Financial Instability Contagion: a quantitative definition and mechanism

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Objective

1. Questions:
   - What is contagion?
   - Why and how does that happen?
   - Can it be prevented?

2. Answers
   - Quantitative definition of (instability) contagion
   - Mechanism
   - Hope so
Outline

1. Previous Result (w/ R. Douady, G. Castellacci)
   - Agent-based dynamical system of wealth
   - Early warning system: Market Instability Indicator
   - Extension to multiple economies
   - Quantitative definition of (instability) contagion

2. Main Result (w/ G. Castellacci)
   - Mechanism of contagion

3. Theory vs. Real Life
   - Working with data

4. Conjectures and wishes (by YC)
Dynamical System of Wealth I

- Divide an economy into $n$ aggregates called agents

- $w_i(t) =$ Wealth of Agent $i$ at time $t$
  - $w_i(t) =$ Equity + Debt = Cash(ables) + Invested Assets
  - $w_i(t) = E_i(t) + D_i(t) = L_i(t) + K_i(t)$

- $w_i(t + 1) =$ Wealth of Agent $i$ at time $t + 1$
  - $w_i(t + 1) = w_i(t) + \text{Internal Growth} + \text{Cash In - Cash Out}$
    - $w_i(t + 1) = w_i(t) + F_{ii}(t) + \sum_{j \neq i} F_{ij}(t) - \sum_{k \neq i} F_{ki}(t)$
    - $F_{ij}(t) =$ fund transferred from $j$ to $i$ at $t$

- Wealth dynamical system $f : \bar{M} \subset \mathbb{R}^n \longrightarrow \bar{M}$,
  $(w_1(t), w_2(t), \ldots, w_n(t)) \longmapsto (w_1(t + 1), w_2(t + 1), \ldots, w_n(t + 1))$
Feedback Loop via Flow of Funds

**Figure:** Feedback loop in a two-economy system. Five agents in each economy, Consumers, Firms, Banks, Government, and Investors, are interconnected by flow of funds.
Dynamical System of Wealth II

- Stable equilibrium: persists perturbation
- Unstable equilibrium: perturbation propagates through feedback loop

**Figure**: One dimensional illustration of stability change
Elasticity Coefficient I

- Elasticity Coefficient $a_{ij}(t) = \frac{\partial F_{ij}(t + 1)}{\partial w_{j}(t)}$

- Different sign of $\Delta w_{j}(t)$ yields different reaction of $F_{ij}(t)$:
  - Post-Crisis Banks: credit reduction vs. hoarding cash
  - Post-Crisis Firms: layoff vs. hire freeze
Elasticity Coefficient II

- Elasticities vs. Jacobian $df(w(t)) = B(w(t))$:
  - $b_{ii} = 1 + a_{ii} - \sum_{k \neq i}^{n} a_{ki}$
  - $b_{ij} = a_{ij}$ for $i \neq j$

- Canonical embedding of local elasticities and Jacobians

$$
A(t) = \begin{pmatrix}
A^{(1)}(t) & A^{(12)}(t) & \ldots & A^{(1s)}(t) \\
A^{(21)}(t) & A^{(2)}(t) & & \\
& \vdots & \ddots & \\
A^{(s1)}(t) & \ldots & & A^{(s)}(t)
\end{pmatrix}
$$

$$
B(t) = \begin{pmatrix}
B^{(1)}(t) & A^{(12)}(t) & \ldots & A^{(1s)}(t) \\
& \ddots & \ddots & \\
& & \ddots & \ddots \\
A^{(s1)}(t) & \ldots & & B^{(s)}(t)
\end{pmatrix}
$$
Market Instability Indicator and Contagion

- **Market Instability Indicator (MII)**

  \[ I(t) = \text{Spectral Radius of } B(w(t)) = \rho(B(w(t))) \]

  - \( I(t) < 1 \): perturbations of the system tend to be absorbed
  - \( I(t) > 1 \): small perturbations tend to increase when propagating

- We say that *instability contagion* occurs if given two times \( 0 < t_0 < t_1 \),
  1. At \( t < t_0 \), \( \max_k \rho(B^{(k)}(t)) < 1 \) and \( \rho(B(t)) < 1 \)
  2. At \( t \in (t_0, t_1) \), \( \max_k \rho(B^{(k)}(t)) > 1 \) and \( \rho(B(t)) < 1 \)
  3. At time \( t > t_1 \) \( B(t) \neq \bigoplus_{k=1}^{s} B^{(k)}(t) \) and \( \rho(B(t)) > 1 \).

- Condition 3 implies \( A^{(ij)}(t) \neq 0 \) \( (i \neq j) \)

  \[ \implies \text{causal nature of contagion} \]
Mechanism of Contagion

- Lower bound of MII

\[ \frac{|\text{tr}(B)|}{n} \leq \max_{\lambda_i \in \sigma(B)} |\lambda_i| = \rho(B) \]

\[ b_{ii} = 1 + a_{ii} - \sum_{k \neq i} a_{ik} \]

- \( w_i(t + 1) = w_i(t) + F_{ii}(t) + \sum_{j \neq i}^n F_{ij}(t) - \sum_{k \neq i}^n F_{ki}(t) \)

  - If \( w_i(t) \) is strictly decreasing in \( t \), then \( b_{ii}(t) = \frac{\partial w_i(t+1)}{\partial w_i(t)} > 0 \)
  - If \( w_i(t) \) is strictly decreasing in \( t \) and concave, then \( b_{ii}(t) > 1 \)
  - When \( w_i \) decreases, \( \sum_{j \neq i} F_{ij} \) tends to drop and \( \sum_{k \neq i} F_{ki} \) goes up
  - When agent \( i \) hits liquidity and solvency constraints, \( F_{ki} \) becomes obliged
  - These push up \( b_{ii} \), thus \( \rho(B) \) as well
Contagion within an Economy I

6-agent model with consumers (1), firms (2), banks (3), government (4),
investors (5), and rest of the world (6).

- Assume consumers are highly leveraged, have little $L_1$
- $K_1$ and $w_1$ are decreasing and concave $\implies a_{11} > 0$
- As $w_1 \downarrow$, eventually $a_{21} \sim 0$, $a_{31} < 0$, $a_{41} > 0$, $a_{51} \sim 0$, and $a_{61} \sim 0$
- $b_{11} \approx 1 + a_{11} - a_{31} - a_{41} \gg 1$
Contagion within an Economy II

- Split the economy into C-B (1) and F-G-I-R (2) partitions
  - $b_{11} \gg 1 \Rightarrow$ likely $\rho(B^{(1)}) \geq \frac{|b_{11} + b_{33}|}{2} > 1$ while $\rho(B(t)) < 1$
  - There is financial instability in Partition (1)
  - Stage (ii) of instability contagion

- If C defaults on payment, then $w_3(t)$ decreases
  - Out of panic, $w_3$ would decrease with acceleration, so $a_{33} > 0$
  - Due to mass-withdrawal, $a_{i3} < 0$ for $i = 1, 2, 5, 6$, risking a bank run
  - $b_{33} = 1 + a_{33} - \sum_{k \neq 3}^{6} a_{k3}$ jumps, driving $\rho(B(t)) \geq \frac{\sum_{i=1}^{6} |b_{ii}|}{6} > 1$
  - Stage (iii) of instability contagion

- Instability contagion has taken place

- In reality: US authorities immediately intervened
Contagion across Two Economies

11-agent model with C1, F1, B1, G1, I1, C2, F2, B2, G2, I2, and R (1 - 11).

- Assume G1 has difficulty paying back B2
  - $w_i$ is decreasing and $a_{ii} > 0$ for $1 \leq i \leq 5$
  - $b_{44} = 1 + a_{44} - \sum_{k \neq 4}^{11} a_{k4} \approx 1 + a_{44} - a_{34} - a_{54} - a_{84} \gg 1$
  - High probability that $\rho(B^{(1)}(t)) > 1$ while $\rho(B(t)) < 1$
  - Stage (ii) of Contagion

- If G1 defaults on payment, then B2 is hit and $w_8(t)$ goes down
  - Due to panic, $b_{88} \gg 1$, possibly driving $\rho(B(t)) \geq \frac{\sum_{i=1}^{11} |b_{ii}|}{11} > 1$
  - Stage (iii) of instability contagion

- Instability contagion has taken place

- In reality: ECB “...whatever it takes...” (in Stage (ii))
Work in Progress and Current Issues

- Data!!
  - Federal Reserve Board, Bureau of Economic Analysis, (FDIC)
  - Not enough details and frequency
  - Selection of Agents: Nonprofit Organizations, Fed

- QE distorted market: we are riding a saddle
  - Regrouping agents are necessary to measure the true effect of QE

- Printed monies: seeds of the next (mega) bubbles?

- Detecting bubbles (after defining them) using MII
Selected References


