Stochastic control methods for risk management and portfolio optimization

Fields Program on Quantitative Finance, April-June 2010

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Lecture Dates Course starts on April 21 at the Fields Institute weekly on Wednesdays from 9:30 to 12:15 and then again from 1:30 to 4:15 for 6.5 weeks.

Guest Lectures It is a great pleasure to have Bruno Bouchard (University Paris Dauphine), Mete Soner (ETH Zurich), and Agnès Tourin (Fields Research Immersion Fellow), giving advanced lectures as an integral part of the course. Bruno will be giving the afternoon sessions of the 5th and the 12th of May. Mete will be giving the afternoon sessions of the 26th of May. Agnès will be giving the afternoon session of June 2.

Outline Decision problems in finance, among many other applications, are usually formulated in terms of optimization in the context of dynamic continuous-time models. This PhD level course addresses the general theory of stochastic control and the most recent connections with partial differential equations (PDEs) and backward stochastic differential equations (BSDEs), together with relevant applications in finance.

We first consider the control problem of Markov diffusions. The verification technique requires elementary technical skills from stochastic calculus, and allows readily to solve the simplest portfolio optimization problem formulated by Merton in 1969. In order to address the absence of a priori regularity, we provide a self-contained introduction to the theory of viscosity solutions of second order PDEs. Then, the dynamic programming approach allows to obtain a characterization of the value function by means of the so-called Hamilton-Jacobi-Bellman equation. This level of technicality is needed for instance to solve the hedging problem under portfolio constraints, gamma, or illiquidity risk.

The second part of the course is dedicated to the theory of backward stochastic differential equations and their connection with stochastic control and semilinear partial differential equations. We provide various applications to hedging, portfolio optimization, and risk measurement.

The third part of the course focuses on numerical probabilistic methods for nonlinear PDEs suggested by BSDEs. The algorithms can be seen as an extension of well-established numerical methods for American options.

The final part of the course adresses the extension of BSDEs to the second order. This allows for a connection with fully nonlinear PDEs, and thus provides a representation of stochastic optimal control problems. A relevant financial application is the problems of hedging under uncertain volatility (and correlation). This extension also opens the door for a more general class of risk measures which account for the volatility risk.

Prerequisites Student are expected to be familiar with Brownian motion, the corresponding stochastic calclus, stochastic differential equations, and the basic modeling concepts in continuous-time finance. A suitable textebook is:

• Shreve, S. *Stochastic Calculus for Finance, Volume II: Continuous-time Finance*, Springer.

References Some references with a simplified presentation are:

• Pham, H. (2009). *Continuous-time Stochastic Control and Optimization with Financial Applications*. Springer, Berlin.

• El Karoui, N., Peng, S. and Quenez, M.-C. (1995). Backward stochastic differential equations in finance. *Mathematical Finance* 7, 1-71.

• Fleming, W.H., Soner, H.M. (1993). *Controlled Markov Processes and Viscosity Solutions*. Applications of Mathematics 25. Springer-Verlag, New York.

• Peng, S. (2007) *G-Brownian motion and dynamic risk measure under volatility uncertainty*, arXiv:0711.2834v1.

Assignment Papers reading and presentation.

Evaluation Papers reading and presentation.