

GIADA BASILE University of Bonn

Convergence of a kinetic equation to a diffusion equation under an anomalous scaling

A linear Boltzmann equation in dimension two is interpreted as the forward equation for the probability density of a Markov process (K(t), i(t), Y(t)) on $(\mathbb{T}^2 \times \{1, 2\} \times \mathbb{R}^2)$. Here (K(t), i(t)) is an autonomous jump process, with waiting times between two jumps with finite expectation but infinite variance, and Y(t) is an additive functional of K, defined as $Y(t) = \int_0^t ds \ v(K_s)$ where $|v| \sim 1$ for small k. In a phononic picture, Y(t) describes the trajectory of a phonon. We prove that the rescaled process $(N \ln N)^{-1/2} Y(Nt)$ converges in distribution to a Brownian motion. As a consequence, the appropriately rescaled solutions of the Boltzmann equation converge to a diffusion equation.

CÉDRIC BERNARDIN CNRS, UMPA, Ens-Lyon

Anomalous diffusion for a class of systems with two conserved quantities

(joint with G. Stoltz)

We introduce a new class of deterministic dynamics, depending on a potential V, with two conserved quantities (say the energy and the volume). We show by numerical simulations that these systems display an anomalous diffusion of energy with a a non-universal scaling exponent depending on V. Then, we perturb the system by a energy-volume conserving noise.

First, we derive, in the smooth regime, hydrodynamic limits of the perturbed system, which are given by an hyperbolic system of two conservation laws. Secondly, we show that even with the stochastic perturbation, the system has an anomalous diffusion of energy, and we prove it rigorously for $V(r) = r^2$.

THIERRY BODINEAU Ecole Normale Supérieure

On long range correlations in non-equilibrium systems

A generic property of non-equilibrium systems maintained in a steady state by contact with several reservoirs of particles at unequal chemical potentials is the presence of long range correlations. In this talk, we will first discuss the equivalence of ensembles in a non-equilibrium framework and the impact of the long range correlations on this issue. In a second part of the talk, we will show for some specific models how the long range correlations become singular when approaching a phase transition.



JEREMY CLARK University of Helsinki

A Brownian limit perturbed by a microscopic periodic potential

I will talk about a one-dimensional model for a massive particle immersed in a gas of light particles and under the influence of a microscopic periodic potential. The starting point is a Markovian description of the massive test particle dynamics given by a linear Boltzmann equation. In the Brownian limit, the influence of the microscopic potential vanishes, and the rescaled momentum of the particle converges to an Ornstein-Uhlenbeck process. On a smaller scale, the total drift in momentum due to the forcing of the potential converges to a Brownian motion time-changed by the local time at zero of the limiting Ornstein-Uhlenbeck process. Our result is related to the literature on limit theorems for additive functionals of null recurrent Markov processes. This is joint work with Loc Dubois.

STEPHAN DE BIEVRE UFR de Mathématiques et Laboratoire CNRS

 $\label{eq:equilibration} Equilibration, \ generalized \ equipartition, \ and \ diffusion \ in \ dynamical \ Lorentz \\ gases$

Joint work with P. Parris, J. Stat. Phys. (2011).

We demonstrate approach to thermal equilibrium in the fully Hamiltonian evolution of a dynamical Lorentz gas, by which we mean an ensemble of particles moving through a *d*-dimensional array of fixed soft scatterers that each possess an internal degree of freedom to which moving particles locally couple. We show that the momentum distribution of the moving particles approaches a Maxwell-Boltzmann distribution at a certain temperature T, provided that they are initially fast and the scatterers are in a sufficiently energetic but otherwise arbitrary stationary state of their free dynamics–they need not be in a state of thermal equilibrium. In our treatment, the temperature T to which the particles equilibrate is *defined* through a fluctuation-dissipation-like relation that emerges naturally from the microscopic Hamiltonian dynamics as a result of dynamical friction; IT obeys a generalized equipartition relation, in which the associated thermal energy $k_{\rm B}T$ is equal to an appropriately defined average of the scatterers' kinetic energy. In the equilibrated state, particle motion is diffusive.



ABHISHEK DHAR Raman Research Institute

Heat conduction in disordered harmonic lattices with energy conserving noise

We study the nonequilibrium steady state of disordered harmonic lattices whose Hamiltonian dynamics is supplemented by a stochastic flipping of the momentum of every particle. This dynamics conserves energy but not momentum. The one-body and pair correlations in this system are shown to be the same as those obtained for a model with self-consistent reservoirs, after an appropriate mapping of parameters. Results are presented, from numerics and simulations of a 1D system, on the dependence of the heat flux on the system size and on the flipping rate.

JEAN-PIERRE ECKMANN Université de Genève

Rattling and freezing in a 1-D transport model

With Lai-Sang Young, I considered a heat conduction model which is an open system in which particles exchange momentum with a row of (fixed) scatterers without recoil.

The main phenomenon for this model is freezing, which is the slowing down of particles with time although there is no dissipation. Most results evade rigorous mathematical analysis, but several features shed light on the intriguingly slow convergence to a steady state in which all but one particle remain active.

TADAHISA FUNAKI University of Tokyo

An evolutional model of Young diagrams with conservation law

I will introduce an evolutional model of Young diagrams, which conserves the area, and then state a conjecture concerning the hydrodynamic limit for the related particle system. As a step to prove such conjecture, the equivalence of ensembles under a spatially inhomogeneous conditioning is shown. The relation of our result to the so-called Vershik curve will be discussed.

GIOVANNI GALLAVOTTI Universita' di Roma "La Sapienza"

BBGKY hierarchy for hard spheres

The structure of the hierarchy will be discussed in connection with its possible use in the theory of stationary heat transport, its solutions and its ambiguities.



THOMAS GILBERT Université Libre de Bruxelles

A two-stage approach to relaxation in locally confined hard sphere systems

I will describe the characterization of heat transfer in models of high-dimensional billiards, which capture the properties of materials that possess both the spatial structure of solids and collisional dynamics of gases. I will discuss the conditions under which the heat conductivity of these models can be universally expressed in terms of the frequency of collisions between gas particles. This universality manifests itself when interactions between gas particles are rare, though not necessarily small. The dynamical properties of these models will be analyzed, and the role of dimensionality emphasized.

ALEXANDER GRIGO Fields Institute

Mixing properties of a heat conduction model

Consider a linear chain of N particles each carrying an energy. The evolution of the energies is governed by a continuous time, pure jump Markov process. The interactions are nearest neighbor ones preserving the energy. A lower bound (in terms of N) is presented for the spectral gap of the Markov generator under the assumption that the stationary distributions are reversible. This is joint work with K. Khanin and D. Szasz.

MARTIN HAIRER University of Warwick

Toy models of extreme non-equilibrium

We present three simple low-dimensional toy models of extreme non-equilibrium statistical mechanics. The common feature of all models is that they exhibit unexpected behaviour in their approach to stationarity and extreme sensitivity to the fine details of the model.



FRANÇOIS HUVENEERS Université de Paris Dauphine

Energy transport for a particle system interacting through rare collisions

(joint work with S. Olla)

We study a one-dimensional hamiltonian chain of masses, perturbed by an energy conserving noise. According to the hamiltonian part of the dynamics, particles move freely in cells and interact with their neighbors through collisions, made possible by a small overlap between near cells. The noise only randomly flips the sign of the velocity of particles. I will first and mainly consider the case where the size ϵ of the overlap region between cells goes to zero, and where time is rescaled by a factor $1/\epsilon$. We have shown in that case that energy evolves autonomously according to some stochastic equation. If only two different energies are present, the limiting process for the energy actually coincides with the symmetric simple exclusion process. This leads us to think that in a such a case, even when $\epsilon > 0$ is kept constant, the process should converge to a heat equation with a constant diffusion coefficient in the hydrodynamic limit. I'll then briefly expose the present state of our research on that harder problem.

MILTON JARA IMPA

Second-order Boltzmann-Gibbs principle and applications

In a recent work with P. Goncalves, we introduced what we called the second-order Boltzmann-Gibbs principle, and we used it in order to show that KPZ equation appears as the scaling limit of density fluctuations of weakly asymmetric, conservative systems in dimension 1. In this talk we will present other applications of of this principle, ranging from functional limit theorems for additive functionals of particle systems to the derivation of a novel equation, which we call fractional KPZ equation. This equation has the same scaling exponents of the KPZ universality class, but it is qualitatively different from the conjectured scaling limit of the KPZ universality class. This fact makes this equation a candidate for the scaling limit of a still unknown universality class, which we conjecture to have some relation with heat anomalous conduction in dimension d=1. Joint work with P. Goncalves.



GIANNI JONA-LASINIO Universita di Roma La Sapienza

Dynamic and static large deviations in nonequilibrium statistical mechanics: an overview

Large deviations have progressively become a major topic in nonequilibrium statistical mechanics both conceptually and as a technical tool. The aim of this talk is to provide a perspective on the work done by various authors over the last decade and discuss some recent contributions.

ANTTI KUPIAINEN University of Helsinki

Diffusion of a Brownian Quantum Particle

We consider a quantum particle interacting with a free field environment and prove its motion is diffusive. Examples of environments are 3d optical or acoustic phonons. Our proof is based on a novel quantum renormalization group. Joint work with W. De Roeck

YOUNG LAI-SANG New York University

Nonequilibrium steady states of certain dynamical models

I will summarize the results of a few studies, part numerical and part analytical, that I started with J-P Eckmann and continued with K Lin and P Balint the last several years. Most of the discussion will pertain to a class of mechanical models characterized by large arrays of rotating disks and many moving particles. Energy exchange occurs at particle-disk collisions. We assume the system is coupled to unequal heat reservoirs, and is in a nonequilibrium steady state. Easy-to-compute algorithms for macroscopic quantities such as energy and particle density profiles are proposed, and relations between memory, finite-size effects, and geometry are discussed. We find numerically that these models, which have chaotic local dynamics, tend quickly and robustly to local thermal equilibria. To demonstrate that LTE cannot be taken for granted, I will discuss briefly a second model with "integrable" dynamics and anomolous behavior.



CLAUDIO LANDIM IMPA

Action functional and quasi-potential for the Burgers equation in a bounded interval

Consider the viscous Burgers equation $u_t + f(u)_x = \varepsilon u_{xx}$ on the interval [0, 1] with the inhomogeneous Dirichlet boundary conditions $u(t,0) = \rho_0$, $u(t,1) = \rho_1$. The flux f is the function f(u) = u(1-u), $\varepsilon > 0$ is the viscosity, and the boundary data satisfy $0 < \rho_0 < \rho_1 < 1$. We examine the quasi-potential corresponding to an action functional, arising from non-equilibrium statistical mechanical models, associated to the above equation. We provide a static variational formula for the quasi-potential and characterize the optimal paths for the dynamical problem. In contrast with previous cases, for small enough viscosity, the variational problem defining the quasi potential admits more than one minimizer. This phenomenon is interpreted as a non-equilibrium phase transition and corresponds to points where the super-differential of the quasi-potential is not a singleton.

JOEL LEBOWITZ Rutgers University

Transport Processes in Macroscopic Systems

I will give an overview of various approaches, heuristic and rigorous, to the derivation from microscopic models of macroscopic equations describing the transport of locally conserved quantities.

RAPHAEL LEFEVERE Université Denis Diderot (P7)

Large deviations of the current in collisional dynamics

We argue that the large deviations of the current in a class of deterministic collisional dynamics display a peculiar behavior.



MARCO LENCI University of Bologna

Quenched results for random Lorentz tubes

A Lorentz tube is Lorentz gas in a *d*-dimensional set which is infinitely extended in one dimension only (e.g., a strip in d = 2, a square cylinder in d = 3, etc.). One expects that, in great generality, a system like this possesses strong chaotic properties—whatever this means for dynamical systems preserving an infinite measure. At the very least, the typical Lorentz tube should be recurrent. By randomly placing scatterers within the tube, in a natural way, we construct *ensembles* of (deterministic) dynamical systems. In other words we define *quenched random Lorentz tubes*. Under some general hypotheses (very mild for d = 2; a little stronger for d > 2) we prove that every system in the ensemble is uniformly hyperbolic and almost every system is recurrent, ergodic, and such that the first-return map to any given scatterer is *K*-mixing.

(Joint papers with G. Cristadoro, M. Seri, M. Degli Esposti, S. Troubetzkoy.)

CARLANGELO LIVERANI U. Roma Tor Vergata

Fourier law and the weak coupling limit

I consider lattices of weakly interacting Hamiltonian systems. In the limit of vanishing interactions one can show that the systems are essentially in local equilibrium and hence one can deduce, in the appropriate time scale, an autonomous equation for the evolution of the energies alone. One can then try to study such an equation to deduce the Fourier law or the heat equation (works in collaboration with S.Olla and D.Dolgopyat).

JANI LUKKARINEN University of Helsinki

Kinetic description of a homogeneous Bose fluid with condensate

We consider the kinetics of a three-dimensional fluid of weakly interacting bosons with supercritical densities. More precisely, we consider the postulated nonlinear Boltzmann-Nordheim equations for this system, in a spatially homogeneous state which has an isotropic momentum distribution. The resulting evolution equations have a surprisingly rich mathematical structure, where proper definitions play an important role. Elaborating on previous results, we propose a definition of the coupled equations for which the thermal equilibrium states are stationary. To test the validity of the equations, we study the global existence and uniqueness of solutions, as a problem about return to equilibrium from a perturbation of a thermal state with a condensate.



STEFANO OLLA CEREMADE

Energy diffusion: linear response and hydrodynamic limits

How to derive macroscopic heat diffusion, governed by heat equation (or, in the stationary situation, Fourier's law), from microscopic hamiltonian dynamics? I will give some mathematical precise statement of what we would like to prove, and what could be the "ingredients" of the proof, in particular linear response in equilibrium. Then I will expose some mathematical results that can be obtained if are allowed some energy conserving stochastic perturbation of the hamiltonian dynamics. I will also expose some surprising numerical results about the "thermal conductivity" of a chain of rotors, when it is really far from equilibrium.

BRIAN RYALS New York University

Nonequilibrium steady state of a 1D mechanical chain

I will discuss a one-dimensional chain of colliding particles that comes from a mechanical system and compare it with a stochastic model. The macroscopic energy profile and local marginals will be examined numerically. Of interest is that the local distributions are not Gibbsian. Deviation from linearity due to unequal particle speeds along the chain will also be discussed.

GUNTER SCHÜTZ Institut für Festkörperforschung

Space-time correlations in the ASEP conditioned on carrying a large flux

We show that in the asymmetric simple exclusion process (ASEP) on a ring, conditioned on carrying a large flux, the particles experience an effective long-range potential which is similar to the effective potential between the eigenvalues of the circular unitary ensemble in random matrices. We obtain the quasi stationary measure, transition probabilities and spatio-temporal correlations using the Bethe ansatz and determinantal free fermion techniques. (joint work with V. Popkov, D. Simon)



IMRE PÉTER TÓTH Technical University of Budapest

A heat conduction model with localized billiard disks and weak interaction forces

I present a heat conduction model motivated by the Gaspard-Gilbert model. It consists of billiard disks confined by fixed scatterers near lattice points, which exchange energy with their neighbours through some weak potential.

The model is studied in the two-stage approach of Gaspard and Gilbert. First we take a weak coupling limit with the appropriate time scaling to obtain a Markov process for the energies, i.e. an interacting particle system. Second, we try to understand the hydrodynamic behaviour of this interacting particle system – which is very similar to the one that Dolgopyat and Liverani obtain as the weak coupling limit of their interacting smooth Hamiltonian systems. The discussion contains many heuristic arguments. I will try to argue why the model is in some sense "better" than the original Gaspard-Gilbert model.

TATIANA YARMOLA Fields Institute

Ergodic properties of some canonical systems driven by thermostats

We consider a periodic bounded horizon dispersing billiard table with scatterers acting as thermostats set at possibly different temperatures. That is, whenever a particle with velocity v hits a thermostat, the tangential component v_t of the particles velocity is instantaneously set to a random value sampled from the distribution with density $\sqrt{\frac{\beta}{\pi}}e^{-\beta v_t}$, where $T = 1/\beta$ represents the thermostat temperature; the perpendicular component of the velocity changes sign. The particles do not interact with each other. For certain classes of such systems, we will discuss existence, uniqueness, and ergodic properties of the stationary measures.

LORENZO ZAMBOTTI Universite Pierre et Marie Curie

Large deviations of stochastic collisional dynamics

We discuss some surprising features of the Donsker-Varadhan large deviations principle of a class of simple stochastic dynamics. We wish to explain some of the results presented in Raphael Lefevere's talk with the study of the large deviations functional of the empirical measure of o class of piecewise-deterministic Markov processes.