Representation Theory and Quantization

Titles and Abstracts

Leticia Barchini, Oklahoma State University

Computing associated cycles of Harish-Chandra modules, techniques and examples

Abstract. We discuss two types of invariants attached to a Harish-Chandra module X, the associated variety and the finer invariant, the associated cycle. The associated variety $AV(X) = \cup \overline{\mathscr{O}_i}$ is a finite union of closures of nilpotent K-orbits. The associated cycle is an integer combination of the $\overline{\mathscr{O}_i}$; the coefficients $m_{\mathscr{O}_i}(X)$ are called the multiplicities. If $\{X(\lambda)\}$ (λ in the integral lattice) is a coherent family of Harish-Chandra modules, then the multiplicities $m_{\mathscr{O}_i}(X(\lambda))$ extend to harmonic polynomials. The problem of computing these polynomials is of current interest. In this talk we first discuss general techniques for approaching this problem. Then we focus on some classes of representations for which such computation is possible. This presentation is based on work with Roger Zierau.

Prakash Belkale, UNC Chapel Hill

Geometric unitarity of KZ/Hitchin connection on conformal blocks in genus 0

Abstract. We prove that the vector bundles of conformal blocks, on suitable moduli spaces of genus zero curves with marked points, for arbitrary simple Lie algebras and arbitrary integral levels, carry geometrically defined unitary metrics (as conjectured by K. Gawedzki) which are preserved by the Knizhnik-Zamolodchikov/Hitchin connection. Our proof builds upon the work of T. R. Ramadas who proved this unitarity statement in the case of the Lie algebra sl(2) (and genus 0) and arbitrary integral level.
We note that unitarity has been proved in all genera (including genus 0) by the combined work of Kirillov and Wenzl in the 90's (this approach does not yield a concrete metric).

Raul Gomez, UC San Diego

Bessel Models for General Admissible Induced Representations: The Compact Stabilizer Case

Abstact. Let G be a Lie group, and let $P = MAN$ be a parabolic subgroup such that N is abelian. Let χ be a generic character of N whose stabilizer in M, M_χ, is compact. In the work of Wallach it is shown that if $\pi = {}^\infty Ind_P^G \sigma_V$, where (σ, V_σ) is a finite dimensional representation of M, then $\dim Wh_\chi(\pi) = \dim V_\sigma$. In my joint work with Wallach, we generalize this result by showing that, if (σ, V_σ) is an admissible, finitely generated, smooth Fréchet representation of M, then $Wh_\chi(\pi) \cong V'_\sigma$ as an M_χ-module.

Bogdan Ion, University of Pittsburg

The Hopf-Poincare-Birkhoff-Witt theorem

Abstract. Virtually all the approaches to the PBW theorem (and its generalizations) are of combinatorial nature, reducing one way or another to the knowledge of generators and relations for the algebras in question. However, as I will explain, the PBW theorem is a basic structural result that in the correct formulation does not require any reference to generators and relations, and holds in vast generality: any Hopf algebra in arbitrary braided tensor category is of PBW type.
The structure of a Hopf algebra appears preeminently, besides Lie theory, in topology and geometry and I will explain how early structural results (Hopf, Samelson, Borel, Quillen, Milnor and Moore, Cartier, Kostant) and results from the quantum groups era (Kashiwara, Lusztig, Rosso) are related to the general result.

Allen Knutson, Cornell University
TBA
Abstract. TBA

Ivan Penkov, Jacobs University Bremen
Koszul categories of $sl(\infty)-, o(\infty)-, sp(\infty)-$modules
Abstract. Various categories of integrable modules over the classical infinite-dimensional Lie algebras $sl(\infty), o(\infty), sp(\infty)$ have been studied in recent years. In this talk we construct a category based on the mixed tensor algebra (the tensor algebra of the natural and conatural modules). The indecomposable injectives in the category turn out to be simply indecomposable direct summands in the tensor algebra. In addition, we show that this category is antiequivalent to the category of locally unitary finite-dimensional modules over a Koszul algebra. This yields a remarkable equivalence of the corresponding categories for $o(\infty)$ and $sp(\infty)$. The talk is based on joint work with Elizabeth Dan-Cohan and Vera Serganova.

Alexey Petukhov, Jacobs University Bremen
Joseph ideals and bounded modules
Abstract. Let $\mathfrak{g}$ be a simple Lie algebra with the adjoint group G. Any two-sided ideal of $\mathrm{U}(\mathfrak{g})$ determines a G-stable subvariety of $\mathfrak{g}^*$. We call ideals associated with the nilpotent orbit of minimal dimension *Joseph ideals*. If $\mathfrak{g}$ is of type A, the set of these ideals (and the set of modules annihilated by these ideals) is closely related with both modules over Weil algebra and S_n-modules. In my talk I will show that many simple bounded $(\mathfrak{g}, \mathfrak{k})$-modules defined by I. Penkov and V. Serganova are holonomic, annihilated by some Joseph ideals of $\mathrm{U}(\mathfrak{g})$ and clarify which role play in this example Weil algebra and the group S_n.

Arturo Pianzola, University of Alberta
Reductive group schemes, Torsors, and Extended Affine Lie Algebras
Abstract. The theory of reductive groups over an arbitrary base was developed by Demazure and Grothendieck in the early 60's. Affine (and extended affine) Lie algebras were to appear later. About a decade ago a surprising relation was found to exist between these two worlds.

The bridge is given by torsors. The talk, which will be for a general algebra audience, will illustrate the concepts and ideas mentioned above.

Vera Serganova, UC Berkeley

Borel-Weil-Bott theorem and Bernstein-Gelfand-Gelfand reciprocity for classical supergroups

Abstract. In 2003 J. Brundan found remarkable connections between the category F of finite-dimensional representations of $GL(m,n)$ and tensor representations of $GL(\infty)$. In the recent paper Brundan and Stroppel develop this idea further using categorification approach and construct a certain Koszul algebra which completely describes the structure of the category F. The essential ingredient of this method is the Bernstein-Gelfand-Gelfand reciprocity law which relates projective and and simple objects in F via so called Kac modules. Unfortunately, Kac modules do not exist for other classical supergroups.
We will show that BGG reciprocity holds if in place of a Kac module one takes a virtual module given by the Euler characteristic of a line bundle on a flag supermanifold. We give a combinatorial algorithm for decomposition of a projective module into a sum of these virtual modules for the orthosymplectic supergroup and explore connections with representations of $GL(\infty)$ on this case.
This talk is based on a joint work with C. Gruson.

Peter Trapa, University of Utah

Spin representation of Weyl groups and nilpotent orbits

Abstract. Fix a semisimple Lie algebra g, Cartan subalgebra h, and let W denote the Weyl group of h in g. Since W acts by orthogonal transformations on the real span of roots of h in g, one can consider the preimage of W in an appropriate Pin group. Its irreducible genuine representations – the so-called spin representations of W – were classified in a case-by-case basis in the work of Schur, Morris, Reade and others. Recently, Dan Ciubotaru discovered a remarkable uniform classification of them (modulo a notion of equivalence) in terms of the nilpotent cone of the Lie algebra g. Ciubotaru's parametrization, which formally resembles Springer theory for W itself, fits perfectly with the theory of the p-adic Dirac operator (developed in joint work with Barbasch and Ciubotaru). This talk will present Ciubotaru's theorem and its consequences for unitary representation of split p-adic groups.

Oded Yacobi, University of Toronto

Polynomial representations of general linear groups and categorifications of Fock space

Abstract. Consider a field F of characteristic p. It is well known that the direct sum of the categories $Rep(S_k)$ categorifies the basic representation of the Kac-Moody algebra g, where g is sl_p when $p > 0$ and sl_∞ when $p = 0$. Moreover, the symmetric groups and general linear groups are related via Schur-Weyl duality. Therefore it is natural to ask for an analogue of the direct sum of categories $Rep(S_k)$ for the general linear groups. We will construct such a category as a limit category of polynomial representations of GL_n, and show that it categorifies

the Fock space representation of g (in the sense of Chuang and Rouquier). Time permitting, we will discuss some consequences of this categorification, such as how to recover the crystal of Fock space from this construction and how, from this perspective, Schur-Weyl duality categorifies the standard projection from Fock space onto the basic representation. This is based on joint work with Jiuzu Hong.

Milen Yakimov, Louisiana State University
Quantum Schubert cells and quantum flag varieties
Abstract. De Concini, Kac, and Procesi defined a family of subalgebras $U_q^w(g)$ of a quantized universal enveloping algebra $U_q(g)$ associated to the elements of the corresponding Weyl group W. They are deformations of universal enveloping algebras of nilpotent Lie algebras and can be interpreted as quantizations of coordinate rings of Schubert cells. We will describe explicitly all torus invariant prime ideals of the algebras $U_q^w(g)$, construct efficient generating sets, and describe the poset of those ideals. We will then apply these results to prove a conjecture of Goodearl and Lenagan on polynormal generating sets of torus invariant primes, prove that the spectrum of $U_q^w(g)$ is normally separated, and describe the dimensions of the strata of the Goodearl-Letzter stratification of Spec $U_q^w(g)$. We will use these results to classify the torus invariant prime ideals of all quantum partial flag varieties.

Roger Zierau, Oklahoma State University
Harmonic spinors on homogeneous spaces
Abstract. In joint work with L. Barchini we consider the cubic Dirac operator on a reductive homogeneous space G/H. When G/H is a riemannian symmetric space it is well-known that the space of square integrable harmonic spinors is an irreducible unitary representation of G. When H is noncompact a notion of square integrable harmonic spinor is defined. We give a condition for the space of square integrable harmonic spinors to be nonzero. The natural invariant hermitian form on G/H (typically an indefinite form) gives an hermitian form (by integrating) on the space of square integrable spinors, which in some situations is positive defnite.