



GENERAL RESEARCH SEMINARS IN DYNAMICAL SYSTEMS

SPEAKER:

DMITRII ANOSOV
The Steklov Institute, Moscow

**On Linear Systems of Ordinary Differential Equations in
Complex Domain**

A Fuchsian system of ordinary linear differential equations is a system having finite number of poles of 1st order over the whole Riemann sphere (and no other singularities). Hilbert's 21st problem (H-21) is: does there exist a Fuchsian system having prescribed singularities and given character of branching of solutions at these singularities (so-called monodromy)? Hilbert himself was convinced that such a system always exists. But this turned out to be a rare case of wrong forecast made by him: recently A.A. Bolibruch constructed a number of counterexamples. Formally this means a negative answer to H-21 as it was stated by Hilbert, and essentially this means that we must change the point of view and try to find conditions for existence or nonexistence of the required Fuchsian system. It would be ideal to have necessary and sufficient conditions. In the general case this ideal is not achieved (yet?), but a number of results on existence or nonexistence are known. Some of them are old, some are new (due to Bolibruch himself and to V. Kostov).

One can formulate a problem similar to H-21 but concerning so-called regular systems instead of Fuchsian ones. (A regular system is a system of ordinary linear differential equations which has only a finite number of singular points and possesses a property that when independent variable x tends to a singular point any solution $y(x)$ grows at most polynomially. (As $y(x)$ may branch logarithmically, this has to be understood modulo evident provisoes). Regular systems constitute a more broad class than Fuchsian ones). This problem is always answered affirmatively, as it was proved by Plemelj in 1906. Proofs of almost all positive results about the original H-21 begin by referring to Plemelj's theorem; after this one tries to modify the regular system provided by this theorem in order to obtain a required Fuchsian system. Another important ingredient of the proofs of the most part of positive and all negative results on H-21 is a relatively new local theory (which supplements a well known theory going back to Poincaré) due to Levelt (1961).

Monday, December 6, 1993

3:30 pm, room 3018

The Fields Institute