Faculty of Science

Closure properties for the class of Cuntz-Krieger algebras

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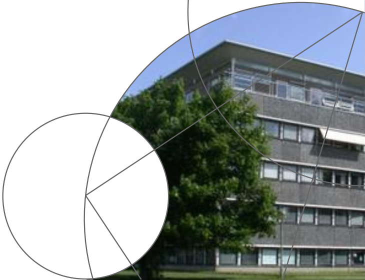

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Corollary (A-Ruiz)

Corners of Cuntz-Krieger algebras are Cuntz-Krieger algebras.

Extensions of purely infinite Cuntz-Krieger algebras

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If 1–3 holds, then A looks like a purely infinite Cuntz-Krieger algebra.

Classification of purely infinite Cuntz-Krieger algebras

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Theorem (Restorff)

Let A and B be purely infinite Cuntz-Krieger algebras with $\mathrm{Prim}(A) \cong \mathrm{Prim}(B)$. Then $\mathrm{FK}_{\mathcal{R}}(A) \cong \mathrm{FK}_{\mathcal{R}}(B)$ implies $A \otimes \mathbb{K} \cong B \otimes \mathbb{K}$.

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Example (Reduced filtered K-theory $\mathrm{FK}_{\mathcal{R}}$)

For a C^*-algebra A with ideal lattice

$$\begin{array}{ccc} & A & \\ J_1 & & J_2, \\ & I & \\ & 0 & \end{array}$$

its $\mathrm{FK}_{\mathcal{R}}(A)$ consists of

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Applying classification to extensions of Cuntz-Krieger algebras

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Theorem (Kirchberg, Meyer-Nest, Bentmann-Köhler)

Let A and B be Kirchberg X-algebras with X an accordion space. Then $\mathrm{FK}(A) \cong \mathrm{FK}(B)$ *implies* $A \otimes \mathbb{K} \cong B \otimes \mathbb{K}$.

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Corollary

Let A be a C^*-algebra with $\mathrm{Prim}(A)$ an accordion space. Then A is a purely infinite Cuntz-Krieger algebra if and only if it looks like one.