Taxation in Matching Markets

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Two-Fold Motivation

1. Look at matching models in between the "non-transferable utility" and the "perfect transfers" cases frequently studied in the literature.
   - Transfers may be imperfect because they are taxed,
   - Or because they are not money, but 'in kind' things that are valued less by the recipient than by the giver.
2. Think about income taxation taking seriously the matching nature of labor markets:
   - Firms have heterogeneous preferences over workers;
   - Workers have heterogeneous preferences over firms.

Literature

Matching Literature:

- Connect Literatures on Matching with and without transfers
  - Gale–Shapley, Conway, McVitie–Wilson, Roth, ...
  - Gale, Shapley-Shubik, Becker, Kelso-Crawford, ...
- Related to matching with contracts/non-linear utility fronteirs
  - Quinzii, Hatfield *et al.*, ...
- These do not consider the transition from non-transferable to transferable utility.

Labor literature

- Most closely related to the effect of taxation on workers' occupational choices:
  - Parker; Sheshinski; Powell and Shan; Lockwood et al.,
- But these do not consider two-sided heterogeneous preferences.

Model

A two-sided matching problem

- Managers, $m \in M$ on one side,
- Workers, $w \in W$ on the other,
- A *match* μ denotes a mapping of each agent to a match partner,

$$\mu(m) \in W \cup \{m\} \quad \forall m \in M,$$
$$\mu(w) \in M \cup \{w\} \quad \forall w \in W,$$

such that $\mu(\mu(i)) = i \ \forall i \in M, W$.

I present the results in the language of one-to-one matching to economize on notation, but they extend to many-to-one matching with substitutable preferences.

Match Utilities

The *match-utility*, α_i^j is the utility i gets from being matched to j.

- Can be positive or negative for either side:
  - Internships that workers would pay to get,
  - Workers that detract from productivity.
- More flexible than the 'surplus function' of Becker et al.

We normalize the utility of being unmatched to zero for all agents, $\alpha_i^i = 0 \ \forall i$.

The total match utility from a match μ is

$$\mathfrak{M}(\mu) = \sum_{i \in M \cup W} \alpha_i^{\mu(i)}$$

Transfers

In addition to caring about their match partners, agents care about the transfers they give or receive.

- We use $t^{m \to w}$ to denote the transfer from m to w.
    - If the manager receives a positive transfer then $t^{m \to w} < 0$.
- With taxation, the transfer the worker receives will be less.
    - The worker's transfer is

$$\xi(t^{m \to w}) \leq t^{m \to w}.$$

- The vector t includes transfers between all *potential* partners:
    - Even those agents that don't match – so agents know the 'price' of that alternative.

Preferences

Each agent only cares about his or her match-partner and transfer.

The utility to an individual of match μ supported by transfer vector t given transfer function $\xi(\cdot)$ is

$$\begin{aligned} u^m([\mu; t]) &= \alpha_m^{\mu(m)} - t^{m \to \mu(m)}, \\ u^w([\mu; t]) &= \alpha_w^{\mu(w)} + \xi(t^{\mu(w) \to w}). \end{aligned}$$

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$$

Focus on stability

- No agent has negative utility.
- No agent prefers a different partner with the associated transfer.

Existence

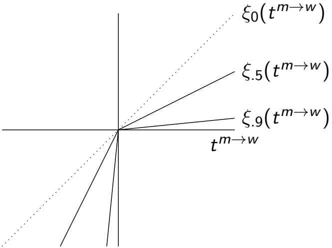
Kelso-Crawford allows for workers to have generic valuations of transfers so their existence results apply here.

- They show that the above definition is equivalent to group stability.
- They show that under *substitutable* preferences, a stable match always exists.
    - Increases in the transfer required to get certain workers will not cause a manager to no longer want workers for whom the required transfer is unchanged.

Proportional Tax

If a manager, m gives a payment $t^{m \to w}$, to worker w when the tax level is τ then the worker receives

$$\xi_\tau(t^{m\to w}) = \begin{cases} (1-\tau)\cdot t^{m\to w} & t^{m\to w} \geq 0, \\ \frac{1}{1-\tau} t^{m\to w} & t^{m\to w} < 0. \end{cases}$$



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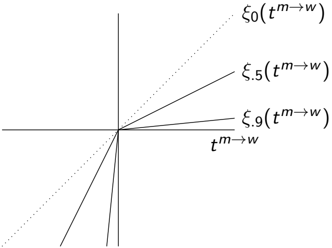
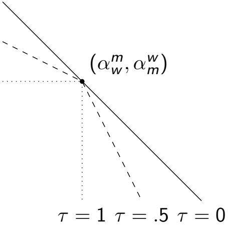


The kink in the transfer function, $\xi_\tau(\cdot)$, generates a kink in the Pareto frontier.



Example

An example illustrates some negative results:

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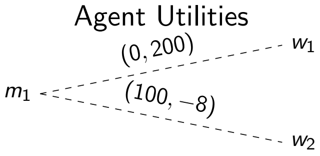

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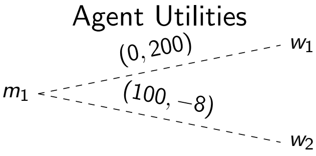

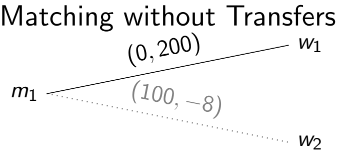

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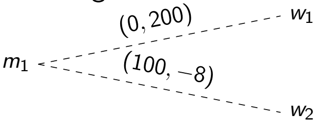
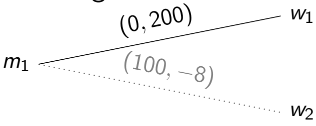
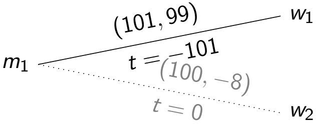
Agent Utilities



Matching without Transfers



Matching with Perfect Transfers



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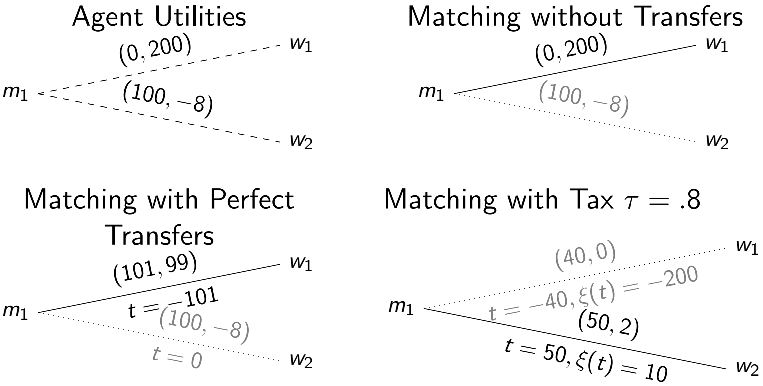

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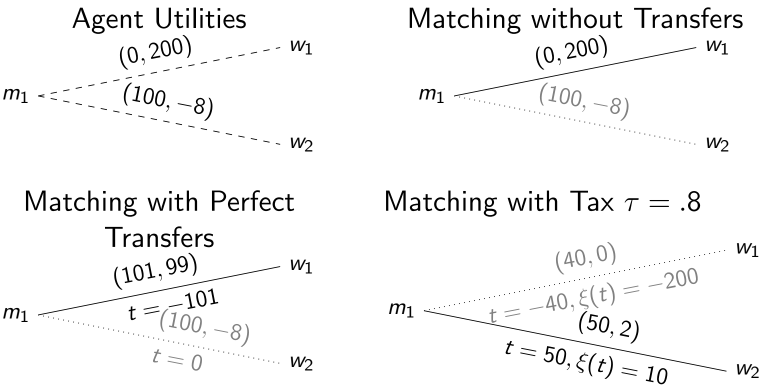

The efficient match is unstable for

$$(100 - 200(1-\tau))(1-\tau) > 8 \quad \iff \quad \tau \in (.6, .9).$$

Result 1: Non-monotonicity

The efficient match (in this case $\mu(m_1) = w_1$) can oscillate between being stable and not being stable.

$\Rightarrow$ Improving transfer efficiency may hurt allocative efficiency.

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$\Rightarrow$ Improving transfer efficiency may hurt allocative efficiency.

Also,

- Individual utilities are non-monotonic in τ.
  - Not just from the match changing.
- The number of agents matched can change with τ.
  - Think of a manager with ε utility of matching with w_2.

Positive Results?

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When will that happen?

Definition

A market is a *wage market* if each worker's match utility of matching to every manager is negative; that is if

$$\alpha_w^m < 0 \qquad \text{for all } w \in W \text{ and } m \in M.$$

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Theorem

In a wage market with proportional taxation, a decrease in taxation (weakly) increases the total match utility of stable matches.
That is, in a wage market, if match $\tilde{\mu}$ is stable under tax $\tilde{\tau}$, match $\hat{\mu}$ is stable under tax $\hat{\tau}$, and $\hat{\tau} < \tilde{\tau}$, then

$$\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) = \sum_{i \in M \cup W} (\alpha_i^{\hat{\mu}(i)} - \alpha_i^{\tilde{\mu}(i)}) \geq 0.$$

Proof

Let $\tilde{t}$ support $\tilde{\mu}$ with tax $\tilde{\tau}$ and $\hat{t}$ support $\hat{\mu}$, with tax $\hat{\tau} < \tilde{\tau}$.

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$$\sum_{m \in M} \left(\tilde{t}^{m \to \hat{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \right) \geq \sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)} \right).$$

Stability conditions for the workers imply that

$$\alpha_w^{\tilde{\mu}(w)} + (1 - \tilde{\tau}) \tilde{t}^{\tilde{\mu}(w) \to w} \geq \alpha_w^{\hat{\mu}(w)} + (1 - \tilde{\tau}) \tilde{t}^{\hat{\mu}(w) \to w},$$
$$\alpha_w^{\hat{\mu}(w)} + (1 - \hat{\tau}) \hat{t}^{\hat{\mu}(w) \to w} \geq \alpha_w^{\tilde{\mu}(w)} + (1 - \hat{\tau}) \hat{t}^{\tilde{\mu}(w) \to w},$$

$$(1 - \hat{\tau}) \sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(w)} - \hat{t}^{m \to \tilde{\mu}(m)} \right) \geq (1 - \tilde{\tau}) \sum_{m \in M} \left(\tilde{t}^{m \to \hat{\mu}(m)} - \tilde{t}^{m \to \tilde{\mu}(m)} \right).$$

Combining the workers' and managers' equations, we find that

$$(1-\hat{\tau}) \sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(w)} - \hat{t}^{m \to \tilde{\mu}(m)} \right)$$
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Since $1-\hat{\tau} > 1-\tilde{\tau}$ (we assumed $\hat{\tau} < \tilde{\tau}$) this implies that

$$\sum_{m \in M} \left(\hat{t}^{m \to \hat{\mu}(m)} - \hat{t}^{m \to \tilde{\mu}(m)} \right) \geq 0.$$

Next, using two of those same equations

$$\alpha_m^{\hat{\mu}(m)} - \hat{t}^{m\to\hat{\mu}(m)} \geq \alpha_m^{\tilde{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)},$$
$$\alpha_w^{\hat{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\hat{\mu}(w)\to w} \geq \alpha_w^{\tilde{\mu}(w)} + (1-\hat{\tau})\hat{t}^{\tilde{\mu}(w)\to w},$$

we find that

$$\begin{aligned}\mathfrak{M}(\hat{\mu}) - \mathfrak{M}(\tilde{\mu}) \geq \sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \\ - (1-\hat{\tau})\sum_{w\in W}\left(\hat{t}^{\hat{\mu}(w)\to w} - \hat{t}^{\tilde{\mu}(w)\to w}\right), \\ = \hat{\tau}\sum_{m\in M}\left(\hat{t}^{m\to\hat{\mu}(m)} - \hat{t}^{m\to\tilde{\mu}(m)}\right) \\ \geq 0.\end{aligned}$$

Result 2: Monotonicity in Wage Markets

This says that allocative efficiency is *decreasing* in the tax rate in wage markets.

- This is another source of dead weight loss from taxation. It is not the extensive or intensive margin, but the allocative margin.
- (Not due to search costs.)

Different than extensive margin.

Other Results

1. Generic uniqueness
2. If a inefficient match, $\tilde{\mu}$ is stable, it must be that workers have higher match utility

$$\sum_{w \in W} \alpha_w^{\tilde{\mu}(w)} > \sum_{w \in W} \alpha_w^{\hat{\mu}(w)}.$$

   - Though they may be worse off due to lower transfers.
3. Individual utility is non-monotonic.
4. There exists a $\underline{\tau}$ such that only an efficient match is stable for $\tau < \underline{\tau}$.

Ongoing work

This analysis focuses on total utility.

- What about agent utility (not including taxes)?
  - Depends on transfer vector
    $\Rightarrow$ Theory has little to say (unless we know the algorithm)

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$\Rightarrow$ Experiments

Experiments

Subjects play same market for different transferability

- Can only hold one offer at a time
- Both sides can make offers
- Spirit of Gale-Shapley with out pinning down outcome

See how outcomes change with the tax rate

- Probability of a stable match
- Agent welfare
- Compare 100% tax to no transfers allowed

Conclusion

For both proportional and lump sum taxation of transfers we have shown:

- Allocative efficiency is increasing in transfer efficiency in markets where all transfers flow in one direction
- Allocative efficiency may (locally) decrease in transfer efficiency in markets where transfers flow in both directions
- Even when transfers are uni-directional, individual utility may decrease when transfer efficiency increases

This implies that taxes in labor markets can cause deadweight loss through misallocation of workers to jobs.

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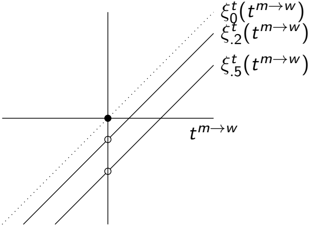
Lump Sum Tax

There are two ways to consider implementing a flat tax.

- Lump sum tax on transfers

$$\xi_f^t(t^{m\to w}) \equiv \begin{cases} t^{m\to w} - f & t^{m\to w} \neq 0 \\ t^{m\to w} & t^{m\to w} = 0. \end{cases}$$



- Lump sum tax on matches:

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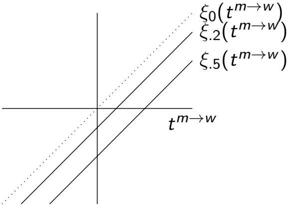


In wage markets they are equivalent.

Lump Sum Tax on Transfers

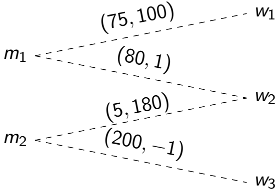
Distortionary:

- Creates a discontinuity at a zero transfer;
- Encourages pairings where the match utility $\alpha_m^w + \alpha_w^m$ is evenly distributed between the two partners ($\alpha_m^w \approx \alpha_w^m$).

Non-monotonicities:

- Of total match utility,
- Of number of agents matched.

Agent Utilities



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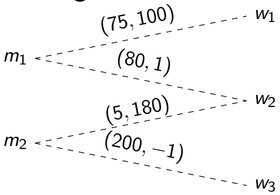
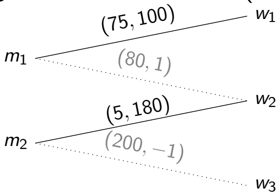
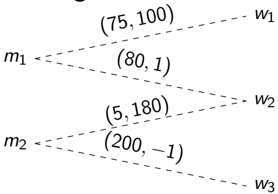


Matching with No Transfers ($f = \infty$)



$U = 360$

Agent Utilities



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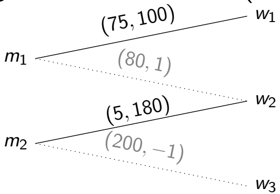

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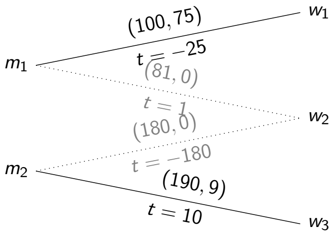
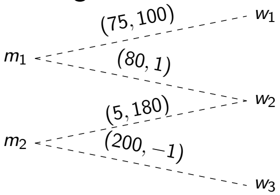
Matching with Perfect Transfers ($f = 0$)

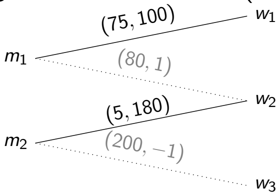


$U = 374$

Agent Utilities



Matching with No Transfers ($f = \infty$)



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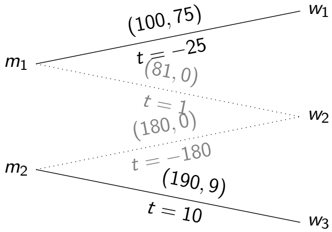
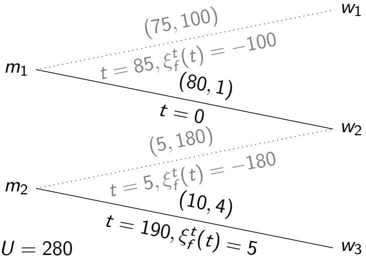
Matching with Perfect Transfers ($f = 0$)



$U = 374$

Matching with Lump Sum Tax ($f = 185$)



$U = 280$

Lump Sum Tax in Wage Markets

In wage markets, taxing transfers is equivalent to taxing all matches.

A lump sum tax *on* matches does not distort among matches, only on the margin of whether to match.

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A lump sum tax *on* matches does not distort among matches, only on the margin of whether to match.

We show that decreasing the lump sum tax on matchings (weakly):

- Increases the number of agents matched at a stable match;
- Increases the total match utility of a stable match;

Also

- A match can only be stable if it maximizes utility for a constraint on the number of agents matched.

Deadweight Loss

What's the deadweight loss?

- Lump Sum Tax
  - Bounded above by the f times the change in the number of people matched.

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- Lump Sum Tax
  - Bounded above by the f times the change in the number of people matched.
- Linear Tax
  - In wage markets, *v*ery loose bound of

$$\tilde{\tau} \sum_{m \in M} \alpha_m^{\hat{\mu}(m)}$$

  - Can't say more without structure on preferences:
    - How much do workers disagree about relative desirability of jobs?
    - How big is surplus as a fraction of wages?
    - (Most attempts to estimate preferences assume agreement.)

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For both proportional and lump sum taxation of transfers we have shown:

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