Quantum Frequency Conversion and Temporal-Mode Multiplexing of States of Light

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Quantum Frequency Conversion (QFC):
The complete or partial exchange of quantum states between two spectral bands.

\[ |\psi\rangle_g |\text{vac}\rangle_b \rightarrow |\text{vac}\rangle_g |\psi\rangle_b \]

\[ |\psi\rangle_g |\phi\rangle_b \rightarrow \alpha |\psi\rangle_g |\phi\rangle_b + \beta |\phi\rangle_g |\psi\rangle_b \]

note: need phase coherence for the latter
Potential Uses of Single-Photon States

A. Many Classical Bits in Single Photon

B. Spectral-Temporal Photonic Qubit

Need Pulse-Shape Multiplexing
Commonly used multiplexing schemes in radio technology

- Frequency-division multiple access (FDMA)
- Time-division multiple access (TDMA)
- Code-division multiple access (CDMA)

FDMA and TDMA use only time-frequency space.
CDMA uses field-orthogonal codes in code space

\[ \int E_n^*(t)E_m(t)dt = \delta_{nm} \]

In radio, field-orthogonal codes are easily demultiplexed. In optical, there is NO known method to demultiplex field-orthogonal codes efficiently.
The Mythical Device

input signals

input shape selector

drop

pass
The Mythical Device with Optional Output Shape Control

Atomic Ensemble Quantum Memory with Temporal-Mode Selectivity
Nonlinear Optical Frequency Conversion:
a potential method to spatially separate field-orthogonal
temporal modes.

Note: The device is a linear-mode transformer.
It treats single-photons packets the same as weak classical (coherent-state) fields.

Methods for Quantum Frequency Conversion

Three-wave mixing in NLO crystal

Methods for Quantum Frequency Conversion

Three-wave mixing in NLO crystal

\[ \chi^{(2)} \]

\[ \omega_2 = \omega_1 + \omega_p \]

large frequency shift

Huang and Kumar, PRL (1992)
Rakher, ... Srinivasan, Nat. Photonics 4, 786 (2010)

Four-wave mixing Bragg Scattering in optical fiber

\[ \chi^{(3)} \]

\[ \omega_2 = \omega_1 + \omega_p - \omega_q \]

allows small frequency shift

McKinstrie, Opt Ex (2005)
McGuinness PRL (2010)

EXPERIMENTS: Dispersion of Step-Index Fiber and Photonic-Crystal Fiber

Zero Dispersion

\[ \frac{d}{d\lambda} \left( \frac{1}{v_g} \right) \]
**EXPERIMENTS:** Dispersion of Step-Index Fiber and Photonic-Crystal Fiber

Zero Dispersion

$$\frac{d}{d\lambda} \left( \frac{1}{v_g} \right)$$

Signal 1 has same group velocity as Pump 1.
Signal 2 has same group velocity as Pump 2.
2. Conversion

Conversion efficiency = 29%
How effective can QFC be in separating field-orthogonal optical codes?
Modeling QFC by Nonlinear Wave Mixing

\[
\begin{align*}
\left( \frac{\partial}{\partial z} + \frac{1}{v_g} \frac{\partial}{\partial t} \right) A_g(z,t) &= i\gamma P(z,t) A_b(z,t) \\
\left( \frac{\partial}{\partial z} + \frac{1}{v_b} \frac{\partial}{\partial t} \right) A_b(z,t) &= i\gamma P^*(z,t) A_g(z,t)
\end{align*}
\]

The equations are linear in \( A_g \) and \( A_b \) signal field operators.
Solution:

\[
\begin{pmatrix} A_g(t) \\ A_b(t) \end{pmatrix}_{OUT} = \int^t dt' \begin{pmatrix} G_{gg}(t,t') & G_{gb}(t,t') \\ G_{bg}(t,t') & G_{bb}(t,t') \end{pmatrix} \begin{pmatrix} A_g(t') \\ A_b(t') \end{pmatrix}_{IN}
\]

All quantum correlations can be calculated from Green functions.

Pump shapes:

\[
P_{TWM}(z,t) = A_p^*(z,t) \\
P_{FWM}(z,t) = A_{p1}^*(z,t) A_{p2}(z,t)
\]

Pump/coupling:

\[
\gamma
\]

Christ, Brecht, Mauerer, Silberhorn (NJP 2013)
Schmidt Mode Decomposition of the Green functions
(singular-value decomposition)

\[
\begin{pmatrix}
A_g(t) \\
A_b(t)
\end{pmatrix}_{OUT} = \sum_n \int^t dt' \begin{pmatrix}
\tau_n v_n(t)V_n^*(t') & i\rho_n v_n(t)W_n^*(t') \\
i\rho_n w_n(t)V_n^*(t') & \tau_n w_n(t)W_n^*(t')
\end{pmatrix}
\begin{pmatrix}
A_g(t') \\
A_b(t')
\end{pmatrix}_{IN}
\]

with \(\rho_n^2 + \tau_n^2 = 1\) \(\rho_n^2\) = conversion, \(\tau_n^2\) =nonconversion

Temporal Schmidt modes reduce problem to low-dimensional state space:

if \[\begin{pmatrix}
A_g(t') \\
A_b(t')
\end{pmatrix}_{IN} = \begin{pmatrix}
a_g V_1(t') \\
a_b W_1(t')
\end{pmatrix}\]

then \[\begin{pmatrix}
A_g(t) \\
A_b(t)
\end{pmatrix}_{OUT} = \begin{pmatrix}
(\tau_1 a_g + i\rho_1 a_b) v_1(t) \\
i\rho_1 a_g + \tau_1 a_b) w_1(t)
\end{pmatrix}\]

Operators undergo a pair-wise beam-splitter transformation.

Figure of Merit for Temporal-mode Selectivity

\[ G_{bg}(t,t') = i \sum_n \rho_n w_n(t) V^*_n(t') \]

Apply to three-wave mixing:

\[ \eta_n = |\rho_n|^2 = \text{conversion efficiency} \]

separability ≡ \( \frac{\eta_{\text{Target}}}{\sum_n \eta_n} \leq 1 \)

\( S \equiv \text{Selectivity} \equiv \text{separability} \times \eta_{\text{Target}} \)

\[ S = \left| \frac{\eta_{\text{Target}}}{\sum_n \eta_n} \right|^2 \leq 1 \]

ideally: \( S = 1 \)

Three-wave Mixing

One pump signal

Optimum case: Pump velocity matches ‘green’ signal velocity. Blue is slower.

Three-wave Mixing - Low conversion efficiency
ultrashort pump pulse

Separability is 0.94, but Selectivity is low when conversion efficiency is low.
Three-wave Mixing - High conversion efficiency
ultrashort pump pulse

Optimum case: Pump velocity matches green signal velocity

\[ S = \text{selectivity} \]

\[ \bar{\gamma} = \gamma / \left( \frac{1}{v_g} - \frac{1}{v_b} \right) \]

max selectivity
Three-wave Mixing - High conversion efficiency
ultrashort pump pulse

Optimum case: Pump velocity matches green signal velocity

\[ S = \left| \frac{\eta_{\text{Target}}}{\sum n \eta_n} \right|^2 \]

\[ \max S \approx 0.85 \]

pump strength, \( \gamma \sqrt{\frac{L}{\left( \frac{1}{v_g} - \frac{1}{v_b} \right)}} \)
Origin of limited Selectivity: oscillations in the Green function make it non-separable.

Consequence of ‘Rabi oscillations’ between blue and green.
Four-wave Mixing

One pump selects the input mode shape;
Other pump determines the output mode shape.

Four-Wave Mixing

Much the same as TWM, with the shape of the medium replaced by the shape of the second pump.

Optimum case: pump 1 velocity matches green signal velocity and pump 2 matches blue signal velocity. Complete collision occurs.

\[ \bar{\gamma} = 0.83 \]

Selectivity \( \sim 0.5 \)

Conversion Efficiency increase \( \bar{\gamma} \)
Four-Wave Mixing

\[ \hat{\gamma} = 1.8 \]

Schmidt modes: input

\[ \gamma = 1.8 \]

\[ \psi_n(t') \]

\[ \phi_n(t') \]

Schmidt modes: output

\[ \gamma = 1.8 \]

\[ \psi_n(t) \]

\[ \phi_n(t) \]

Conversion Efficiency

Selectivity \sim 0.7
Four-Wave Mixing

\[ \eta_n = |\rho_n|^2 = \text{conversion efficiency} \]

\[ S \equiv \text{Selectivity} \equiv \text{separability} \times \eta_{\text{Target}} \]

\[ \text{separability} \equiv \frac{\eta_{\text{Target}}}{\sum_n \eta_n} \leq 1 \]

\[ S = \frac{|\eta_{\text{Target}}|^2}{\sum_n \eta_n} \leq 1 \]

Cannot exceed 0.85
Still Mythical: a drop device with 100% Selectivity
Atomic Ensemble Quantum Memory with Temporal-Mode Selectivity

Selectivity cannot exceed ~ 0.85


Quantum conversion between near-frequency channels, for Quantum Internet


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