Equilibrium Commodity Trading

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sharp common price movements since 2002

- oil, copper, zinc, tin, soybeans
- prices doubled/halved within a year

Figure: from Tang and Xiong 2010
commodity futures markets

Figure: from Tang and Xiong 2010

- sharp increase (and variations) of the open interest
- ETF’s
is there a link?

Figure: from Singleton WP 2012

- looks like open-interest and prices move together
model features and results

features

• commodity production, consumption, and speculation are endogenously determined
• asymmetric information model
• futures are used for both hedging and speculation

results

• commodity supply is a channel by which speculators on the futures market impact the spot market
• both open-interest and futures price are informative (consistent with Hong and Yogo (2012), JFE)
• If accuracy of private information is low, more speculators makes both production and spot prices more volatile.
• More speculators typically increases correlation between financial and commodity markets (consistent with Silvennoinen and Thorp (2013), JIFMIM)
model features and results

features

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## Literature

### Classic

*Scheinkman and Schechtman RES '83, Deaton and Laroque RES '92, Deaton and Laroque JPE '96, Hirshleifer RFS '88, Hirshleifer JPE '88, Hirshleifer ECMA '90, Hong JF '00, Merton JF '87, Routledge, Seppi, and Spatt JF '00*

### Recent: (also) empirical

*Acharya, Lochstoer, and Ramadorai JFE '13, Christoffersen, Jacobs, and Li WP '13, Hamilton WP '09, Hamilton and Whu WP '12, Hong and Yogo JFE '12, Knittel and Pindyck WP '13, Singleton WP '12, Tang and Xiong FAJ '12, Silvennoinen and Thorp JIFMIM '13*

### Recent: theoretical

*Basak and Pavlova WP '13, Basak and Pavlova WP '13, Ekeland, Lautier, and Villeneuve WP '13, Sockin and Xiong WP '13, Goldstein, Li, and Yang RFS '13*

### Derivatives on underlying

*Frey and Stremme MF '97, Platen and Schweizer MF '98, Sircar and Papanicolaou AMF'97, Schoenucher and Wilmott SIAM JoAM '00, Grossman JB '88, Genotte and Leland AER '90*
1 futures markets for hedging
2 futures markets for hedging and learning
futures markets for hedging

suppliers

futures market

financiers

endowment

end-users

spot market

output

productivity shock

\[ k(\tilde{\epsilon} + \tilde{\tau}) \]

CARA-normal setting

\[ \left( \tilde{z}, \tilde{\tau}, \tilde{\epsilon} \right) \sim \mathcal{N} \]

extraction costs:

\[ q \mapsto \kappa q \]
futures markets for hedging

suppliers

futures market

financiers

endowment

end-users

spot market

output

productivity shock

CARA-normal setting

extraction costs:

\[ k(\tilde{\epsilon} + \tilde{\tau}) \]

\[ \tilde{z}, \tilde{\tau}, \tilde{\epsilon} \sim N \]

\[ c \mapsto -e^{-\gamma c} \]

\[ q \mapsto k q \]

\[ \alpha_f \]

\[ (mass \ \nu) \]
futures markets for hedging

suppliers

futures market

futures market

financiers

mass \( \nu \)

endowment

\( e_\epsilon \tilde{\epsilon} + \tilde{z} \)

productivity shock

end-users

output

\( k(\tilde{\epsilon} + \tilde{\tau}) \)

\( \tilde{z}, \tilde{\tau}, \tilde{\epsilon} \sim \mathcal{N} \)

\( c \mapsto -e^{-\gamma c} \)

cARA-normal setting

extraction costs: \( q \mapsto \kappa q \)
futures markets for hedging

suppliers

futures market

financiers

endowment

end-users

spot market

output

productivity shock

CARA-normal setting

extraction costs:

\[
\begin{align*}
\hat{z}, \hat{\tau}, \hat{\epsilon} &\sim \mathcal{N} \\
c &\mapsto e^{-\delta c}
\end{align*}
\]

\[q \mapsto \kappa q\]
futures markets for hedging

futures market
(t = 0)

financiers
(mass \( \nu \))

endowment
\( e_\epsilon \tilde{\epsilon} + \tilde{z} \)

productivity
shock

suppliers

\( \alpha_p \)

q

spot market

k

end-users

output
\( k(\tilde{\epsilon} + \tilde{\tau}) \)

CARA-normal setting

\[
\begin{pmatrix}
\tilde{z}, \tilde{\tau}, \tilde{\epsilon} \\
\epsilon \\
\end{pmatrix} \sim N \\
\mapsto -e^{-\gamma c}
\]

extraction costs:
\( q \mapsto \kappa q \)
futures markets for hedging

suppliers

futures market
(t = 0)

financiers
(mass ν)

end-users

endowment
e_ε ˜ε + ˜z
(t = 1)

productivity shock

spot market
(t = 1)

output
k( ˜ε + ˜τ)

CARA-normal setting
\( \tilde{z}, \tilde{\tau}, \tilde{\epsilon} \sim N \)
\( c \mapsto -e^{-\gamma c} \)

extraction costs: \( q \mapsto \kappa q \)
futures markets for hedging

suppliers

futures market
$(t = 0)$

financiers
$(mass \nu)$

endowment
$e_\epsilon \tilde{\epsilon} + \tilde{\tau}$
$(t = 1)$

end-users

spot market
$(t = 1)$

productivity shock

output
$k(\tilde{\epsilon} + \tilde{\tau})$
$(t = 2)$

endowment

output

suppliers

futures market
$(t = 0)$

financiers
$(mass \nu)$

endowment
$e_\epsilon \tilde{\epsilon} + \tilde{\tau}$
$(t = 1)$

end-users

spot market
$(t = 1)$

productivity shock

output
$k(\tilde{\epsilon} + \tilde{\tau})$
$(t = 2)$

CARA-normal setting

extraction costs: $q \mapsto \kappa q$
futures markets for hedging

suppliers

futures market $(t = 0)$

financiers

endowment $e_{\epsilon} \tilde{\epsilon} + \tilde{z}$ $(t = 1)$

end-users

spot market $(t = 1)$

output $k(\tilde{\epsilon} + \tilde{\tau})$ $(t = 2)$

endowment $e_{\epsilon} \tilde{\epsilon} + \tilde{z}$ $(t = 1)$

productivity shock

CARA-normal setting

extraction costs: $q \mapsto \kappa q$

extraction costs: $q \mapsto \kappa q$
supplier’s problem

\[
\sup_{q, \alpha_s} E \left[ U_s (\tilde{w}) \mid F_{0,s} \right] \quad \text{u.c.} \quad \tilde{w} = q (\tilde{p} - \kappa) + \alpha_s (\tilde{p} - F)
\]

Note 1 large enough horizon for the supply level to be adjusted

dend-user’s problem

\[
\sup_k E \left[ U_e (\tilde{w}) \mid \tilde{\epsilon}, \tilde{p} \right] \quad \text{u.c.} \quad \tilde{w} = k (\tilde{\tau} - \tilde{p}R)
\]

financier’s problem

\[
\sup_{\alpha_f} E \left[ U_f (\tilde{w}) \mid F_{0,f} \right] \quad \text{u.c.} \quad \tilde{w} = \alpha_f (\tilde{p} - F) + \tilde{e}
\]
restrictions on the parameters

positive supply

extraction costs $\kappa$ not too high

$$\kappa \leq \frac{1}{R} \left( \mu - \frac{1}{\gamma_f} + \frac{1}{\nu \gamma_s} \epsilon e \sigma_e^2 \right)$$
equilibrium

definition: rational expectations equilibrium (REE)

futures price $F$, distribution for $\tilde{p}$, individual strategies

- markets clear
- individual strategies optimal
- rational expectations (supply)
rational expectations

aggregate supply

market clearing

L[\bar{p}]

individual supply

∑

individual problem

proposition

∃! equilibrium
If the financiers are negatively exposed to the commodity price risk, i.e. $\sigma_{e,p} \leq 0$, or if the extraction costs are low enough, then

- the financiers buy futures contracts
- when the mass $\nu$ of financiers increases
  - the supply increases
  - the expected spot price decreases

Conversely, if both $\sigma_{e,p} > 0$ and the extraction costs are high enough, then

- the financiers sell futures contracts
- when the mass $\nu$ of financiers increases
  - the supply decreases
  - the expected spot price increases
expected utilities

proposition

an increase of the mass $\nu$ of financiers is

- beneficial to the end-users, if $\sigma_{e,p} \leq 0$ or $\kappa$ low enough.
- detrimental to the end-users, if $\sigma_{e,p} > 0$ and $\kappa$ high enough.
- detrimental to the financiers
- ambiguous for the suppliers

ambiguous for the suppliers because

- each supplier sells more of the commodity
- but they collectively increase the supply
- and do not internalize the adverse effect on prices

corollary

- welfare improving
numerical results
futures markets and information

futures markets allow to

• speculate according to one’s view regarding spot prices/demand
• learn about the views of others
• learn about non public information
futures markets for hedging and learning

futures market \((t = 0)\)

financiers

endowment \(\tilde{\epsilon} \tilde{\epsilon} + \tilde{z}\) \((t = 1)\)

suppliers

\(\tilde{\kappa}\)

\(\alpha_p\)

\(\tilde{\kappa}\)

\(q\)

spot market \((t = 1)\)

end-users

\(k\)

output \(k(\tilde{\epsilon} + \tilde{\tau})\) \((t = 2)\)

\(\tilde{\epsilon} + \tilde{\epsilon}_r\)

\(\tilde{\epsilon}\)

CARA-normal setting

\[
\begin{pmatrix}
\tilde{z}, \tilde{\tau}, \tilde{\epsilon}, \tilde{\epsilon}_r, \tilde{\epsilon}_r
\end{pmatrix} \sim \mathcal{N} \\
\text{supp}(\tilde{\kappa}) = [\kappa, \overline{\kappa}] \\
c \mapsto -e^{-\gamma c}
\]

individual problems
\( \exists ! \) equilibrium

proposition

- \( \exists ! \) equilibrium
- it is linear
- futures price reveals the extraction costs
- open-interest partially reveals the signal
spot price and endowment results

proposition

• an increase of the mass \( \nu \) of financiers only decreases the variance of the spot price if the signal is accurate enough

\[
\partial_\nu \text{Var}[p] < 0 \iff \sigma_r^2 \sigma_{e\epsilon}^2 < \frac{d_1 \sigma_e^2}{R^2 \nu \gamma}
\]

• the correlation between spot price and endowment increases with \( \nu \) if and only if the average exposure of the endowment to the commodity price risk is non-positive

\[
\partial_\nu \rho_{e,p} \geq 0 \iff \mu_e \leq 0.
\]

note financiers can have a destabilizing effect
the exposure of the financiers is driven by

exogenous hedging motives

• $\sigma^2_r \sigma^2_{e_e}$ “large”
• supply driven by exogenous factors
• spot market “contaminated”
• variance of spot price increases with $\nu$

superior information

• $\sigma^2_r \sigma^2_{e_e}$ “small”
• futures markets synchronize demand and supply
• variance of spot price decreases with $\nu$

\[
\frac{\sigma_p}{\sigma_{p,\text{ref}}} \begin{cases} 
0.99 & \text{hedging} \\
1.00 & \text{critical} \\
1.01 & \text{info}
\end{cases}
\]
more accurate signals make the expected spot price less sensitive to $\nu$

more capital for absorbing shocks makes a larger impact in a riskier world
relative expected utility of the end-users

- effect of $\nu$ on expected returns dominates the effect on variance
expected utility of the end-users

- with few financiers: better to have them well informed
- with many financiers: better to have them not too well or too badly informed
expected utility of the suppliers

• as in the symmetric information model, ambiguous for the suppliers, who do not internalize the price effect
• also true in the signal dimension
conclusion

• study the impact of more investors trading on futures markets
• role of production channel highlighted
  • more hedging and production
  • expected prices decrease
  • ambiguous effect on volatility
• open-interest and futures price can provide distinct information
• correlation between commodity spot and endowment typically increases with the mass of financiers
Thank you
### Calibration

**Base Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>2</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1</td>
</tr>
<tr>
<td>$R$</td>
<td>1.035</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>38.74</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>28.58</td>
</tr>
<tr>
<td>$q$</td>
<td>580.4</td>
</tr>
<tr>
<td>Elast. of Demand</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**With Learning**

- $\sigma_{e_{\epsilon}} = \frac{1}{20} \mu_{e_{\epsilon}}$
- Hedging: $\sigma_r = 1.5 \times \sigma_{r,\text{crit.}}$
- Speculation: $\sigma_r = 0.5 \times \sigma_{r,\text{crit.}}$

---

> numerical results 0  numerical results 1
rational expectations

\[ \sum \text{aggregate supply} + \text{futures price} \]

\[ \mathcal{L}[\tilde{p}|\mathcal{F}_{0,f}] \]

individual supply

\[ \mathcal{L}[\tilde{p}|\mathcal{F}_{0,s}] \]

open-interest

private signal

private hedging motives