Minicourses

Bernhard Krötz (Paderborn)

Lecture 1: Volume growth on homogeneous spaces

Abstract: (joint with Sayag and Schlichtkrull) After a brief introduction to homogeneous spaces for a real Lie group, we address the issue of volume growth and derive a version of the Riemann-Lebesgue Lemma.

Lecture 2: Real spherical spaces I: Geometry

Abstract: (joint with Knop, Sayag and Schlichtkrull) We now focus on a special class of a homogeneous spaces, called real spherical. Real spherical spaces feature remarkable geometric structures. We will explain: the local structure theorem, finite orbit decompositions, spherical roots and the little Weyl group and the large scale structure (polar decomposition).

Lecture 3: Real spherical spaces II: Harmonic Analysis

Abstract: (joint with Knop and Schlichtkrull) This lecture is about the basic structure of the Plancherel Theorem for a real spherical space. We determine the support of the Plancherel Theorem and give a sharp bound on the multiplicities.

Yiannis Sakellaridis (Rutgers)

Lecture 1: Geometry of toric and spherical varieties, and the theory of asymptotics

Abstract: Most of the interesting spaces in harmonic analysis can be modelled "at infinity" by simpler spaces, which we will call "boundary degenerations". For example, a hyperboloid looks asymptotically like a cone. The theory of asymptotics provides a passage between functions on the original space and functions on its degenerations.

Lecture 2: Plancherel decomposition and scattering theory

We will review the general formalism of the Plancherel formula, and then use its uniqueness to relate the continuous part of the Plancherel formula for a space X to the Plancherel formula of its boundary degenerations. This can be thought of as a "unitary" version of the theory of asymptotics. Considerations having to do with the "preservation of energy" force relations between the L^2-spaces of the various degenerations, the so-called scattering relations.

Lecture 3: Local harmonic analysis and periods of automorphic forms.

Abstract: The Plancherel formula on a homogeneous space $X = G/H$ can be used to normalize local periods, i.e. H-invariant functionals on G-representations. The normalization requires a "Langlands-type" description of the Plancherel decomposition of $L^2(X)$, generalizing the local Langlands conjectures in the case X = a reductive group. These normalized local periods are then conjectured, and known in some cases, to be the local Euler factors of global periods of automorphic forms. The combination of local and global conjectures can be seen as some "relative Langlands program".

Conference Talks

Langlands parameters for simple supercuspidal representations of classical groups

Moshe Adrian (Toronto)

Gross, Reeder, and Yu have recently constructed a family of supercuspidal representations, called "simple supercuspidal representations". In this talk, I will report on progress to explicitly determine the Langlands parameter of such a supercuspidal representation, for the classical groups.

Base change for non tempered representations

Ioan Badulescu (Poitiers)

Arthur and Clozel proved the base change for tempered representations of the linear group, for cyclic extensions of local fields. That base change is a correspondence $\pi \mapsto \sigma$ of representations determined by the fact that π and σ satisfy the Shintani relation for characters. For general representations I will distinguish "σ is a base change of π" from "σ and π satisfy the Shintani relation for characters". I will show that base change implies Shintani relation for unitary representations. I will explain why we cannot expect that to be true in general. This is a joint work with Guy Henniart.

Invariant Means and Harmonic Analysis on Direct-Limit Groups

Matthew Dawson (CIMAT, Mexico)

In this talk we will discuss several applications of amenability theory to representation theory and harmonic analysis on direct limits of compact Lie groups. In particular, we will address questions of unitarizability of representations and present a new formulation of Olshanski's theory of spherical functions on direct limits of compact Gelfand pairs in terms of invariant means. Coauthors: Gestur Ólafsson

Stable vectors in the Moy-Prasad filtration

Jessica Fintzen (Harvard)

Reeder and Yu gave recently a new construction of certain supercuspidal representations of p-adic reductive groups (called epipelagic representations). Their construction relies on the existence of stable vectors in the first Moy-Prasad filtration quotient under the action of a reductive quotient. We will explain these ingredients and present a theorem about the existence of such stable vectors for all primes p. This builds on a result of Reeder and Yu about the existence of stable vectors for large primes. The above work forms part of a joint research project with Beth Romano.

Uniform in p estimates for orbital integrals

Julia Gordon (UBC)

Abstract: For reductive p-adic groups, it is a well-known theorem of Harish-Chandra that the orbital integrals, normalized by the square root of the discriminant, are bounded (for a fixed test function). However, it is not easy to see how this bound behaves if we let the p-adic field vary (for example, if the group G is defined over a number field F, and we consider the family of groups $G_v = G(F_v)$, as v runs over the set of finite places of F), and how it varies for a family of test functions. Using a method based on model theory and motivic integration, we prove that for a fixed test function, the bound on orbital integrals can be taken to be a fixed power (depending on G) of the cardinality of the residue field, and also obtain a uniform bound for the family of generators of the spherical Hecke algebra playing the role of the test functions. This statement has an application to the recent work of S.-W. Shin and N.Templier on counting zeroes of L-functions. This project is joint work with R. Cluckers and I. Halupczok.

Ladder representations and Galois distinction

Max Gurevich (Technion)

The space $X = \mathrm{GL}_n(E)/\mathrm{GL}_n(F)$, for a quadratic extension E/F of p-adic fields, serves as an approachable case for the study of harmonic analysis on p-adic symmetric spaces on one hand, while having ties with Asai L-functions on the other.

It is long known that a $\mathrm{GL}_n(F)$-distinguished representation of $\mathrm{GL}_n(E)$ must be contragredient to its own Galois conjugate. Conversely, a conjecture often attributed to Jacquet states that the last-mentioned condition is close to being sufficient for distinction.

We show the conjecture is valid for the class of ladder representations which was recently explored by Lapid and Minguez. Along the way, we will suggest a reformulation of the conjecture which concerns standard modules in place of irreducible representations.

Representations of the n-fold Covering Group of the Special Linear Group and Their K-types

Camelia Karimianpour (Ottawa)

In this talk, we will discuss the irreducibly and the K-types of various principal series representations of SL_2 over a p-adic field. We also explicitly compute the distribution of certain K-types of a non-regular unramified principal series representation of the aforementioned group into its irreducible components.

Twisted endoscopy from the sheaf-theoretic perspective

Paul Mezo (Carleton)

Suppose G is a connected reductive algebraic group defined over the real numbers $\mathbb{R}$. The Langlands correspondence partitions the set of irreducible representations of $G(\mathbb{R})$ into finite sets of equivalence classes called L-packets. Shelstad associated the L-packets of G to the L-packets of endoscopic groups, which are in a sense smaller than G. The association is an identity involving distribution characters of the L-packets. Kottwitz and Shelstad later enriched the theory of endoscopy by introducing an automorphism of G. This "twisted" theory of endoscopy allows for more endoscopic groups and character identities. Meanwhile, an alternative perspective to the theory of endoscopy was developed by Adams, Barbasch and Vogan. They recast the theory into the framework of sheaves on a complex variety. I will sketch how the theory of twisted endoscopy may be incorporated into their framework.

Positive energy representations of gauge groups
Karl–Hermann Neeb (FAU Erlangen)

We report on a joint project with Bas Janssens on unitary representations of gauge groups $\mathrm{Gau}_c(P)$ (compactly supported gauge transformations) of a principal bundle P with compact structure group K over a smooth manifold. We consider representations which are covariant with respect to a smooth $\mathbb{R}$-action by automorphisms on the bundle P for which the corresponding flow on M has no fixed points. If M is compact and the flow on M is periodic, then all irreducible representations of this type are finite tensor products of representations of twisted loop groups obtained by restrictions to finitely many orbits in M. If M is not compact, then locally finite sets of orbits in M lead to infinite tensor products which can be controlled by suitable C^*-algebras.

Reflection positivity: representation theory meets quantum field theory
Gestur Ólafsson (LSU)

Reflection positivity is one of the axioms of constructive quantum field theory as they were formulated by Osterwalder and Schrader 1973/1975. The goal is to build a bridge from a euclidean quantum field to a relativistic quantum field by analytic continuation to imaginary time. In terms of representation theory this can be formulated as transferring representations of the euclidean motion group to a unitary representation of the Poincaré group via c-duality of symmetric pairs.

We start by discussing c-duality and then introduce the notion of reflection positive representations. We discuss the special case of the real line, that is reflection positive 1-parameter groups. We then give examples of reflection positive representations and reflection positive kernels. Finally we discuss recent results on integration of infinitesimally unitary representations. This is joint work with K-H Neeb as well as P. Jorgensen and S. Merigon.

On the theory of automorphic forms on loop groups

Manish Patnaik (Alberta)

The theory of automorphic forms on loop groups exhibits both parallels and some surprising differences from the usual finite-dimensional theory. We first review one of the motivations for developing such an infinite-dimensional theory stemming from the Langlands-Shahidi method for analyzing automorphic L-functions on finite dimensional Lie groups. Then we focus on some related constructions in the loop setting: cuspidal Eisenstein series, Hecke algebras, spherical functions and their asymptotics, and the negative Eisenstein series. Joint work in parts with: A. Braverman, H. Garland, D. Kazhdan, and S.D. Miller

Spherical subgroups of Kac-Moody groups

Guido Pezzini (FAU Erlangen)

Spherical subgroups of finite type of a Kac-Moody group have been recently introduced, in the framework of a research project aimed at bringing the classical theory of spherical varieties to an infinite-dimensional setting.

In the talk we discuss their definition and some of their properties. We will introduce a combinatorial object associated with such a subgroup, its homogeneous spherical datum, which satisfies the same axioms as in the finite-dimensional case.

Decay and regularity on spherical spaces

Eitan Sayag (BGU)

This lecture is about two basic issues in analysis on Spherical Spaces: the decay of generalized matrix coefficients on real spherical spaces and the Regularity of generalized (spherical) characters on p-adic Spherical spaces.

I will report on joint works with Bernhard Krötz and Henrik Schlichtkrull regarding decay of functions and about a joint work with Avraham Aizenbud and Dmitry Gourevitch regarding the regularity of certain distributions.

After reviewing the necessary background, we will discuss the main results, say a little about the techniques and elaborate on few applications of these results to problems originating in arithmetic, especially to the problem of counting lattice points in the realm of real Spherical spaces.

The main results include quantitative generalizations of Howe-Moore phenomena in the real case and a qualitative generalizations of Howe/Harish-Chandra character expansions in the p-adic case. The techniques relies on Bernstein center in the p-adic case and the theory of ODE in the real case.

Specifying smooth vectors for semibounded representations by single elements and applications

Christoph Zellner (FAU Erlangen)

For a semibounded representation of a (possibly infinite-dimensional) Lie group we show that the space of smooth vectors is determined by one generator. Using results on smoothing operators by Neeb and Salmasian this entails direct integral decomposition of semibounded representations into irreducible ones. There are also applications to positive energy representations of certain Lie groups.