumed allele frequencies of the markers.

In summary, this new “fuzzy” concept has the potential to improve inference about genetic linkage in pedigrees, where the decision to further explore genomic regions could be based not only on p-values but also on the evidence (or uncertainty) embodied in the p-value itself.

Laurent Briollais (Samuel Lunenfeld Research Institute)
ON MARCH 13 AND 14, DONALD SAARI of the University of California at Irvine presented two lectures in the first Fields-Carleton Distinguished Lecture Series. This series of lectures by renowned mathematicians will be held at Carleton University, sponsored by the Fields Institute and the School of Mathematics and Statistics of Carleton University. The Director of Fields, Barbara Keyfitz, was on hand to inaugurate the event.

Saari is UCI Distinguished Professor of Mathematics and Economics and Director of the Institute of Mathematical Behavioral Sciences at Irvine. (He is also professor of Logic and Philosophy of Science.) His series was entitled Fascinating Mathematics for the Social Sciences.

The first lecture, “The geometry of departmental discussions” was a public lecture, attended by over one hundred people including many students. From the opening joke about how his wife was worried about having to be Saari for the rest of her life, to his Phil Donahue-esque way of taking questions and answering them at the end, Saari is a consummate public lecturer who manages to fill the auditorium with an energy and excitement that leaves people wanting to hear more.

As an introductory example, Saari showed how to “rig” an election so that under apparently fair voting an unpopular candidate would win, and he related symmetry to election paradoxes. In the main part of his lecture, he explained mathematically, with a bit of geometry and some elementary group theory, why no matter how wonderful a proposal you might come up with, no matter how many hours of labour you put into the proposal to try and win everybody over, somebody will assuredly try and improve it. One of his theorems identifies voting procedures, based on the number of voters, that will ensure, for a given number of issues, that a core point will exist – that is, in order for there to exist an unbeatable proposal. In the question period, he was asked to give additional previously unimagined areas where mathematics plays a key role.

Saari’s second lecture was about “Developing a qualitative evolutionary game theory”. Saari emphasized that, in the social sciences, the differential equations used to model dynamics are often of questionable credibility, and so he recommended an alternative: approach the system qualitatively and identify robust results that would be true no matter which differential equation is used. For example, you can determine, based only on limiting conditions and the assumption of causality, whether a situation will lead to equilibrium or a threshold. Using index theory, he was able to extend a one-dimensional result that he proved at the blackboard to a more complex situation, and to say quite a bit about the existence of equilibria. The potential of this approach, to produce insight into the consequences of social and economic policies, rather than to find only formulas that no one believes, was well illustrated and exciting to the mathematicians and the economists in the audience alike.

Ben Steinberg (Carleton)

NOTED

PHELIM BOYLE has been named the 2005 IAFE Financial Engineer of the Year. Phelim is an organizer of the quantitative finance seminar.

BARBARA KEYFITZ has won the 2006 Esther Farfel award, the University of Houston’s highest honour.

The AARMS summer schools – modelled after the celebrated schools of Perugia/Italy – were initiated in 2002 by Edgar Goodaire (Memorial University), who directed the schools in St. John’s from 2002 to 2004. In 2005 they moved to Dalhousie University in Halifax and will continue there until 2007, under the directorship of Tony Thompson (see www.aarms.math.ca/summer/2006/). The principal aim of the schools is, on the one hand, to broaden the graduate course offerings for students in the Atlantic region, and on the other – perhaps at least as important – to expose our students to graduate students from different cultures, and with different backgrounds, from around the world. There are some statistics: between 2002 and 2005, close to 40 students from Newfoundland, Nova Scotia, New Brunswick and Prince Edward Island attended the schools; an almost identical number of students came from four other Canadian provinces, and there were over 50 students from the USA and 12 European countries. The course offerings range from algebra, number theory, elliptic curve cryptography, algebraic and fractal geometry to computational combinatorial mathematics, partial differential equations, wavelet theory and numerical applications, mathematics of finance, massive networks and internet mathematics, and on to mathematical biology, mathematical statistics and statistical genomics. The instructors have been first-rate research mathematicians and equally motivated teachers: counting this year’s summer school, they represent, in addition to seven leading universities across Canada, departments from universities in Australia, Brazil, England and Scotland, Italy, and the USA. Promising senior undergraduate students are encouraged to attend the summer schools, and it is clear that such high-level and broad course offerings provide a marvelous incentive to continue their studies at the graduate level.

AARMS workshops also play an important role in the training of graduate students. They not only expose them to cutting-edge mathematical research but also allow informal discussions with the regional and international speakers. Since mathematical sciences departments have hired a significant number of new faculty – most of whom are supervising graduate students – over the last five years or so, these workshops are instrumental in building a cohesive mathematical community in our region, as they create interaction and research collaborations. A key requirement for a workshop to get financial support from AARMS is that a substantial part of the budget go towards funding of attending graduate students. To complement the above statistic: since 2002 there have been 10 workshops at seven of the “smaller” Atlantic universities, in addition to some 15 workshops at the “big ones”. Of the many examples, I would like to mention the two AARMS sessions, Graph Theory and Combinatorics and Robust and Computationally Intensive Statistical Models held during the 2005 APICS meeting at Acadia University (APICS stands for Atlantic Provinces Council for the Sciences). Organized, following a “challenge” by the AARMS director to the head of department, by two dynamic local faculty members and attracting close to 40 people each, they were representative of many other similar workshops in that they were built around an ideal blend of invited speakers, senior and junior researchers, PDFs, and many graduate students whose contributions were truly impressive. All this (and much more – e.g. the AARMS PDF support program) was made possible when, in June 2002, the directors of CRM (Jacques Hurtubise), the Fields Institute (Ken Davidson), and PIMS (Nassif Ghoussoub) convinced senior administrators at Memorial University of Newfoundland, Dalhousie University, and the University of New Brunswick to match the Institutes’ proposed annual long-term funding package for AARMS. In the meantime, Acadia University has joined these “AARMS universities”. I am very pleased to acknowledge that the strong commitment of the three Institutes to AARMS has impressively continued under their new Directors – François Lalonde, Barbara Lee Keyfitz and Ivar Ekeland – and bodes well for the future of Mathematics in Canada.

Hermann Brunner (Memorial University)
One Saturday a month, the Fields Mathematics Education Forum brings together mathematicians, teachers, and other interested individuals to discuss issues of common interest. On January 28, 2006, the forum was entirely devoted to exploring the complex and non-transparent relationships between media/popular culture and mathematics/mathematics education.

After an opening presentation by Miroslav Lovric (McMaster University), who outlined a number of issues to address, attention turned to the guest presenters and panelists: Louise Brown (one of two reporters covering education for the Toronto Star, on occasion writing about math and math curriculum issues), Peter Calamai (the Ottawa-based national science reporter for the Toronto Star, and a past member of NSERC’s advisory board), Keith Devlin (executive director of Stanford University’s Center for the Study of Language and Information, and author of several popular columns on mathematics), Eric McMillan (managing editor of a number of Toronto community newspapers) and Siobhan Roberts (a freelance writer based in Toronto, and author of a biography of Donald Coxeter).

Keith Devlin discussed how he ended up writing a mathematics column with the Manchester Guardian, which started his rich experience with a wide variety of media including radio, film and television. He left the audience some clues about how to write for non-mathematicians, and how to steer mathematical stories into the press.

Louise Brown and Peter Calamai shared their experiences in reporting on science, education, and mathematics. They gave the audience a sense of what topics capture the interest of editors. Besides a newsworthy topic, there has to be a narrative – stories are more than presenting facts. They can describe how people do mathematics, but they can’t teach the mathematics itself.

Siobhan Roberts talked about her goals in writing about mathematics, and particularly the challenge of sharing interesting mathematical personalities with the public. Eric McMillan referred to popular images of mathematicians, and described his experience of how reporters find stories or are pitched them by contact.

Roughly forty people shared this fascinating glimpse of the interface between two worlds, and an energetic discussion continued long after. Follow up material will be available through the forum website.

Miroslav Lovric (McMaster)
RCI Public Lecture – Miroslav Lovric

For generations, the Royal Canadian Institute for the Advancement of Science (RCI) has sponsored public lectures in science. Lately these have included a number of mathematics talks, arranged with the assistance of the Fields Institute. This relationship has a certain historical precedent, since John Charles Fields was in fact the RCI’s president from 1919 to 1925.

On January 22, 2006 Miroslav Lovric of McMaster University presented an RCI public lecture in this series. His talk was titled Infinity: the most fascinating of all ideas, and drew a sellout crowd to the University of Toronto’s medical sciences auditorium. He described the cultural significance of the infinite and infinitesimals, including examples of these as unreachable limits. Turning to mathematics, he discussed the apparent paradoxes accompanying arithmetic with infinite sums. Moving on to prime numbers he contrasted Skewe’s number (something very large but not infinite), with Goldbach’s conjecture (verified for a large number of cases, but not proved for all its infinitely many cases). Zeno’s paradox then led to a discussion of geometric series. Infinity enters geometry through tilings, which allowed him to discuss mosaics, Islamic art, and the work of Escher. Linguistically, he compared ideas of infinity in European culture with others, for example, those of India or the Maori.

Throughout, Lovric challenged his audience with puzzles and questions. Some of these led naturally to a discussion of cardinality, and how one rigorously defines the infinite. Cantor’s work was described, including the idea that there are various types of infinity. Turning to the continuum hypothesis, this gave him a chance to briefly mention Gödel’s work and independence.

He closed with the universe itself, the question of whether it is infinite or not, and some connections of this question to other aspects of his talk, including irrational numbers and curvature in differential geometry. The audience – which ranged in age from children to university students to seniors – responded with many enthusiastic questions at the end of the talk, and for long afterwards as well. Slides and a video of the talk can be found on the RCI website.

Tom Salisbury (Fields)

First Canadian Genetic Epidemiology and Statistical Genetics Workshop

This workshop, held at the Fields Institute March 30-31, 2006, was sponsored by the CIHR Institutes of Genetics and Population and Public Health, Genome Canada, the NCE in the Mathematics of Information Technology and Complex Systems, the National Program on Complex Data Structures, the Fields Institute, IBM and Affymetrix, Inc.

Complex traits, such as susceptibility to diabetes, cancer or tuberculosis, that vary in human and natural populations are determined by multiple genetic and environmental factors that interact with one another in complicated ways. Many challenges in the study of complex traits relate to population and study design issues. In particular, data with high dimension and complex structure present statistical challenges related to inference and computation. These challenges include incompletely observed data, inference for data with complex dependencies and high-volume testing of statistical hypotheses. Despite the incomplete understanding of these issues, human genetics is advancing quickly into uncharted areas such as whole genome association studies and the study of gene-environment interactions using population cohorts. This has led to a high demand worldwide for expertise in genetic epidemiology and statistical genetics. This demand is both exciting and daunting for the Canadian research community. On one hand, there are many opportunities to combine new technologies with unique Canadian resources including founder populations, large families, experienced clinical assessment teams, and the universal health care system. On the other hand, the pool of researchers with expertise is limited in number, and dispersed across the country, hampering efforts to develop formal graduate programs.

Two scientific sessions addressed analytic and design issues, the first in genome-wide association studies and population-based designs, and the second specific to studies on population resources in Canada. A third scientific session focused on novel approaches in genetic epidemiology. There were also poster presentations and break-out sessions on Canadian opportunities, capacity building and training, and retention of highly qualified personnel. A compelling example of the personal impact of this research was provided by Dr. Terry-Lynn Young of the Memorial University of Newfoundland. She described work that localized a genetic mutation in families from outport communities of Newfoundland that had been struck by a sudden and fatal form of heart disease. The research led to a genetic test which, so far, has identified 97 at-risk individuals, allowing them to be fitted with heart defibrillators. Already, 27 of these defibrillators have fired, saving 27 lives!

Jinko Graham (SFU)
A new volume in our Communications Series, *Nonlinear Dynamics and Evolution Equations*, will appear shortly. Based on lectures given during the International Conference on Nonlinear Dynamics and Evolution Equations at Memorial University in St. John’s on July 6-10, 2004, it is edited by Hermann Brunner and Xiao-Qiang Zhao (Memorial University), and Xingfu Zou (Western Ontario). There are nine survey papers, introducing the reader to and describing the current state of the art in major areas of dynamical systems, ordinary, functional and partial differential equations, and applications of such equations in the mathematical modelling of various biological and physical phenomena.

The Clay Mathematics Institute and the AMS have published the volume *Harmonic Analysis, The Trace Formula, And Shimura Varieties*, edited by James Arthur, David Ellwood and Robert Kottwitz. Its stated goal is to provide an entry point into modern theory of automorphic forms, embodied in what has come to be known as the Langlands program, which proposes fundamental relations that tie arithmetic information from number theory and algebraic geometry with analytic information from harmonic analysis and group representations. The volume is centered around the trace formula and Shimura varieties and is based on the courses given at the Clay Mathematics Institute Summer School held at the Fields Institute in the summer of 2003. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

The Fields books described here, as well as all earlier Fields publications, are now available at the front desk of the Institute, free of shipping charges and also at a discount for AMS members.

*Carl Riehm (McMaster)*

NEW ENTRIES IN EACH OF OUR TWO SERIES, Fields Institute Communications and Fields Institute Monographs, are appearing this spring. M. Aguiar and S. Mahajan have contributed *Coxeter Groups and Hopf Algebras* to our Monograph Series. This is the first text book on this subject, and arises from the BIRS meeting on Combinatorial Hopf Algebras two years ago. It was G.-C. Rota who first observed that various combinatorial objects possess natural product and coproduct structures. The enumeration and classification of these structures give rise to an associated graded Hopf algebra. Typically, it is a graded vector space where the homogeneous components are spanned by finite sets of combinatorial objects of a given type and the algebraic structures are given by constructions on those objects. These algebras are called combinatorial Hopf algebras. They have found applications in such diverse areas as renormalization in quantum field theory, the geometry of polytopes, and the representation theory of the Hecke algebras, and the Aguiar-Mahajan book provides a lucid introduction to the subject.

Since January, additional Fields activities took place that space precludes reporting on in this issue. These included workshops on *Combinatorial Inverse Systems* held Jan. 13-15, 2006 (organizers S. Faridi, C. Hohlweg, & M. Zabrocki); *Mathematical Aspects of Quantum Adiabatic Approximation* held Feb. 9-11, 2006 at Perimeter Institute (organizers A Ambainis & M.B. Ruskai); and *Lie Algebras* held March 24-26, 2006 at the Univ. of Ottawa (organizer E. Neher). A *Weekend Workshop on Algebraic Varieties* ran March 4-5, 2006; *Southern Ontario Dynamics Day* took place on April 7, 2006 (organizers M. Cojocaru and A. Lawniczak); the *Ontario Combinatorics Workshop* ran April 21-22, 2006 (organizers M. Molloy and P. Danziger); and the *3rd Young Mathematicians’ PDE conference* was held April 28-29, 2006. R.S. Berry gave the 2006 Nerenberg Lecture at the Univ. of Western Ontario on March 21, 2006.

The Institute’s many ongoing research seminars and graduate courses were active all winter, including a number of thematic mini-courses. A new risk management course is being offered at Fields, by PRMIA. Medical researchers and mathematicians interacted at three seminars of the Centre for Mathematical Medicine. Practitioners and academics met four times for the Quantitative Finance Seminars, and John Fraser (CRO of Hydro One) addressed the PRMIA Risk Management seminar. The Industrial Optimization Seminar met three times, the April 4, 2006 session of which consisted of a workshop on *IMRT Radiation Therapy Treatment Planning*. The Fields Mathematics Education Forum held four Saturday meetings, and the Fields Board of Directors met March 23, 2006 to set the Institute’s next budget.

*Tom Salisbury (Fields)*
Yair Minsky, of Yale University, presented a sequence of three talks entitled “Curve complexes, surfaces and 3-manifolds.” Minsky studies the relationships between 3-dimensional topology, hyperbolic geometry, Riemann surfaces, and dynamics. During the past few years Minsky has focused his work on a proof (joint with Brock and Canary) of the famous Ending Lamination Conjecture (ELC), a rigidity statement about certain hyperbolic 3-manifolds that was originally posed by Thurston.

A primary tool in their proof of the ending lamination conjecture is the curve complex $C(S)$ of a surface $S$. This is a simplicial complex with vertices corresponding to the homotopy classes of closed curves in $S$, edges corresponding to pairs of homotopy classes of closed curves that can be represented by disjoint curves, and more generally with $n$-simplices corresponding to $(n+1)$-tuples of homotopy classes of curves whose representatives can be chosen mutually disjoint.

When considered as a metric space with all edges of length 1, Masur and Minsky proved that $C(S)$ is $\delta$-hyperbolic in the sense of Canon and Gromov. That is, for an appropriate definition of geodesics, the geodesic triangles are $\delta$-thin: each edge of a triangle $ABC$ is within a $\delta$ neighborhood of the other two edges. Spaces with this property provide a generalization of classical hyperbolic space and, in particular, $\delta$-hyperbolic spaces also have a natural compactification. In the case of the curve complex, Minsky explained how the points at the boundary of the curve complex correspond to an appropriate class of laminations on $S$. Hence there is a unique geodesic in $C(S)$ joining a pair of distinct laminations in the boundary of $C(S)$. This geodesic plays an essential role in Minsky’s proof of ELC as well as in the more recent work that he presented at Fields.

The mapping class group of a surface $S$ is its group of self-homeomorphisms up to isotopy equivalence. One can always consider a finitely generated group as a geometric object by taking its Cayley graph and assigning length 1 to each of the edges. Minsky explained that for interesting groups, the geometric properties of the Cayley graph and algebraic properties of the group are often related. For the mapping class group, he presented recent work with Behrstock that relates the geometry of the mapping class group (its geometric rank) to its algebraic properties (its algebraic rank.) Through an intermediate geometric space, the space of markings on $S$, they use the hyperbolic structure of the complex of curves in combination with bounds on the projection from $C(S)$ to $C(T)$ for a subsurface $T$ of $S$ in order to establish the equality algebraic rank = geometric rank.

Minsky concluded with interesting remarks on Thurston’s hyperbolization theorems. He is interested in making these theorems “effective,” that is, providing a geometric realization, or a geometric approximation for a hyperbolizable manifold in a constructive way. Suppose that $M$ is a 3-manifold that fibers over the circle with fiber $S$ that satisfies the hypotheses of Thurston’s theorem. Minsky explained how the gluing map from $S$ to $S$ determines a geodesic in $C(S)$. He proposes that using this geodesic and geometric building blocks from his proof of the ELC can provide a geometric model for the manifold that encodes its injectivity radius, the Margulis tubes, and other geometric data, data which are assured to exist by Thurston’s hyperbolization theorem, but which may be difficult to obtain explicitly.

Yair Minsky

Roland Roeder (Fields Institute)
Call for Proposals, Nominations, and Applications

For detailed information on making proposals or nominations, please see the website: www.fields.utoronto.ca/proposals

General Scientific Activities*

Proposals for short scientific events in the mathematical sciences should be submitted by October 15 or March 15 of each year, with a lead time of at least one year recommended. Proposals will be considered at other times as funds permit. Activities supported include workshops, conferences, seminars, and summer schools. If you are considering a proposal, we recommend that you contact the Director, Barbara Keyfitz (bkeyfitz@fields.utoronto.ca) or Deputy Director, Tom Salisbury (salt@fields.utoronto.ca).

Thematic Programs *

Deadlines for letters of intent and proposals for semester or year-long programs at the Fields Institute are August 31 and March 15 each year. Organizers are advised that a lead time of several years is required, and are encouraged to submit a letter of intent prior to preparing a complete proposal. They may consult the directorate about their projects in advance to help structure their proposal.

Postdoctoral Opportunities

Applications are invited for postdoctoral fellowship positions for the 2007-2008 academic year. The thematic program on Operator Algebras will take place at the Institute from July-December 2007, while the thematic program on New Trends in Harmonic Analysis will run from January-June 2008. Qualified candidates who have recently completed a PhD in a related area of the mathematical sciences are encouraged to apply. The fellowships provide for a period of engagement in research and participation in the activities of the Institute. They may be offered in conjunction with partner universities, through which a further period of support may be possible. One recipient will be awarded the Institute’s prestigious Jerrold E. Marsden Postdoctoral Fellowship. Applicants seeking postdoctoral fellowships funded by other agencies (such as NSERC or international fellowships) are encouraged to request the Fields Institute as their proposed location of tenure, and should apply to the address below for a letter of invitation. Additional support is available from NSF to support junior US visitors to these programs. Applications are encouraged from all qualified candidates, particularly aboriginal peoples, persons with disabilities, members of visible minorities and women.

The deadline for postdoctoral applications for the 2007-2008 programs is December 7, 2006.

Postdoctoral opportunities also exist at some of the Fields Institute’s sponsoring universities. Consult http://www.fields.utoronto.ca/proposals/#pdf for details.

CRM–Fields–PIMS Prize

Nominations are invited for this joint prize in recognition of exceptional achievement in the mathematical sciences. The candidate’s research should have been conducted primarily in Canada or in affiliation with a Canadian university.

Nominations for the CRM-Fields-PIMS Prize should reach the Institute by October 1, 2006.

Distinguished Lecture Series In Statistical Science (DLSS)

Nominations are being solicited for the seventh Fields Institute Distinguished Lecture Series in Statistical Science, to be given in Spring, 2007. The awardee will be an internationally prominent statistical scientist, who will give two lectures (one general, one specialized) at the Fields Institute.

Nominations for the DLSS should reach the Institute by October 1, 2006.

*A note on diversity. In proposing any activity, applicants are requested to consider the mandate of the Institute to broaden and enlarge the community. Applicants should explain how they plan to include women and members of visible minority groups in the proposed activity. As well, they should ensure that the proposed participant lists include scientists representing a range of career levels, types of institutions and geographical locations in Canada and abroad.

Please send applications, nominations, and proposals to:

The Director, Fields Institute
222 College Street, Toronto, Ontario, M5T 3J1 Canada
MAY TO SEPTEMBER 2006

at Fields unless otherwise indicated

Detailed information: www.fields.utoronto.ca/programs

Thematic Programs

HOLOMORPHIC DYNAMICS, LAMINATIONS, AND HYPERBOLIC GEOMETRY
Organizers: B. Kleiner, M. Lyubich, Y. Minsky, M. Shub and M. Yampolsky

MAY 23-27, 2006
Workshop on Hyperbolic Geometry

CRYPTOGRAPHY
Organizers: I. Blake, A. Menezes, M. Mosca, K. Murty, A. Stein, R. Scheidler, R. Venkatesan, H. Williams

JUNE 19 - JULY 7, 2006
Summer School on Computational Number Theory and Applications to Cryptography
Rocky Mountain Mathematics Consortium, held at the University of Wyoming

SEPTEMBER 18-20, 2006
Algebraic curves in cryptography: The 10th Workshop on Elliptic Curve Cryptography (ECC 2006)

SEPTEMBER 25-27, 2006
Coxeter Lecture Series: Gerhard Frey (Institut für Experimentelle Mathematik, Universität Duisburg-Essen)

GEOMETRIC APPLICATIONS OF HOMOTOPY THEORY
(Starts January 2007)
Organizers: G. Carlsson, D. Christensen, R. Jardine

General Scientific Activities

MAY 5, 2006
SNAP Math Fair

MAY 10-12, 2006
Numerical, Mathematical and Modeling Analysis related to Fluid Dynamics in Hydrogen Fuel Cells
held at the University of Ottawa

MAY 11-12, 2006
Southern Ontario Statistics Graduate Student Days 2006
held at McMaster University

MAY 12-13, 2006
Ottawa-Carleton Discrete Mathematics Workshop
held at Carleton University

MAY 14-16, 2006
Workshop on Covering Arrays: Constructions, Applications and Generalizations
held at Carleton University

MAY 15-19, 2006
Random Walks in Random Environments

MAY 15-17, 2006
Workshop on Probabilistic Symmetries and their Applications
held at the University of Ottawa

JUNE 1, 2006
Actuarial Research Day
held at the University of Western Ontario

JUNE 1-3, 2006
Carleton Applied Probability Workshop
held at Carleton University

JUNE 8-10, 2006
Xenakis Legacies Conference
held at the University of Guelph, Fields, and Perimeter Institute

JUNE 8-11, 2006
Digital Mathematical Performance
held at the University of Western Ontario

JUNE 10-20, 2006
Summer School on Mathematical Modeling of Infectious Diseases
held at York University

JULY 3-22, 2006
Valuation Theory and Integral Closures in Commutative Algebra: Summer School and Conference
held at the University of Ottawa

JULY 5-9, 2006
Canadian Undergraduate Mathematics Conference
held at McGill University

JULY 10-14, 2006
Symmetry and Spaces: A Conference on the Occasion of Gerry Schwarz’s 60th Birthday

JULY 24-AUG. 4, 2006
Workshop on Computational and Combinatorial Commutative Algebra

JULY 24-27, 2006
6th Annual MOPTA Conference – Modeling and Optimization: Theory and Applications
held at the University of Waterloo

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I now spend more time talking to people who are not mathematicians than at any other time in my adult life. Part of my job is to be an ambassador to government agencies, commerce and industry, newspapers, and anyone who is curious about what we do.

This wider world asks, “What is mathematical research? Is there anything in mathematics that isn’t known yet?”

How can we tell people about the research done at Fields, and in mathematics generally? How do we convey the passion so many mathematicians feel about their work, and explain why and how our research needs to be supported?

Of course, mathematicians are not unique. Avant-garde art and music (including Bach and Mozart in their day) are often not appreciated by the public. However, it’s difficult for any piece of mathematics less earth-shaking than Fermat’s Last Theorem to be “news”, because when a result is new, its importance may not be clear, and by the time everything has been validated, it’s no longer news. The problem of time-lag seems to be characteristic of mathematics. In other sciences, timely announcements of break-through results are made every day.

Within mathematics, the seven Clay Mathematics Institute “million dollar problems” have provided a list of challenges, and one of them (computational complexity, often phrased as “P=NP”) even made it onto Science magazine’s list of the top twenty-five critical knowledge gaps, while the other six (including the Poincaré conjecture and regularity of solutions of the Navier-Stokes equations) were included in a longer list of 125 important questions that the editors of Science think may be answered in the next 25 years.

The mathematical case for famous conjectures is that they have resisted the efforts of the world’s most talented mathematicians, often for generations, and that work on them, even when stopping short of the prize, has stimulated beautiful constructions and has revealed deep and far-reaching connections between different fields (topology and differential equations in the case of the Poincaré conjecture). Great problems inspire us and our students.

The tension between “mathematics, queen of the sciences” and “mathematics, handmaiden of the sciences” is ages old. On the one hand, we are proud that our discipline finds use in so many other fields (that we call “applications”). Mathematics commands respect because knowledge of mathematical techniques is needed in almost every branch of science and engineering. Yet often the mathematics most useful here is the simplest, and elaborations miss the point or carry researchers off in unproductive directions. If one argues that creativity lies in the value added by the researcher, and is greatest where the research is most unconventional or least mechanical, then the art of finding the most appropriate or most enlightening model for a physical, biological or social system is creativity of the highest order. The question is whether this is mathematical research. And surely that is answered by noting that many people who do it well are trained in mathematics. More and more, from hospitals that now hire mathematicians as part of their research teams to the portrayal of mathematicians in the TV show Numb3rs, society is recognizing that not only mastery of the techniques of mathematics but also the special nature of mathematical thinking are part of mathematics. Perhaps the queen and the handmaid are not so far apart.

Facts are good but stories are better. It’s a fact that mathematics underlies design of low-energy trajectories for navigation of the solar system. It becomes a story when one reveals the improbable connections between the technology, chaotic dynamics, and the result, that a cheap way to get to Mars might be to swing around an asteroid. It’s an even better story when one talks about the people (including Fields founder Jerrold Marsden) who discovered these connections.

When astronomy finds a new comet, it gets in the news and supports all of physics. We need to show the world our comets.

Barbara Lee Keyfitz (Fields Institute)