New advances and challenges in measuring systemic risk

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Quantitative Finance
Retrospective Workshop
October 27-30, 2013
Introduction: Systemic risk and macro-prudential policy

• As a consequence of the financial crisis, policy makers have focused on systemic risk and macro-prudential policy
• No consensus yet on the definitions of both concepts
• Systemic Risk: ECB/Fed: “a risk of financial instability so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially”
• Macro-prudential policy: introduces a macro perspective into the prudential regulation of financial institutions; aims at preventing systemic risk and addressing the procyclicality of the financial system
• Main issue: measuring and allocating systemic risk
Measuring systemic risk

- Financial Indicators on Risk and Stability (FIRST), by Avesani (2005)
- Expected number of defaults (END), by Chan-Lau and Gravelle (2005)
- Conditional Value-at-Risk (CoVaR), by Adrian and Brunnermeier (2008)
- Conditional probability of failure of a large fraction of financial institutions, by Giesecke and Kim (2009)
- Banking Stability Measures (in particular BSI and JPoD), by Segoviano and Goodhart (2009)
- Distress insurance premium (DIP) of the banking system and an institution’s marginal contribution to it, by Huang et al. (2009) and Huang et al. (2010)
- Marginal expected shortfall (MES), by Acharya et al. (2010) and disentangled into volatility, correlation and tail risk by Brownlees and Engle (2010)
- Measure institutions’ systemic importance using Shapley value methodology, by Tarashev et al. (2010) and Drehmann and Tsatsaronis (2011)
- Information contained in central bank communication to help measure stress in financial markets (specific focus on the euro area), by Grimaldi (2010)
- Composite Indicator of Systemic Stress (CISS) (specific focus on the euro area), by Hollo et al. (2010)
- Systemic risk diagnostics (simultaneous failure of a large number of bank and non-bank intermediaries), by Schwaab et al. (2010)
- Bank Vulnerability (AV), by Greenwood et al. (2011)
- Contingent Claim Analysis (CCA), by Gray and Jobst (2011)
- Non-Core liabilities ratio (Hahm et al., 2011)
- Contagion Index, Cont et al. (2012)
- Granger causality on principal components, by Billio et al. (2012)
- A Survey of Systemic Risk Analytics (31 measures), by Bisias et al. (2012)
- Multi-CoVaR and Shapley value, by Cao (2013)
- Domino Effects when Banks Hoard Liquidity, Fourel et al. (2013)
- A Network View on Money Market Freezes, Abassi et al. (2013)
- Evaluating Early-Warning indicators of banking crises, Drehmann and Juselius (2013)
Recent developments in Economic Theory

Heterogeneity / inequalities

- Agent-based models
- Hyperbolic Discounting
- Global Games
- Inaction

Behavioral Finance / economics

- Information Asymmetries
- Behavioral

Heterogeneous agents models

- Rational Inattention
- Rational Expectations
- Rationality / information

Representative agent

Bounded/Limited Rationality

Time Inconsistency and inattention

Dispersed Information

Imperfect Information

Learning

Global Games

Bayesian Learning

General Equilibrium models
Outline of the presentation

1/ Network analysis to assess contagion risks: application to the EU CDS market

2/ Insights from a contagion (agent based) model (EU CDS Market)

3/ A new DSGE model to assess macro-prudential policies
References

• Clerc et al. (2013): the topography of the EU CDS market and its implications for contagion risks, Banque de France-ESMA, mimeo.


• ESRB report on CDS, directed by M. Brunnermeier and L. Clerc, ESRB Occasional Paper Series, N°4, September 2013.

1- Interconnections on the CDS market

- Context: ESRB Expert Group on CDS
- Access to data collected by Trade repositories
- Mandate of this group: (1) Data issues (ESMA), (2) Scope for contagion (3) On-going regulation and new regulatory initiatives
- main market characteristics & developments over time
- Potential for contagion in CDS networks
  - Structure of the networks of CDS exposures: patterns & structural changes
  - Identification of key market players using centrality metrics
  - Financial resiliency of the potential “super-spreaders”
What is a CDS?

- A Credit Default Swap is a derivative financial instrument used to hedge against the default risk of a given reference entity (whose debt is the underlying asset)
  - Buyer holds the insurance, seller takes the risk. Buyer receives a positive pay-out if a credit event occurs & pays periodic premiums to seller in return

- Main characteristics
  - CDS are traded OTC
  - Pay-offs are highly asymmetric and asymmetry increases in times of stress
  - A contract can end in several ways (besides a credit event)
    - Novation, compression cycles, early termination clauses
  - The most common way is to enter into an “offsetting deal” with another trader
  
    ➔ offsetting deals create networks of CDS exposures
How complex is the network of exposures?
How do we analyze interconnections?

- Exploratory analysis of the structure of the networks of credit exposures determined by trades of single name CDSs on EU reference entities (or involving at least one EU party as seller/buyer)
  - Data = notional values of CDS positions outstanding on each week Friday as recorded in DTCC. Parties are anonymous
  - CDS market participants are the nodes; net bilateral exposures form directed links

- We study time series of weekly network metrics – at system & at node level
  - 213 directed networks from January 4, 2008 to January 27, 2012
  - Four different network representations: three “sectoral” (Financials, Non-Financials, Sovereigns), one overall network – all CDS positions included
What do we do?

• Rank institutions that are possible *super-spreaders*, explore correlations, variation of rankings over time, and try to single out non-dealer/non-bank players

• Match banks’ CDS exposures to balance sheet items to assess their financial resilience

• Caveat: we focus on counterparty risk!
  - Why *aggregate* market representations?
    - Our main interest is the default of participants, not of ref entities. Consistent with the outcome of current market practices & risk-mitigation mechanisms (e.g. *close-out netting*)
  - Why *notional & net* amounts?
    - Net notional values represent the *max possible net fund transfer* between net sellers & net buyers of protection that could be required upon occurrence of a credit event
  - *Limits* for a throughout analysis of risks in CDS positions / no data yet on collateral
Related literature

• Lack of data, limited literature so far on CDS networks
  • Markose et al (2012)

• Recent network models of CDS contagion
  • Heise and Khun (2012) : corporates & financials
  • Guillemey and Peltonen (2012): study SOV default & spillovers to EU banks. Use both CDS positions & portfolios of underlying credit exp., allow for risk mitigation. Model calibration using 2011 data on EU capital exercise shows relatively minor role of CDS exp. for contagion versus major role of sudden increases in collateral requirements on multiple correlated exp. and risk mitigating mechanisms

• More similar to approach
  • Brunetti and Gordy (2012): network topology analysis of CDS market for US ref. entities. Similar results but the only work on two “snapshots” of data (2 days in 2010): not published but referred to by Yellen (2013)
Main market developments

Growth in the CDS market, 2004-2012

Gross and net notional outstanding on EU references (2008-2012)
Network analysis: order & size
Market participants – number, by type

Hedge funds represent 40% of the total number of buyers in 2012, asset managers 33%, banks 18%. The remaining 10% is made up of FS, 10 pension plans, 7 insurance companies. Two CCPs appear since Sept and Dec 2009. On the sell side, it is again HF, AM, and banks that dominate the market, each with a share of 30%
Market participants: Market shares

AM are more active than HF...but sales remain < 1.5% of total.
Network analysis: a short intro

- A network is defined by two sets: \( N = \{1, ..., n\} \), the set of nodes, and \( L \) the set of ordered pairs of elements \((i,j)\) called links that connect the nodes
  - Net bilateral buyers or sellers are the nodes; a directed link is defined if an institution is a net buyer of protection from another. Each link has a weight \((w_{ij})\), given by the size of the net bilateral position of the net seller vis-à-vis net buyer

- A network may be represented by its adjacency matrix \( G(g) = \{g_{ij}\} \), i.e. the \( N \)-square matrix that keeps track of directed links
  - If a node \( i \) has a direct link to node \( j \) then \( g_{ij} = 1 \); \( g_{ij} = 0 \) otherwise
  - If \( i \) and \( j \) are not directly linked, i.e. \( g_{ij} = 0 \), they may nonetheless be connected if there is a path from \( i \) to \( j \). A path is an ordered sequence of nodes \([i_0, i_1, ..., i_k]\) starting from \( i \) and terminating at \( j \) (i.e. \( i_0 = i \) and \( i_k = j \)) such that \( g_{i_s} = 1 \) for all \( 0 < s < k-1 \)
  - We also consider the weighted adjacency matrix \( W(g) = \{w_{ij}\} \)
Network visualisation – EU sovereigns

CDS network on EU SOV on 27.1.2012
182 nodes & 716 links; links>USD 100 million

Core CDS network on EU SOV on 27.1.2012
27 nodes & 104 links; links>USD 1 billion

The size of the firms is proportional to their activity: 15 bank-dealers stand out, exposed for more than USD 3bn as sellers & as buyers; most institutions cannot be distinguished. Zooming into the core (104 largest net bilateral exp, i.e. 45% of tot notional outstanding) we can single out the G15 (green), a non-dealer (blue), and a non-dealer/ non-bank (red). The largest exp is between 2 dealers; 2nd & 3rd largest link an AM and a bank to two dealers.
The average distance between any pair of firms was of 2.51 links ($\pm 0.02$) and the diameter of 5 $\rightarrow$ CDS networks are highly compact, shocks can rapidly transmit even to the “farthest away” participant.
Network analysis: in- and out-degree

**Left:** few nodes- *hubs* sell protection to many participants, most nodes to few. **Right:** most buyers buy from few net sellers; few buyers buy from many.

*Hubs:* 10 net bilateral sellers to more than 100 counterparties. 6 net bilateral buyers from more than 100 counterparties.
How stable is the network?

Random network

Scale free network

Peripheral nodes

Hubs

degree correlation & fit to a power law ($\alpha$)

<table>
<thead>
<tr>
<th>(USD EQ)</th>
<th>$\alpha$</th>
<th>Kolmogorov-Smirnoff test statistics</th>
<th>Results</th>
<th>Size of the tail</th>
<th>Size in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan-08</td>
<td>460,000,000</td>
<td>1.60</td>
<td>0.0710</td>
<td>fail to reject</td>
<td>68/223</td>
</tr>
<tr>
<td>Jan-09</td>
<td>884,718,544</td>
<td>1.62</td>
<td>0.1075</td>
<td>fail to reject</td>
<td>56/213</td>
</tr>
<tr>
<td>Jan-10</td>
<td>513,575,000</td>
<td>1.55</td>
<td>0.0870</td>
<td>fail to reject</td>
<td>71/259</td>
</tr>
<tr>
<td>Jan-11</td>
<td>123,024,009</td>
<td>1.48</td>
<td>0.0593</td>
<td>fail to reject</td>
<td>123/327</td>
</tr>
<tr>
<td>Jan-12</td>
<td>163,500,000</td>
<td>1.53</td>
<td>0.0611</td>
<td>fail to reject</td>
<td>124/366</td>
</tr>
</tbody>
</table>

Results of Kolgomorov and Smirnov test of goodness-of-fit to a theoretical power law ($\alpha$): $P(k) \sim k^{-\alpha}$, for large values of $k$. 1.5 < $\alpha$ < 3.
Network analysis: in sum…

- The analysis suggests that CDS exposures trace “scale-free” networks: net sellers are the hubs.

- Scale-free property strongly correlates with network robustness to failure. A hierarchical structure allows for fault tolerant behaviour.
  - If failures occur at random and the vast majority of nodes are those with few counterparties, the probability that a hub will be affected is almost negligible. Even in case of hub-failure, the network will remain connected thanks to remaining hubs.
  - However, in case a shock hits few major hubs together, the network could possibly lose its connectedness hence its capacity to function.

- Robust-yet-fragile property of complex networks ➔ ensuring safety of the hubs ensures safety of the system.
Who are the “Super-spreaders”?

<table>
<thead>
<tr>
<th>Rank 2011</th>
<th>Largest net bilateral CDS sellers</th>
<th>Largest net bilateral CDS buyers</th>
<th>Largest net multilateral CDS sellers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ranking</td>
<td>Expo./TCE</td>
<td>Ranking</td>
</tr>
<tr>
<td>1</td>
<td>Bank 312*</td>
<td>45%</td>
<td>Bank 497*</td>
</tr>
<tr>
<td>2</td>
<td>Bank 622*</td>
<td>23%</td>
<td>Bank 356*</td>
</tr>
<tr>
<td>3</td>
<td>Bank 765*</td>
<td>56%</td>
<td>Bank 317*</td>
</tr>
<tr>
<td>4</td>
<td>Bank 497*</td>
<td>41%</td>
<td>Bank 765*</td>
</tr>
<tr>
<td>5</td>
<td>Bank 1045*</td>
<td>48%</td>
<td>Bank 622*</td>
</tr>
<tr>
<td>6</td>
<td>Bank 1172*</td>
<td>41%</td>
<td>Bank 148*</td>
</tr>
<tr>
<td>7</td>
<td>Bank 186*</td>
<td>26%</td>
<td>Bank 276*</td>
</tr>
<tr>
<td>8</td>
<td>Bank 148*</td>
<td>23%</td>
<td>Bank 136*</td>
</tr>
<tr>
<td>9</td>
<td>Bank 317*</td>
<td>55%</td>
<td>Bank 1172*</td>
</tr>
<tr>
<td>10</td>
<td>Bank 136*</td>
<td>9%</td>
<td>Bank 1045*</td>
</tr>
<tr>
<td>11</td>
<td>AM 860</td>
<td>N.A.</td>
<td>Bank 954*</td>
</tr>
<tr>
<td>12</td>
<td>Bank 356*</td>
<td>24%</td>
<td>CCP 565</td>
</tr>
<tr>
<td>13</td>
<td>Bank 821</td>
<td>66%</td>
<td>Bank 553*</td>
</tr>
<tr>
<td>14</td>
<td>Bank 553*</td>
<td>8%</td>
<td>Bank 289</td>
</tr>
<tr>
<td>15</td>
<td>Bank 276*</td>
<td>7%</td>
<td>Bank 186*</td>
</tr>
<tr>
<td>16</td>
<td>CCP 565</td>
<td>N.A.</td>
<td>Bank 1176*</td>
</tr>
<tr>
<td>17</td>
<td>Bank 954*</td>
<td>10%</td>
<td>Bank 782</td>
</tr>
<tr>
<td>18</td>
<td>HF 508</td>
<td>N.A.</td>
<td>Bank 804</td>
</tr>
<tr>
<td>19</td>
<td>Bank 1176*</td>
<td>32%</td>
<td>Bank 304</td>
</tr>
<tr>
<td>20</td>
<td>Bank 656</td>
<td>67%</td>
<td>AM 873</td>
</tr>
</tbody>
</table>
Super-spreaders & financial stability

Buffer of common equity per US dollar of assets

- Std dev - Top 18
- Top 18 bank super-spreaders
- Std dev - Other banks
- Top 9 Other banks

- 2008
- 2009
- 2010
- 2011
How do network statistics correlate with market price information?

Table 10: Correlations between market-price and exposure-based measures

<table>
<thead>
<tr>
<th></th>
<th>(06 Jan 2012)</th>
<th></th>
<th>(08 Jan 2010)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eigenvector</td>
<td>Betweenness</td>
<td>Exposure</td>
<td>Indegree</td>
</tr>
<tr>
<td>Contr-CoVaR</td>
<td>0.542</td>
<td>0.545</td>
<td>0.688</td>
<td>0.635</td>
</tr>
<tr>
<td>Exp-CoVaR</td>
<td>0.031</td>
<td>-0.069</td>
<td>0.106</td>
<td>-0.013</td>
</tr>
<tr>
<td>Contr-CoCDS</td>
<td>0.043</td>
<td>-0.277</td>
<td>0.048</td>
<td>-0.300</td>
</tr>
<tr>
<td>Exp-CoCDS</td>
<td>-0.184</td>
<td>-0.247</td>
<td>-0.214</td>
<td>-0.305</td>
</tr>
<tr>
<td>CDS-val</td>
<td>-0.237</td>
<td>-0.312</td>
<td>-0.204</td>
<td>-0.330</td>
</tr>
<tr>
<td>MES-val</td>
<td>0.138</td>
<td>-0.089</td>
<td>0.251</td>
<td>-0.013</td>
</tr>
<tr>
<td>Rel.</td>
<td>0.639</td>
<td>0.583</td>
<td>0.712</td>
<td>0.643</td>
</tr>
<tr>
<td>Cap.shortfall</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market-val</td>
<td>0.107</td>
<td>0.266</td>
<td>0.202</td>
<td>0.220</td>
</tr>
</tbody>
</table>
2/ Main insights from a contagion model

Five transmission channels from sovereign to banks are featured in the model:
1. direct losses on sovereign bond holdings
2. write-downs on other (AFS) sovereign exposures;
3. direct CDS repayments triggered by the credit event;
4. increased collateral requirements to cope with higher CDS spreads on other non-defaulted reference entities;
5. contagious propagation of counterparty failures.

Calibration made using public data released by the European Banking Authority (EBA) on 65 major European banks related to the EU 2011 Capital Exercise. The dataset includes both sovereign bond and CDS holdings at a bank level for 28 European sovereign entities, while bilateral CDS exposures are estimated and their market values simulated. Additional balance sheet data are retrieved from Bloomberg. Exogenous sovereign default scenarios are studied for four stressed euro area countries (Italy, Ireland, Spain, Portugal) for a wide range of recovery rates.
Actual and simulated networks

Actual network

Simulated network
Assessing contagion risk from the CDS market

The number of bank failures through various propagation channels following a simulated sovereign credit event.

<table>
<thead>
<tr>
<th>Recovery rate</th>
<th>Direct bond losses</th>
<th>Correlated bond losses</th>
<th>Collateral shortage</th>
<th>Contagious insolvency</th>
<th>Contagious illiquidity</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ireland</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0,1</td>
<td>0</td>
<td>10 (83%)</td>
<td>2 (17%)</td>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>0,5</td>
<td>0</td>
<td>0</td>
<td>3 (100%)</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0,9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Italy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0,1</td>
<td>6 (24%)</td>
<td>17 (68%)</td>
<td>2 (8%)</td>
<td>0</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>0,5</td>
<td>1 (8%)</td>
<td>10 (77%)</td>
<td>2 (15%)</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>0,9</td>
<td>0</td>
<td>0</td>
<td>2 (100%)</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td><strong>Portugal</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0,1</td>
<td>3 (43%)</td>
<td>2 (29%)</td>
<td>2 (29%)</td>
<td>0</td>
<td>1 (33%)</td>
<td>7</td>
</tr>
<tr>
<td>0,5</td>
<td>0</td>
<td>1 (33%)</td>
<td>1 (33%)</td>
<td>0</td>
<td>1 (33%)</td>
<td>3</td>
</tr>
<tr>
<td>0,9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Spain</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0,1</td>
<td>5 (21%)</td>
<td>17 (71%)</td>
<td>2 (8%)</td>
<td>0</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>0,5</td>
<td>2 (15%)</td>
<td>9 (69%)</td>
<td>2 (15%)</td>
<td>0</td>
<td>0</td>
<td>13</td>
</tr>
<tr>
<td>0,9</td>
<td>0</td>
<td>0</td>
<td>3 (100%)</td>
<td>0</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

Note: The table shows the estimated number of banks failing following a simulated sovereign credit event, while the percentages in parentheses indicate the relative share of each failure channel.
3/ A new DSGE to assess macroprudential policy

- This is an on-going work, which takes place in the context of a Macro-prudential Research Network (MaRs) of the European System of Central Banks

- Objectives
  - Provide an analytical framework for macro-prudential policies (both positive and normative)
  - Provide a “quantitative” framework for assessing the effectiveness and the impact of macro-prudential policies / instruments
Theoretical model – broad overview

- Heterogeneous households (savers and borrowers): non trivial lending and borrowing decisions (housing)
- Entrepreneurs: non trivial borrowing decisions to finance investment
- Optimizing financial intermediaries who allocate their wealth together with funds raised from saving households across two lending activities (mortgage and corporate lending)
Scope for macro-prudential policies

- Three interconnected net worth channels: amplification, propagation and credit/leverage cycles
- Limited liability for household and entrepreneurs (possibility of defaulting in equilibrium) + non-contingent debt contracts
- Non-trivial default risks in all classes of borrowing (including for banks)
- Deposit insurance + limited liability: encourages risking up: capital requirements
Macro-prudential instruments

- Banks are subjects to capital requirements and face limits to raise inside equity
- Distortions due to deposit insurance: alter the pricing of loans
- Loan to value ratios to limit risk taking
- Sectoral capital requirements
- Counter-cyclical buffer

Extensions

- Regulatory treatment of securitization
- Liquidity requirements
Mechanisms and steady state

- Mechanism punchline:
  - when banks have insured liabilities, they lend cheaply and leverage and borrower default is excessive

- Welfare punchline
  - capital requirements in the steady state align incentives and prevent excessive risk taking (+ve effect)
  - capital requirements tighten credit supply and reduce output (-ve effect)
  - at low capital requirements, the +ve effect dominates, otherwise the -ve effect dominates
Where do we stand in the literature?

- Standard debt contracts for all borrowers
- Households
  - patient/impatient as in Iacoviello
  - household default as in Solomon (2010) and DKR
- Firms
  - 2-period OLG with bequests
  - default as in BGG (like in CMR and DKR)
- Banks
  - 2-period OLG with bequests
  - default as in BGG
  - insured deposits
  - regulatory capital requirement
Where do we stand in the literature?

- Unobservable banker monitoring effort
  - Christiano and Ikeda (2013)

- Choice of bad projects under high leverage
  - Martinez-Miera and Suarez (2012)
  - Collard, Dellas, Diba and Loisel (2012)
  - Gallo and Thomas (2013)

- Here the distortion comes from too much (or too little) credit for firms and/or households
Presentation of the Model: Patient households

- The dynasty of patient households are the savers in the economy.
- They maximise the following objective function

$$E_t \left[ \sum_{i=0}^{\infty} (\beta^s)^{t+i} \left[ \log (c_{t+i}^s) + v_{t+i}^s \log (h_{t+i}^s) - \frac{Q_{t+i}^s}{2} (l_{t+i}^s)^2 \right] \right]$$

subject to the intertemporal budget constraint as follows

$$c_t^s + q_t^h h_t^s + d_t \leq w_t l_t^s + q_t^H h_{t-1}^s + R_{t-1}^D d_{t-1} - T_t + \Pi_t$$  \hspace{1cm} (1)

- First order conditions

$$c_t^s : \quad 1/c_t^s = \lambda_t^s$$
$$h_t^s : \quad q_t^H \lambda_t^s = v_t^s / h_t^s + \beta^s E \lambda_{t+1}^s q_{t+1}^H$$
$$l_t^s : \quad Q_t^s l_t^s = w_t \lambda_t^s$$
$$d_t : \quad \lambda_t^s = \beta^s E \lambda_{t+1}^s R_t^D$$
Impatient households

- The dynasty of impatient households are the borrowing households in the economy.

- They maximise the following objective function

\[
E_0 \sum_{t=0}^{\infty} (\beta^m)^t \left[ \log(c_t^m) + \nu_t^m \log(h_t^m) - \frac{q_t^m}{2} (l_t^m)^2 \right]
\]

subject to the intertemporal budget constraint as follows

\[
c_t^m + q_t^H h_t^m - b_t^m \\
\leq w_t l_t^m + \int_0^\infty \max \left\{ \omega_t^m q_t^H h_{t-1}^m - R_{t-1}^m b_{t-1}^m, 0 \right\} dF^m(\omega^m) \\
\equiv n_t^m,
\]
Household default

- Households experience idiosyncratic (mean = 1) shocks \( \omega_t^m \) to their housing value (as in Solomon (2010) and DKR): default whenever house value is less than required repayment

\[
\omega_t^m q_t^H h_{t-1}^m < R_{t-1}^m b_{t-1}^m
\]

- Defines a critical value of \( \omega_t^m \)

\[
\omega_t^m \leq \bar{\omega}_t^m = x_{t-1}^m / R_t^H
\]

where

\[
R_t^H \equiv q_t^H / q_{t-1}^H
\]

is the ex post aggregate realized gross return on housing, and

\[
x_t^m \equiv \frac{R_t^m b_t^m}{q_t^H h_t^m}
\]

is household leverage chosen in the previous period.
Household default (2)

- Housing net worth

\[ \Phi^m_t \equiv \int_0^\infty \max \left\{ \omega_t^m q^H_t h_{t-1}^m - R_{t-1}^m b_{t-1}^m, 0 \right\} dF^m(\omega^m) \]

- Using the same intermediate notation as in BGG, we can more compactly write

\[ \Phi^m_t = (1 - \Gamma^m(\bar{\omega}_t^m)) R_t^H q_{t-1}^H h_{t-1}^m, \]  \hspace{1cm} (3)

where

\[ \Gamma^m(\bar{\omega}_t^m) = \bar{\omega}_t^m \int_{\omega_t^m}^\infty f(\omega^m) d\omega^m + \int_{\bar{\omega}_t^m}^{\omega_t^m} \omega^m f(\omega^m) d\omega^m \]
Credit supply to households

- Banks supply loans to households as long as the profits from these loans deliver the bank’s desired rate of return on equity:

\[ E_t \max \left[ \omega^H_{t+1} \tilde{R}^H_{t+1} b_t^m - R_t^D d_t, 0 \right] \geq \rho_t \phi^H_t b_t^m. \]  \hspace{1cm} (4)

where \( \omega^H_{t+1} \) is a mortgage-bank-specific loan quality shock and \( \tilde{R}^H_{t+1} \) is the loan return. Using the usual BGG notation we have:

\[ (1 - \Gamma^H(\omega^H_{t+1})) \tilde{R}^H_{t+1} b_t^m \geq \rho_t \phi^H_t b_t^m. \]  \hspace{1cm} (5)

- Intuition: mortgage loan profits must deliver the bank’s required expected rate of return on equity \( \rho_t \).
- Limited liability distortions allow banks to meet rate of return benchmark with lower lending rates.
The impatient household problem

\[
\max E_t \left[ \sum_{i=0}^{\infty} (\beta^m)^{t+i} \left[ \log (c_{t+i}^m) + \nu_{t+i}^m \log (h_{t+i}^m) - \frac{q_{t+i}^m}{2} (l_{t+i}^m)^2 \right] \right]
\]

subject to the budget constraint of the dynasty,

\[
c_t^m + q_t^H h_t^m - b_t^m \leq w_t l_t^m + \left( 1 - \Gamma^m \left( \frac{x_{t-1}^m}{R_t^H} \right) \right) R_t^H q_{t-1}^H h_{t-1}^m,
\]

and the participation constraint of the bank,

\[
(1 - \Gamma^H(\omega_{t+1}^H)) \tilde{R}_{t+1}^H b_t^m \geq \rho_t \phi_t^H b_t^m,
\]

where

\[
\tilde{R}_{t+1}^H = \left( \Gamma^m \left( \frac{x_t^m}{R_{t+1}^H} \right) - \mu^m G^m \left( \frac{x_t^m}{R_{t+1}^H} \right) \right) R_{t+1}^H q_t^H h_t^m
\]
Entrepreneurs

- Simplified version of BGG: Entrepreneurs live for two periods
- In second period of life, maximise

\[
\max_{c_{t+1}^e, n_{t+1}^e} (c_{t+1}^e)^{\lambda^e} (n_{t+1}^e)^{1-\lambda^e}
\]  \hspace{1cm} (13)

subject to:

\[
c_{t+1}^e + n_{t+1}^e \leq W_{t+1}^e
\]

Optimizing behavior yields

\[
c_{t+1}^e = \lambda^e W_{t+1}^e
\]  \hspace{1cm} (14)

\[
n_{t+1}^e = (1 - \lambda^e) W_{t+1}^e
\]  \hspace{1cm} (15)
Hence in first period of life maximise:

$$\max_{k_t, b_t^e, R_t^F} E_t(W_{t+1}^e)$$

(16)

subject to the period $t$ resource constraint

$$q_t^K k_t - b_t^e = n_t^e,$$

(17)

the definition

$$W_{t+1}^e = \max \left[ \omega_{t+1}^e \left( r_{t+1}^k + (1 - \delta) q_{t+1}^K \right) k_t - R_t^F b_t^e, 0 \right],$$

(18)

and the bank’s participation constraint

$$E_t(1 - \Gamma^F(\omega_{t+1}^F)) \tilde{R}_{t+1}^F = \rho_t \phi_t^F,$$

(19)
The corporate contracting problem

- Again we use the BGG notation $\Gamma^e (\omega_t^{e+1})$
- The corporate contracting problem solves:

$$\max_{x_t^e, k_t} E_t \left[ \left( 1 - \Gamma^e \left( \frac{x_t^e}{R_t^K} \right) \right) R_{t+1}^K q_t^K k_t \right]$$

subject to the participation constraint of the bank:

$$E_t (1 - \Gamma^F (\omega_t^{F+1})) \tilde{R}_{t+1}^F = \rho_t \phi_t^F,$$  \hspace{1cm} (20)

where

$$\tilde{R}_{t+1}^F = \left( \Gamma^e \left( \frac{x_t^e}{R_t^K} \right) - \mu^e G^e \left( \frac{x_t^e}{R_t^K} \right) \right) R_{t+1}^K q_t^K k_t$$
Bankers live for two periods. In second period of life, maximise

$$\max_{c_{t+1}^b, n_{t+1}^b} \left( c_{t+1}^b \right)^{\chi^b} \left( n_{t+1}^b \right)^{1-\chi^b}$$

subject to:

$$c_{t+1}^b + n_{t+1}^b \leq W_{t+1}^b.$$

Optimizing behavior yields

$$c_{t+1}^b = \chi^b W_{t+1}^b$$

$$n_{t+1}^b = (1 - \chi^b) W_{t+1}^b.$$
Hence in first period of life, maximise:

$$\max_{e^F_t} E_t(W_{t+1}^b) = E_t(\tilde{\rho}_{t+1}^F e^F_t + \tilde{\rho}_{t+1}^M (n^b_t - e^F_t)).$$

(24)

First order condition wrt $e^F_t$:

$$E_t\tilde{\rho}_{t+1}^F = E_t\tilde{\rho}_{t+1}^M = \rho_t,$$

(25)

Aggregate evolution of banker net worth:

$$N_{t+1}^b = \left(1 - \chi^b\right) \left(\tilde{\rho}_{t+1}^F E_t^F + \tilde{\rho}_{t+1}^M \left(N_t^b - E_t^F\right)\right).$$

(26)
Banks

- Banks are one-period lived firms which raise equity from bankers and deposits from patient households.
- Banks specialise in either mortgage or corporate loans. Corporate banks’ profits are given by:

\[
\pi_{t+1}^F = \max \left[ \omega_{t+1} \tilde{R}_{t+1}^F b_t^e - R_t^D d_t^F, 0 \right],
\]

- Regulatory capital constraint:

\[
e_t^F \geq \phi_t^F b_t^e,
\]

(27)

- Bank default

\[
\bar{\omega}_{t+1}^F = (1 - \phi_t^F) \frac{R_t^D}{\tilde{R}_{t+1}^F},
\]

(28)

- Ex post rate of return on equity:

\[
\tilde{\rho}_{t+1}^F = \frac{1 - \Gamma^F(\bar{\omega}_{t+1}^F)}{\phi_t^F} \tilde{R}_{t+1}^F.
\]

(29)
Capital production firms

- **Investment**
  
  \[ l_t = k_t - (1 - \delta) k_{t-1} \]

  requires resources
  
  \[ \left[ 1 + g \left( \frac{l_t}{l_{t-1}} \right) \right] l_t \]

  where \( g \left( \frac{l_t}{l_{t-1}} \right) \) is the investment adjustment cost function.

- **Firm is owned by the patient households** \(\implies\) choose investment \( l_t \) in order to maximize

  \[
  E_t \sum_{t=\tau}^{\infty} \frac{\lambda_t^S}{\lambda_t^s} \left\{ q^K_t l_t - \left[ 1 + g \left( \frac{l_t}{l_{t-1}} \right) \right] l_t \right\},
  \]

  - **FOC:**

  \[
  q^K_t = 1 + g \left( \frac{l_t}{l_{t-1}} \right) + \frac{l_t}{l_{t-1}} g' \left( \frac{l_t}{l_{t-1}} \right) - E_t \phi^P_{t,t+1} \left( \frac{l_{t+1}}{l_t} \right)^2 g' \left( \frac{l_{t+1}}{l_t} \right). \]
Market clearing conditions

- Aggregate bank capital constraint

\[(1 - \chi^b) W_t^b = \phi_t^F \left[ q_t^K k_t - (1 - \chi^e) W_t^e \right] + \phi_t^M \left( \frac{q_t^H h_t^m x_t^m}{R_t^m} \right).\]

- Deposit market

\[d_t = (1 - \phi_t^F) \left[ q_t^K k_t - (1 - \chi^e) W_t^e \right] + (1 - \phi_t^M) \left( \frac{q_t^H h_t^m x_t^m}{R_t^m} \right).\]

- Labour market

\[(1 - \alpha) \frac{y_t}{W_t} = l_t^s + l_t^m.\]

- Goods market: long and ugly expression

- Capital market: entrepreneur demand equals capital firm supply

\[q_t^K k_t = n_t^e + b_t^e.\]
The importance of Bank capital: steady state

- The bank’s required rate of return on equity is given by the following expression:

\[
E_t \tilde{\rho}^F_{t+1} = \frac{E_t \left[ 1 - \Gamma^F (\omega^F_t) \right]}{\phi^F_t} \tilde{R}^F_{t+1}.
\]

- Leverage does two things:
  - amplifies the rate of return on equity for given loan spreads (standard effect)
  - increases the probability of bankruptcy - amplifies profits through higher limited liability subsidy
Higher generalized capital ratio in the steady state
Higher generalized capital ratio in the steady state
IRF to a productivity shock (TFP)
IRF to a productivity shock (TFP)
On-going and future work

• We are currently introducing new macroprudential tools (like the countercyclical buffer and sectoral requirements)

• We consider further extensions
  o Liquidity to factor in liquidity regulation, like the LCR
  o Securitization
  o Nominal rigidities to study the interaction with monetary policy
Conclusion

- Preventing systemic risk remains a challenging task:
  - Requires appropriate/accurate measurement
  - A better understanding of its transmission channels
  - The development of adequate macro-prudential policies
  - Need more structural approaches

- Main challenges remain related to data issues
  - Access to proprietary/supervisory data
  - Issue of aggregation of data collected by trade repositories
  - Data gaps
References

- Clerc et al. (2013): the topography of the EU CDS market and its implications for contagion risks, Banque de France-ESMA, mimeo.