An Options Pricing Approach to Ramping Rate Restrictions at Hydro Power Plants

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Motivation

- Main features: no carbon emissions; low operating costs; ability to meet peak demands; significant operational flexibilities; high reliability.
- Negative effects on downstream environment (Edwards *et al.* (1999))
  - affect the in-stream flow rates, reservoir levels, water temperatures, and therefore change the chemical and physical composition of the released water.
  - impact the beach and bank erosion, beach and backwater formation, which can affect shore areas that provide critical wildlife habitat for native fishes, and other aquatic flora and fauna.
- Scruton *et al.* (2003): "hydro-peaking often results in rapid changes in river discharge and associated habitat conditions over very short time scales (less than a day, or multiple peaks per day) and changes can be moderate or as large as several orders of magnitude."
- Smokorowski *et al.* (2009): "in Ontario both electricity producers and the Ontario Ministry of Natural Resources are interested in testing whether restricting ramping rates through turbines at hydroelectric facilities can provide ecological benefits while, at the same time, minimize production losses."

Ramping Problem

Without restrictions ⇒ maximize the profit by adjusting the magnitude, timing, duration, and rate of change of flow (**ramping rate**).

- Prices/demands are high ⇒ ramp up
- Prices/demands are low ⇒ ramp down

With restrictions ⇒ reduce the efficiency, profitability, and ability to react to changes in electricity demand and price.

- Ramping rate restrictions ⇒ provide environmental benefits by protecting downstream habitat.
- Increased restrictions ⇒ rely more on fossil fuel fired plants ⇒ entail greater Green House Gas emissions.
- Impose the appropriate restrictions ⇒ study the tradeoff among protecting aquatic ecosystems, optimally operating hydropower, and other externalities.

Need both economic and environmental studies to address the ramping issue to guide for policies on ramping restrictions.
Require realistic, statistically sound and parsimonious electricity models to value power plants and then study the associated ramping problem.

Related Literature

- Economics literature
  - deterministic models: Veselka *et al.* (1995), Edwards *et al.* (1999), Edwards (2003), Harpman (1999), and Niu and Insley (2013).
  - stochastic models: Chen and Forsyth (2008).
- Real options literature
  - jump diffusion: Thompson *et al.* (2004) and Chen and Forsyth (2008).
  - regime switching: Chen and Forsyth (2010), Heydari and Siddiqui (2010), and Chen and Insley (2012).
- Jump diffusion models
  - Deng (1999), Escribano *et al*. (2011), Geman and Roncoroni (2006), Weron *et al.* (2004), Weron (2008), Benth *et al*. (2007), and Benth *et al*. (2008).
- Regime switching models
  - early results: Ethier and Mount (1998), Bierbrauer *et al*. (2004), Weron et al. (2004), and De Jong (2006).
  - recent development: Mount *et al*, (2006), Weron (2009), and Janczura and Weron (2009, 2010).

This Paper

In this paper

1. Present a theoretical valuation framework of a stochastic control problem for hydro operations using a regime switching model for electricity prices.
2. Solve a coupled PDE numerically using a fully implicit finite difference approach.
3. Empirically investigate a medium sized prototype hydro plant using estimated parameters for both the base regime and the spike regime.
4. Examine the sensitivity of the hydro operation and profit to different levels of ramping restrictions.

Not in this paper

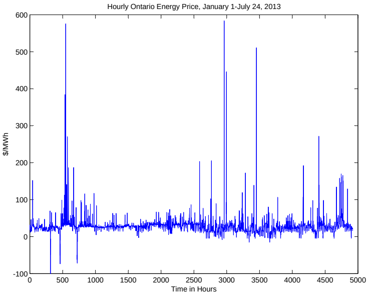
- We do not address the environmental gains to the aquatic ecosystem, nor the environmental costs of alternate thermal power generation.
- If ramping rate restrictions were applied to a significant portion of the hydro generation capacity in a particular province or state, then the impact on the entire grid would need to be considered.

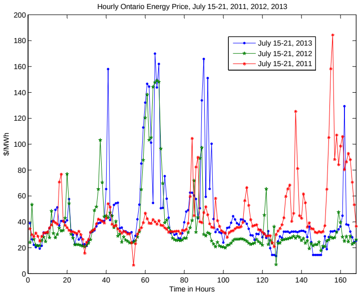
Preview of Results

- The value of the hydro power plant is higher in the spike regime than the value in the base regime.
- In most scenarios, the optimal control is of "bang-bang" type: ramping up or down at the maximum allowed rates.
- Profits are negatively affected by ramping restrictions in both the single regime and regime switching models.
- The profit is less sensitive to ramping restrictions in the regime switching model compared to the single regime model.
- Profits are significantly affected by less than **16%** for the single regime model and less than **9%** for the regime switching model in the case of the most severe ramping constraints.
- However we also find a range of less severe ramping constraints for which profits are impacted by less than **3%** for the single regime model and less than **2%** for the regime switching model.

Hourly Ontario Energy Price

Left: Hourly Ontario Energy Price, January 1-July 24, 2013.
Right: Hourly Ontario Energy Price, July 15-21, 2011, 2012, 2013.





Jump Diffusion vs Regime Switching

In Thompson *et al.* (2004), the **jump diffusion model** can be represented as the following general form

$$dP = \mu(P,t)dt + \sigma(P,t)dW + \sum_{\varsigma=1}^{N} \psi_{\varsigma}(P,t,J_{\varsigma})dq_{\varsigma}.$$

Consider the general N-state process for the **regime switching model**

$$dP = \mu^{\imath}(P,t)dt + \sigma^{\imath}(P,t)dZ + \sum_{\jmath=1}^{N} P(\xi^{\imath\jmath} - 1)dX_{\imath\jmath}.$$

The equation of motion for water

$$dw = H(r,w)a(\ell - r)dt.$$

The ramping control variable z

$$dr = zdt.$$

The up-ramping and down-ramping constraints

$$-r^{d} \leq z \leq r^{u}.$$

Pricing Equation

This stochastic dynamic non-linear optimization problem can be stated as

$$\max_r E^Q \left[\int_0^T e^{-\rho\tau} H(r,w) q(r,h(w))(P-c) d\tau \right].$$

subject to

$$Z(r) \subseteq [-r^d, r^u].$$
$$r^{min} \leq r \leq r^{max}.$$
$$w^{min} \leq w \leq w^{max}.$$

The value of the power plant in state $\imath$ satisfies the following HJB-PDE

$$\frac{\partial V^\imath}{\partial \tau} = \sup_{z \in Z(r)} (z \frac{\partial V^\imath}{\partial r}) + H(r,w) a(\ell - r) \frac{\partial V^\imath}{\partial w} + \frac{1}{2} (\sigma^\imath)^2 (P,t) \frac{\partial^2 V^\imath}{\partial P^2}$$
$$+ (\mu^\imath(P,t) - \Lambda^\imath \sigma^\imath(P,t)) \frac{\partial V^\imath}{\partial P} + H(r,w) q(r,h(w))(P-c) - \bar{r} V^\imath$$
$$+ \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda^Q_{\imath\jmath} (V^\jmath - V^\imath).$$

Boundary Conditions

For $V^{\imath}$ in regime $\imath$ at the terminal time, we use the following zero profit

$$V^{\imath}(P, w, r, \tau = 0) = 0.$$

Solve the equation in $(P, w, r) \in [0, P^{max}] \times [w^{min}, w^{max}] \times [r^{min}, r^{max}]$.
At $w_{min}, w_{max}, r_{min}, r_{max}$, we solve the PDE along the corresponding boundaries.
Take the limit of the PDE equation as $P \to 0$ and for $P \to \infty$ apply $V^{\imath}_{PP} = 0$.

$$\frac{\partial V^{\imath}}{\partial \tau} = C_0 V^{\imath} + BV^{\imath} + \sup_{z \in Z(r)} (z \frac{\partial V^{\imath}}{\partial r}) + H(r, w) a(\ell - r) \frac{\partial V^{\imath}}{\partial w} - H(r, w) q(r, h(w)) c; \; P \to 0$$

$$C_0 V^{\imath} = \alpha^{\imath} K^{\imath} \frac{\partial V^{\imath}}{\partial P} - (\bar{r} + \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda^{Q}_{\imath\jmath}) V^{\imath}.$$

$$\frac{\partial V^{\imath}}{\partial \tau} = C_1 V^{\imath} + BV^{\imath} + \sup_{z \in Z(r)} (z \frac{\partial V^{\imath}}{\partial r}) + H(r, w) a(\ell - r) \frac{\partial V^{\imath}}{\partial w} + H(r, w) q(r, h(w))(P - c); \; P =$$

$$C_1 V^{\imath} = [\alpha^{\imath}(K^{\imath} - P) - \Lambda^{\imath} \sigma^{\imath} P] \frac{\partial V^{\imath}}{\partial P} - (\bar{r} + \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda^{Q}_{\imath\jmath}) V^{\imath}.$$

Numerical Scheme

- Equally spaced grids in the P, w and r directions for the PDE discretization: $[P_0, P_1, \ldots, P_{i_{max}}]$, $[w_0, w_1, \ldots, w_{j_{max}}]$ and $[r_0, r_1, \ldots, r_{k_{max}}]$.
- Discrete timesteps: $0 = 0\Delta\tau < \ldots < \aleph\Delta\tau = T$; the nth timestep: $\tau^n = n\Delta\tau$.
- The exact solution of the pricing equation: $V^{\imath}(P_i, w_j, r_k, \tau^n)$; an approximation of the exact solution: $V^{\imath,n}_{i,j,k}$.
- The standard finite difference methods to discretize the operator: $C_0 V^{\imath}$, $CV^{\imath}$ and $C_1 V^{\imath}$.
- Let $(C_\varepsilon V)^{\imath,n}_{i,j,k}$ denote the discrete value of the differential operators $C_0 V^{\imath}$, $CV^{\imath}$ or $C_1 V^{\imath}$ at a node (P_i, w_j, r_k, τ^n).
- The operators can be discretized using central, forward, or backward differencing in the P direction.
- Let $(B_\varepsilon V)^{\imath,n}_{i,j,k}$ be an approximation of the operator $BV^{\imath}$ at a mesh node (P_i, w_j, r_k, τ^n). For $BV^{\imath} = \sum_{\substack{\jmath=1 \\ \jmath\neq\imath}}^{N} \lambda^Q_{\imath\jmath} V^{\jmath}$, we have $(B_\varepsilon V)^{\imath,n}_{i,j,k} = \sum_{\substack{\jmath=1 \\ \jmath\neq\imath}}^{N} \lambda^Q_{\imath\jmath} \chi(P_i \bar{\xi}^{\imath\jmath}(P_i), V^{\jmath,n}_{i^*_\jmath,j,k}, V^{\jmath,n}_{i^*_\jmath+1,j,k})$.

Numerical Scheme (Cont.)

Using a semi-Lagrangian time-stepping, in regime $\imath$ we get

$$\frac{DV^\imath}{D\tau} \quad = \quad \frac{\partial V^\imath}{\partial \tau} - z\frac{\partial V^\imath}{\partial r} - H(r,w)a(\ell - r)\frac{\partial V^\imath}{\partial w}.$$

Now the PDE can be rewritten as

$$\frac{DV^\imath}{D\tau} = CV^\imath + BV^\imath + H(r,w)q(r,h(w))(P-c).$$

Let $\zeta^{\imath,n+1}_{i,j,k}$ denote the value of the control variable z at the mesh node $(P_i, w_j, r_k, \tau^{n+1})$. Then we can approximate the value of $\frac{DV^\imath}{D\tau}$ by

$$\Big(\frac{DV}{D\tau}\Big)^{\imath,n+1}_{i,j,k} = \frac{1}{\Delta\tau}(V^{\imath,n+1}_{i,j,k} - V^{\imath,n}_{i,\hat{j},\hat{k}}) + \text{ truncation error }.$$

where $V^{\imath,n}_{i,\hat{j},\hat{k}}$ is an approximation of $V^\imath(P_i, w^n_{\hat{j}}, r^n_{\hat{k}}, \tau^n)$ obtained by linear interpolation with $w^n_{\hat{j}}$ and $r^n_{\hat{k}}$ given by

$$w^n_{\hat{j}} = \min[\max[w_j + H(r_k, w_j)a(\ell - r_k)\Delta\tau, w^{\min}], w^{\max}].$$

$$r^n_{\hat{k}} = r_k + \zeta^{\imath,n+1}_{i,j,k}\Delta\tau.$$

Numerical Scheme (Cont.)

Substituting $(C_\varepsilon V)^{\imath,n+1}_{i,j,k}$ and $(B_\varepsilon V)^{\imath,n+1}_{i,j,k}$ into the PDE, we get

$$
\begin{aligned}
& -\Delta\tau\gamma_i^\imath V^{\imath,n+1}_{i-1,j,k} + [1+\Delta\tau\left(\gamma_i^\imath + \beta_i^\imath + (\bar{r} + \sum_{\substack{\jmath=1\\ \jmath\neq\imath}}^{N} \lambda^Q_{\imath\jmath})\right)] V^{\imath,n+1}_{i,j,k} - \Delta\tau\beta_i^\imath V^{\imath,n+1}_{i+1,j,k} \\
&= \sup_{\zeta^{\imath,n+1}_{i,j,k}\in Z_k} V^{\imath,n}_{i,\hat{j},\hat{k}} + \Delta\tau H(r_k, w_j) q(r_k, h(w_j))(P_i - c) \\
&+ \Delta\tau \sum_{\substack{\jmath=1\\ \jmath\neq\imath}}^{N} \lambda^Q_{\imath\jmath} \chi(P_i \bar{\xi}^{\imath\jmath}(P_i), V^{\jmath,n+1}_{i^*_\jmath,j,k}, V^{\jmath,n+1}_{i^*_\jmath+1,j,k}) \text{ if } P_i \in (0, P^{max}).
\end{aligned}
$$

Need to solve a discrete local optimization problem

$$\sup_{\zeta^{\imath,n+1}_{i,j,k}\in Z_k} V^{\imath,n}_{i,\hat{j},\hat{k}}$$

Write and solve

$$R_k = \left\{ r^n_{\hat{k}} \middle| r^n_{\hat{k}} = r_k + \zeta^{\imath,n+1}_{i,j,k}\Delta\tau, \forall\zeta^{\imath,n+1}_{i,j,k} \in Z_k \right\}.$$

$$\sup_{r^n_{\hat{k}}\in \hat{R}_k} V^{\imath,n}_{i,\hat{j},\hat{k}}$$

Numerical Scheme (Cont.)

In matrix form

$$[I + M^{\imath}]V_{j,k}^{\imath,n+1} = \bar{V}_{\hat{j},\hat{k}}^{\imath,n} + \Delta\tau \bar{H}_{j,k}(P - \iota c) + \Delta\tau \Xi(V_{j,k}^{\imath,n+1}).$$

$$[M^{\imath} V_{j,k}^{\imath,n+1}]_{ith\ row} = \Delta\tau[-\gamma_i^{\imath} V_{i-1,j,k}^{\imath,n+1} + \left(\gamma_i^{\imath} + \beta_i^{\imath} + (\bar{r} + \sum_{\substack{\jmath=1 \\ \jmath\neq\imath}}^{N} \lambda_{\imath\jmath}^{Q})\right) V_{i,j,k}^{\imath,n+1} - \beta_i^{\imath} V_{i+1,j,k}^{\imath,n+1}]$$

$$\Xi(V_{j,k}^{\imath,n+1}) = \begin{bmatrix} \tilde{V}_{j,k}^{1,n+1} & \tilde{V}_{j,k}^{2,n+1} & \cdot & \tilde{V}_{j,k}^{\imath,n+1} & \cdot & \tilde{V}_{j,k}^{N,n+1} \end{bmatrix} \begin{bmatrix} \lambda_{\imath 1}^{Q} \\ \cdot \\ 0 \\ \cdot \\ \lambda_{\imath N}^{Q} \end{bmatrix}$$

$$\tilde{V}_{j,k}^{\jmath,n+1} = \begin{bmatrix} \tilde{V}_{0,j,k}^{\jmath,n+1} \\ \cdot \\ \tilde{V}_{i,j,k}^{\jmath,n+1} \\ \cdot \\ \tilde{V}_{i_{max},j,k}^{\jmath,n+1} \end{bmatrix}$$

and 0 is the $\imath th$ element of the column vector. The *ith* element of vector $\tilde{V}_{j,k}^{\jmath,n+1}$ for $\jmath = 1, \ldots, N$ is given by $\chi(P_i \bar{\xi}^{\imath\jmath}(P_i), V_{i_j^*,j,k}^{\jmath,n+1}, V_{i_j^*+1,j,k}^{\jmath,n+1})$.

CIR+Shifted Lognormal Model

Janczura and Weron (2009) use German EEX spot price from 2001-2009 to estimate the following model

- **CIR** (Cox-Ingersoll-Ross) process for the **base regime**

$$dP = \eta(\mu_1 - P)dt + \sigma_1\sqrt{P}dZ.$$

- **Shifted lognormal distribution** for the **spike regime**

$$\log(P - m) \sim N(\mu_2, \sigma_2^2),\ P > m.$$

The spike regime has higher mean and variance than those in the base regime and assigns zero probability to prices below the median m.

The empirical experiments are based on the Abitibi Canyon Generation Station, located on the Abitibi River in Ontario.

Parameter Values

Table: Parameter Values Used for the Regime Switching Case

$dP = \eta(\mu_1 - P)dt + \sigma_1\sqrt{P}dZ; \log(P - m) \sim N(\mu_2, \sigma_2^2),\ P > m.$

Parameter	Value	Parameter	Value
μ_1	47.194 EUR/MWh	P_2^{max}	200 EUR/MWh
η	0.36	P_1^{min}	0 EUR/MWh
σ_1	0.73485	P_2^{min}	48 EUR/MWh
σ_2	0.83066	w^{max}	17000 acre-feet
m	46.54 EUR/MWh	w^{min}	7000 acre-feet
$\bar{r}$	0.05 annually	r^{max}	15000 CFS
T	168h	r^{min}	2000 CFS
c	20 EUR/MWh	r^u	3000 CFS-hr
g	32.15 feet/square-second	r^d	3000 CFS-hr
ρ	1000 kg/cubic-meter	ℓ	6671 CFS
ξ^{12}	1.6470	b	0.0089
ξ^{21}	0.6072	λ_{12}^Q	0.0089
Λ_1	-0.2481	λ_{21}^Q	0.8402
Λ_2	-0.2481	e	0.87
μ_2	3.44	a	0.0826
$Pr(1)$	0.9896	$Pr(2)$	0.0104
q^{max}	336 MW	q^{min}	0 MW
P_1^{max}	200 EUR/MWh	CPC	19000 CFS

Results

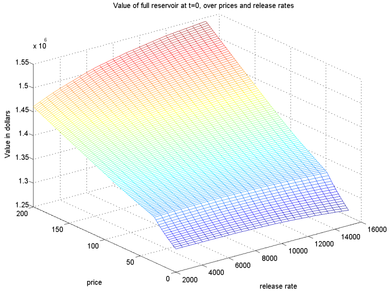
Table: Regime Switching Case

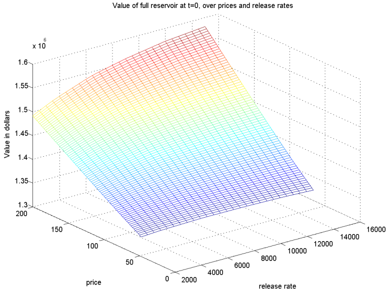
Case	No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Total Profit and Change of Total Profit in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh					
HF, TP	1386100	1380900	1372100	1353300	1320500
HF, CP	N/A	-0.4	-1.0	-2.4	-4.7
FF, TP	1384700	1378400	1367100	1331400	1262800
FF, CP	N/A	-0.5	-1.3	-3.9	-8.8
Total Profit and Change of Total Profit in Regime 1 at Time 0 When the Initial Price is 80 EUR/MWh					
HF, TP	1419000	1413100	1403300	1383000	1351300
HF, CP	N/A	-0.4	-1.1	-2.5	-4.8
FF, TP	1421200	1415400	1405700	1377300	1315500
FF, CP	N/A	-0.4	-1.1	-3.1	-7.4
Total Profit and Change of Total Profit in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh					
HF, TP	1439300	1433200	1423000	1399900	1364000
HF, CP	N/A	-0.4	-1.1	-2.7	-5.2
FF, TP	1441900	1436000	1425800	1394500	1334200
FF, CP	N/A	-0.4	-1.1	-3.3	-7.5
Total Profit and Change of Total Profit in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh					
HF, TP	1591400	1583000	1569200	1530700	1481100
HF, CP	N/A	-0.5	-1.4	-3.8	-6.9
FF, TP	1604400	1598700	1589000	1561900	1522000
FF, CP	N/A	-0.4	-1.0	-2.7	-5.1

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level; TP means total profit; CP means percentage change of total profit compared to the no ramping restrictions scenario .

Results (Cont.)

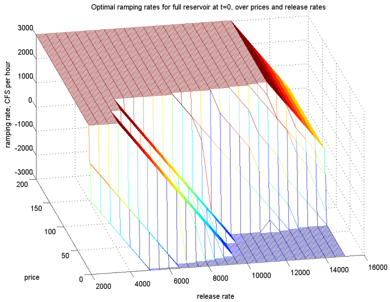
Left: Value of full reservoir at t=0, over prices and release rates (Base regime).
Right: Value of full reservoir at t=0, over prices and release rates (Spike regime).

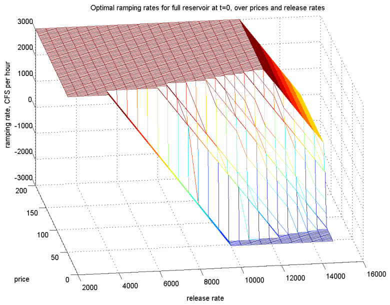




Results (Cont.)

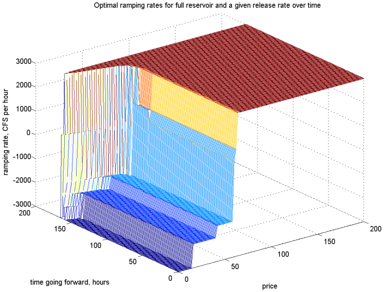
Left: Optimal ramping rate for full reservoir at t=0, over prices and release rates (Base regime).
Right: Optimal ramping rate for full reservoir at t=0, over prices and release rates (Spike regime).

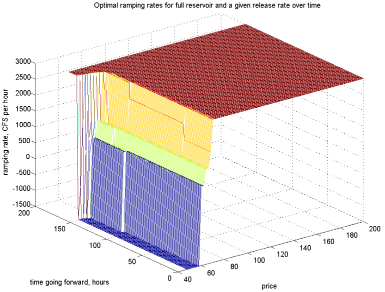




Results (Cont.)

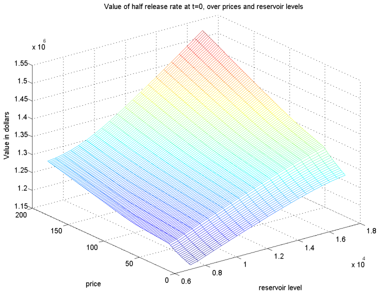
Left: Optimal ramping rate for full reservoir and a given release rate over time (Base regime).
Right: Optimal ramping rate for full reservoir and a given release rate over time (Spike regime).

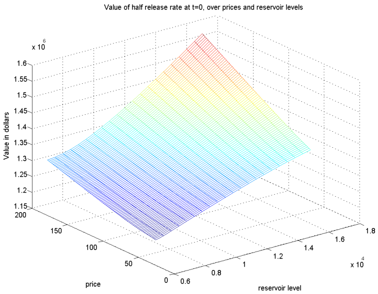




Results (Cont.)

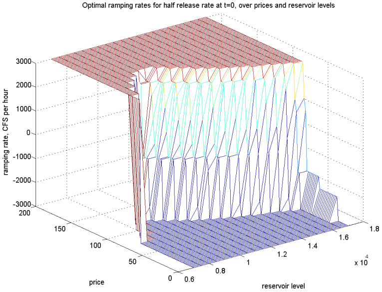
Left: Value of half release rate at t=0, over prices and reservoir levels (Base regime).
Right: Value of half release rate at t=0, over prices and reservoir levels (Spike regime).

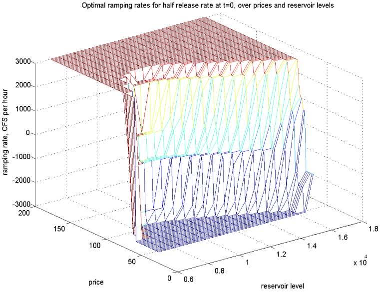




Results (Cont.)

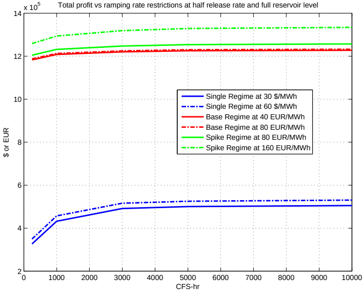
Left: Optimal ramping rate for half release rate at t=0, over prices and reservoir levels (Base regime).
Right: Optimal ramping rate for half release rate at t=0, over prices and reservoir levels (Spike regime).

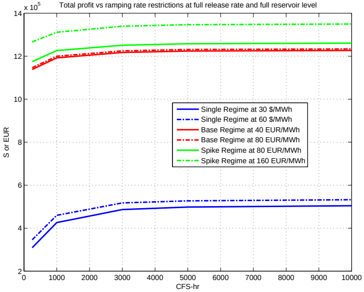




Results (Cont.)

Left: Total profit vs ramping rate restrictions at half release rate and full reservoir level.
Right: Total profit vs ramping rate restrictions at full release rate and full reservoir level.





Sensitivity Analysis

Case	Power Plant Value	Ramping Impact on Power Plant Value
Single Regime Model		
K_0=27 \$/MWh, ϕ=15, σ=0.2 (Benchmark)	N/A	N/A
K_0=47 \$/MWh, ϕ=15, σ=0.2 (I)	Higher (Compared to Benchmark)	Smaller (Compared to Benchmark)
K_0=47 \$/MWh, ϕ=0, σ=0.2 (II)	Higher (Compared to Benchmark)	Smaller (Compared to Benchmark)
K_0=47 \$/MWh, ϕ=0, σ=0.2 (II)	Lower (Compared to (I))	Smaller (Compared to (I))
K_0=47 \$/MWh, ϕ=15, σ=0.4 (III)	Higher (Compared to Benchmark)	Smaller (Compared to Benchmark)
K_0=47 \$/MWh, ϕ=15, σ=0.4 (III)	Higher (Compared to (I))	Larger (Compared to (I))
Regime Switching Model		
η=0.36, λ_{12}^Q=0.0089, λ_{21}^Q=0.8402, σ_1=0.73485, σ_2=0.83066 (Benchmark)	N/A	N/A
η=0.72, λ_{12}^Q=0.0089, λ_{21}^Q=0.8402, σ_1=0.73485, σ_2=0.83066 (IV)	Lower (Compared to Benchmark)	Smaller (Compared to Benchmark)
η=0.36, λ_{12}^Q=0.02, λ_{21}^Q=0.7402, σ_1=0.73485, σ_2=0.83066 (V)	Higher (Compared to Benchmark)	Larger (Compared to Benchmark)
η=0.36, λ_{12}^Q=0.0089, λ_{21}^Q=0.8402, σ_1=0.93, σ_2=1.43 (VI)	Higher (Compared to Benchmark)	Larger (Compared to Benchmark)

Note: Other benchmark parameter values for the single regime case and regime switching case are given in Table 1 and 2 respectively and the corresponding results for these two cases are reported in Table 3 and 4 respectively.

Conclusions

1. This paper provides
   - a regime switching framework for hydro plant valuation.
   - a comprehensive analysis of the economics of ramping rate restrictions at hydro plants to protect ecosystem.
2. This study gives
   - insights into our understanding of ramping related issues for a hydro power station, including the desirable choice of ramping restrictions and possible policy recommendations.
3. Results from this research will
   - facilitate the implementation of environmental regulations designed to promote the integrity of river systems.
   - provide a set of planning tools regulators and industry can use to negotiate the optimal ramping rate for environmental and economic benefits.