State-Price Densities in the Commodity Market and Its Relevant Economic Implications

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(Incomplete and all comments are welcome.)

Motivation

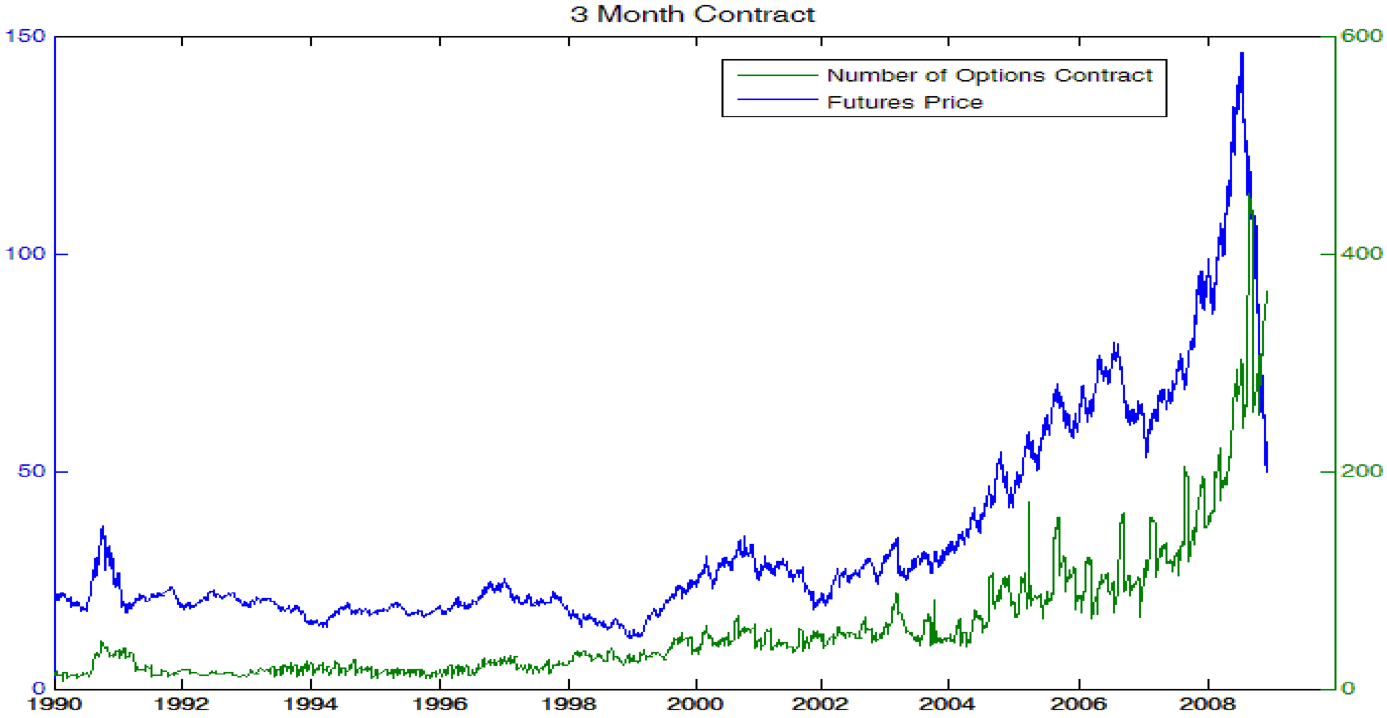
"From a pricing perspective, SPDs are "sufficient statistics" in an economic sense - they summarize all relevant information about preferences and business conditions for purposes of pricing financial securities."

--- Aït-Sahalia and Lo (1998, JF)

- SPDs reflect the beliefs of investors about the likelihood of possible states and their preferences towards these states.
- SPDs have important implications for derivative pricing.
- SPDs provide information about how the commodity market is segmented from other financial asset markets.

Motivation

Commodity derivatives market grows very fast: OTC commodity derivatives contracts is $6.4 trillion in 2006, about 14 times the value in 1998. (Bank for International Settlements, 2006)



Crude oil futures price and number of options written on 3-month contract.

Agenda

- Introduction
  - Concept Definition
  - Main Findings
  - Literature Review
- Methodology
  - Econometric Methodology
- Data
- Empirical Results
- Discussion

Concept Definition

State Price Densities (SPDs) ξ is defined as

$$W_0 = E[\xi_T W_T]$$

(Arrow-Debreu price of per unit of probability.)

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A call option can therefore be priced as:

$$\begin{aligned} C(F_t, K, t, T) &= E[\xi_T (F_T - K)^+ | \mathcal{F}_t] \\ &= \int_K^\infty \widehat{\xi}_T(x)(F_T - K) P(F_T = x | \mathcal{F}_t) dx \end{aligned} \tag{2}$$

Concept Definition

For a call option,

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$$\begin{aligned} C(F_t, K, t, T) &= e^{-r(T-t)} E^Q[(F_T - K)^+] \\ &= e^{-r(T-t)} \int_K^\infty (F_T - K) P^Q(F_T = x | \mathcal{F}_t) dx \end{aligned} \quad (3)$$

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Projection of ξ onto the crude oil futures market is

$$\widehat{\xi}(F_T; \mathcal{F}_t) = e^{-r(T-t)} \frac{P^Q(F_T | \mathcal{F}_t)}{P(F_T | \mathcal{F}_t)} \quad (4)$$

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$$P^Q(F_T | \mathcal{F}_t) = e^{r(T-t)} \frac{\partial^2 C(F_t, K, t, T)}{\partial K^2} \Big|_{K = F_T} \quad (5)$$

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Main Findings

Risk Neutral Densities

- Strongly deviate from the normal distribution;
- Skewness could be *either negative or positive* depending on maturity, slope and volatility;
- Distributions tend to be negatively skewed for short maturity contracts and positively skewed for long maturity contracts.

State Price Densities

- U-Shape SPDs. Investors assign high values for states with very high or low returns.
- Shape of SPDs varies with volatilities, implying the importance of incorporating stochastic volatility into options pricing.

Literature Review

Equity Index Options

- RNDs: Backus et al. (1997), Aït-Sahalia and Lo (1998,JF), Aït-Sahalia and Lo (2000, J. of Econometrics (JEs)), Jackwerth (2000, RFS), Aït-Sahalia and Duarte (2003, JEs), Yatchewa and W. Härdle (2006, JEs), Härdle and Hlávka (2009, JEs).
- SPDs: Rosenberg and Engle (2002, JF), Bakshi, Madan and Panayotov (2009, JFE)

Interest Rate Derivatives

- RNDs: Beber and Brandt (2006, JME)
- RNDs and SPDs: Li and Zhao (2009, RFS)

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Grey-colored papers use parametric methods.

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Green-colored papers find U-shape pricing kernel or SPDs..

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Econometric Methodology

1. For a given level of conditional variables v={slope, volatility}, I collect those observations of call options on different dates whose conditional variables are within a window:

$$W(v,h) = \{1 \leq i \leq n, v_i \in [v - h^*, v + h^*]\}.$$

(The optimal window size h* is chosen by iteration)

2. I filter the data according to slope and convexity constraints.

$$\min_m \sum_i (m_i - c_i)^2,$$

s.t.

$$0 \geq \frac{c_{i+1} - c_i}{x_{i+1} - x_i} \geq -1, i = 1, ..., n-1$$

$$\frac{c_{i+1} - c_i}{x_{i+1} - x_i} \geq \frac{c_i - c_{i-1}}{x_i - x_{i-1}}, i = 2, ..., n-1$$

Econometric Methodology

3. With the filtered data, I use the locally linear approach to estimate risk neutral densities (Aït-Sahalia and Duarte, 2003).

$$\min_{\beta_0,\beta_1} \sum_{i=1}^{n} \{m_i - \beta_0(x,v) - \beta_1(x,v) \times (x - x_i)\}^2 K_h(x_i - x, v_i - v)$$

where $K_h(x_i - x, v_i - v)$ is a joint kernel function and h is the bandwidth.

$$\frac{\partial^2 \widehat{m}(x,v)}{\partial x^2} = \frac{\partial \beta_1(x,v)}{\partial x}.$$

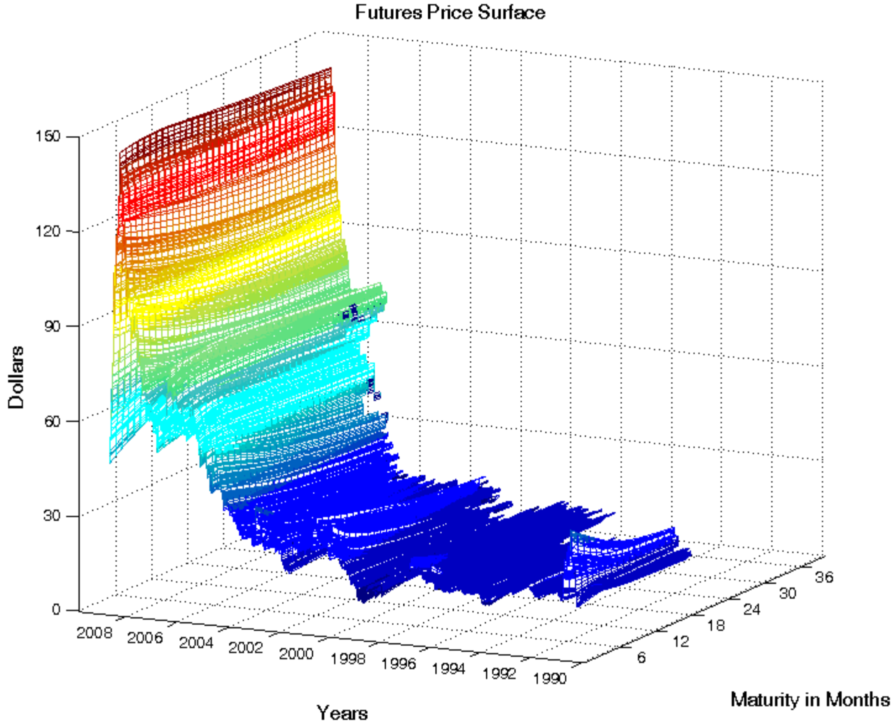
We choose the optimal bandwidth by:

$$\min_{h} \int \min_{h_x(v)} E^Q[\widehat{m}_h(x,v) - \widehat{m}_g(x,v)]^2 dP(v)$$

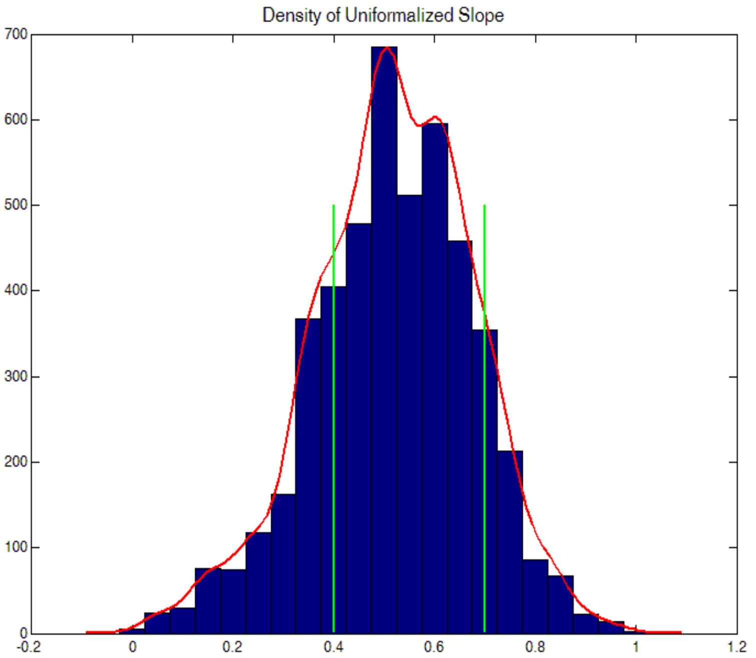
4. I get the physical densities using the standard kernel method.

Data

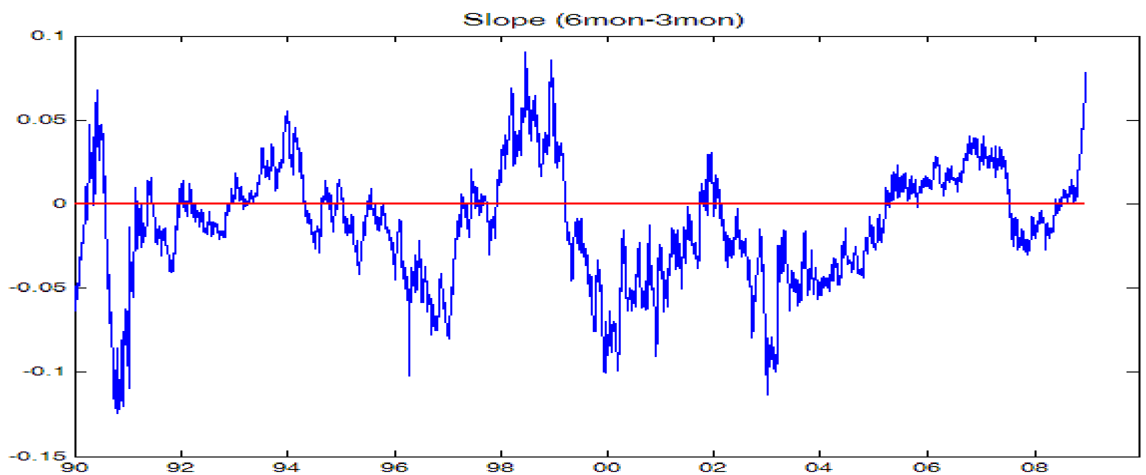
- Crude oil futures and options from NYMEX
  - Jan 02, 1990 – Dec 03, 2008
- Focus on 4 maturities: 3mn, 6mn, 12mn and 24mn.
- Choose call options data according to:
  - Jan 02, 1990 – Dec 14, 2006, keep data with open interest > 100 and price > $0.01;
  - Dec 15, 2006 – Dec 03, 2008, keep data with price > $0.01 (no open interest available).
- Conditional factors are calculated from futures:
  - Slope(t) = log[P(t;6mn)/P(t;3mn)]; (Kogan, Livdan and Yaron,2009,JF)
  - Volatility is from a leverage effect GARCH model for each maturity of futures;
  - All factors are adjusted to have a uniform distribution on [0,1].



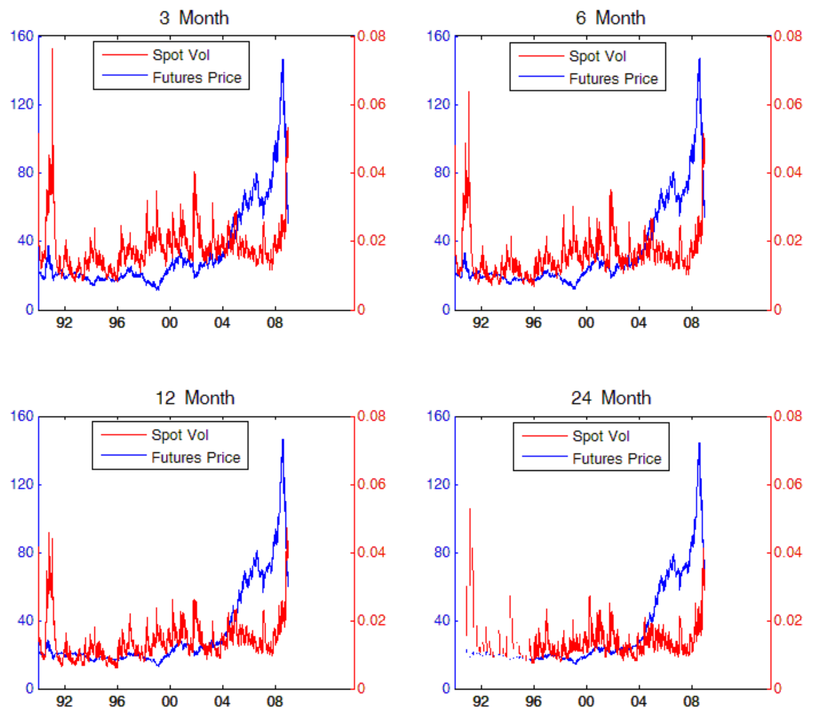
Futures Price Surface
Dollars
150
120
90
60
30
0
2008
2006
2004
2002
2000
1998
1996
1994
1992
1990
Years
6
12
18
24
30
36
Maturity in Months



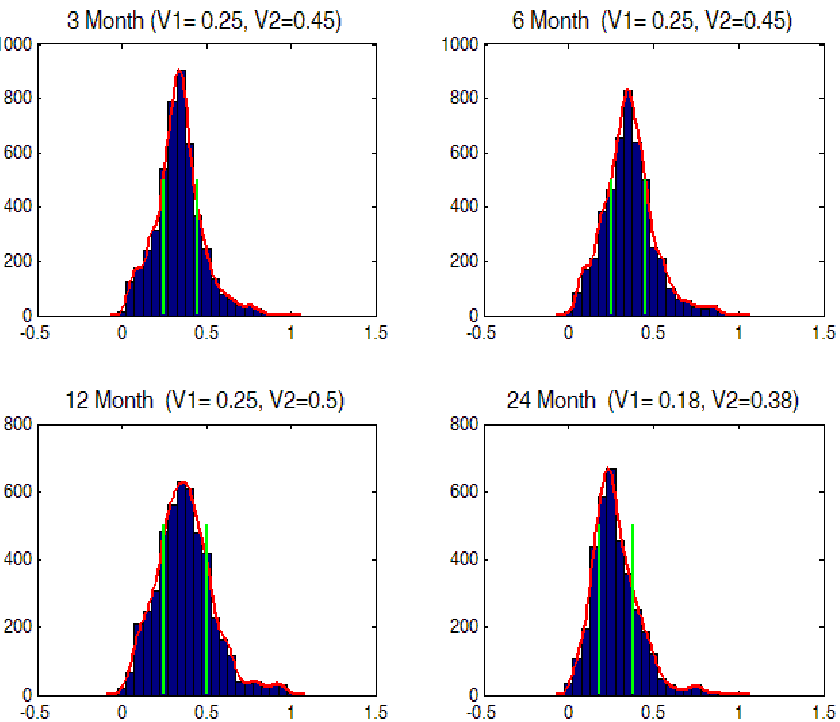
Density of Uniformalized Slope
700
600
500
400
300
200
100
0
-0.2
0
0.2
0.4
0.6
0.8
1
1.2



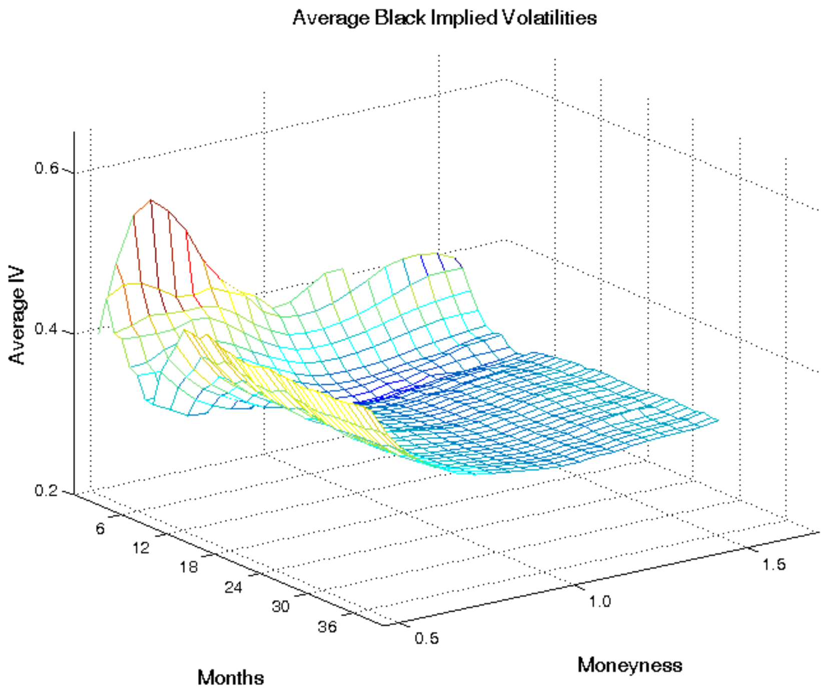
Slope (6mon-3mon)
0.1
0.05
0
-0.05
-0.1
-0.15
90
92
94
96
98
00
02
04
06
08



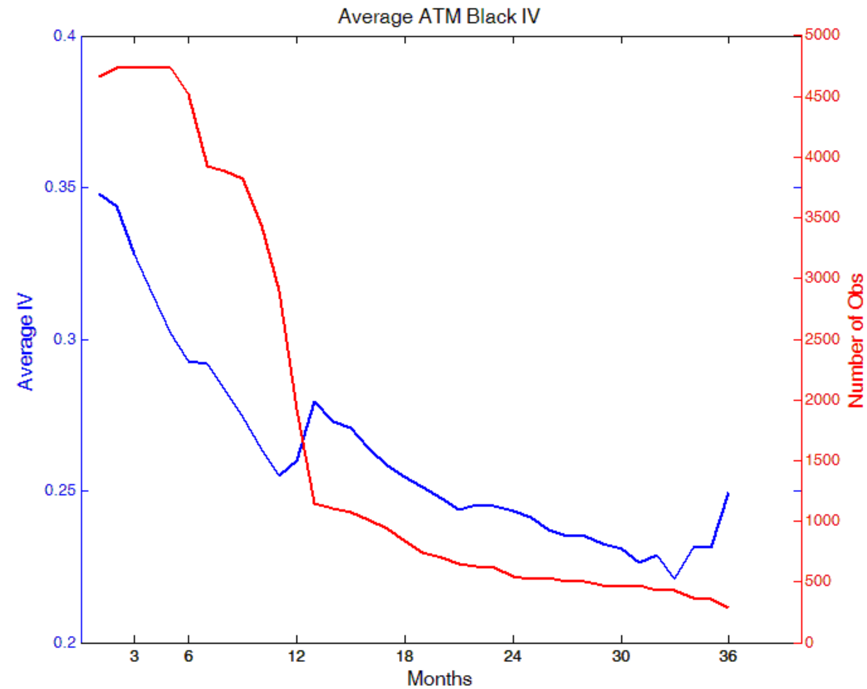
Time series of futures price and conditional volatility



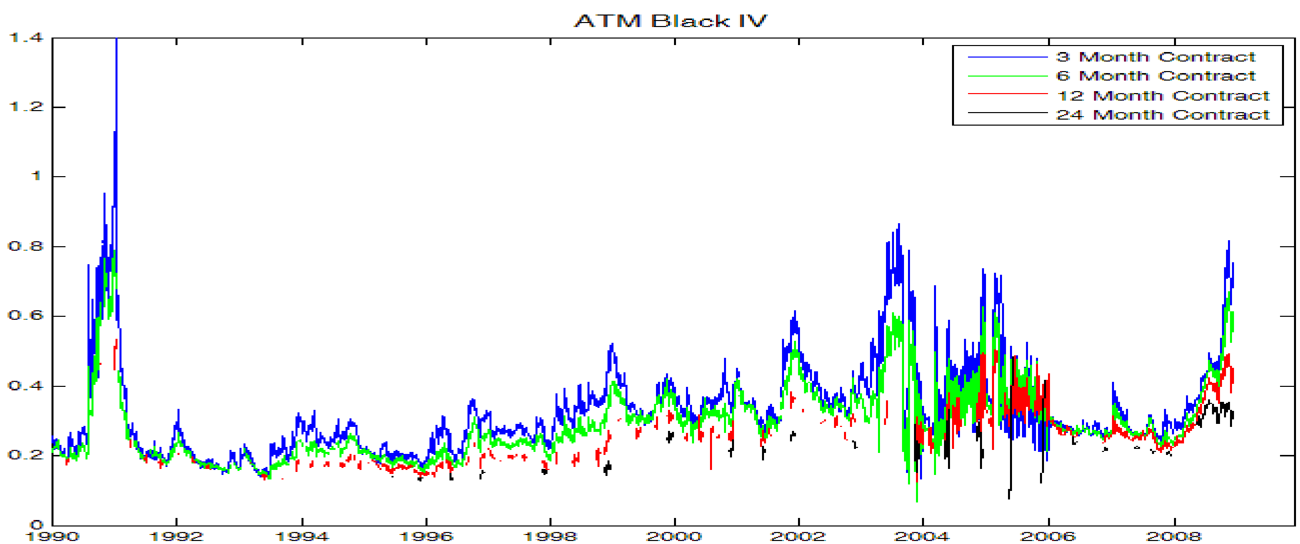
Density of conditional volatility (uniformed)



Average Black Implied Volatilities
Average IV
0.6
0.4
0.2
6
12
18
24
30
36
Months
0.5
1.0
1.5
Moneyness



Average ATM Black IV
Average IV
0.4
0.35
0.3
0.25
0.2
3
6
12
18
24
30
36
Months
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
Number of Obs

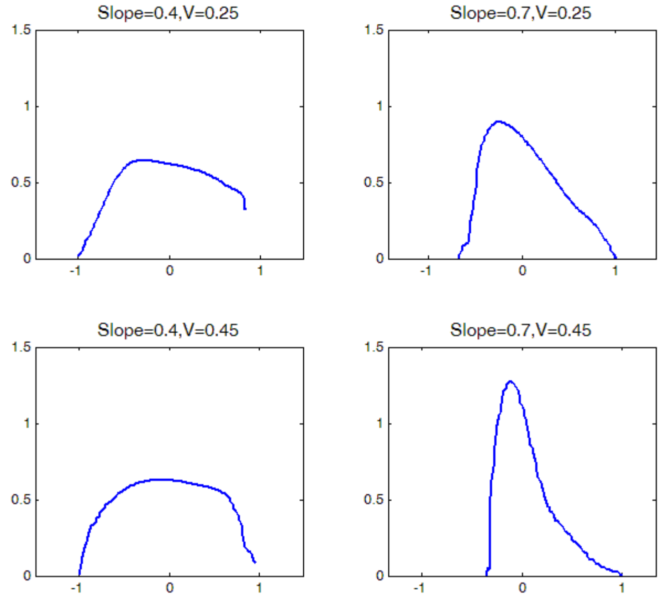


ATM Black IV
3 Month Contract
6 Month Contract
12 Month Contract
24 Month Contract
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008

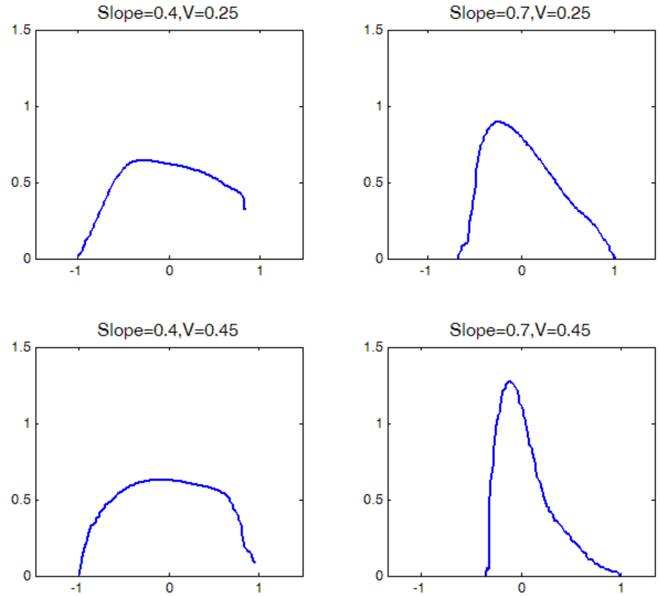
Empirical Results

- Risk Neutral Densities for 3mn, 6mn, 12mn and 24mn contract conditional on slope and volatility factors
- Physical Densities for 3mn, 6mn, 12mn and 24mn contract conditional on slope and volatility factors
- State Price Densities for 3mn, 6mn, 12mn and 24mn contract conditional on slope and volatility factors

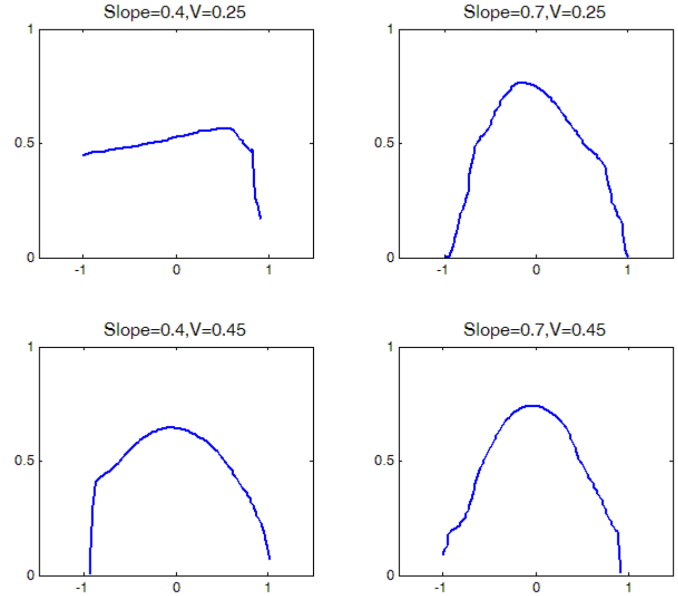
3 Month



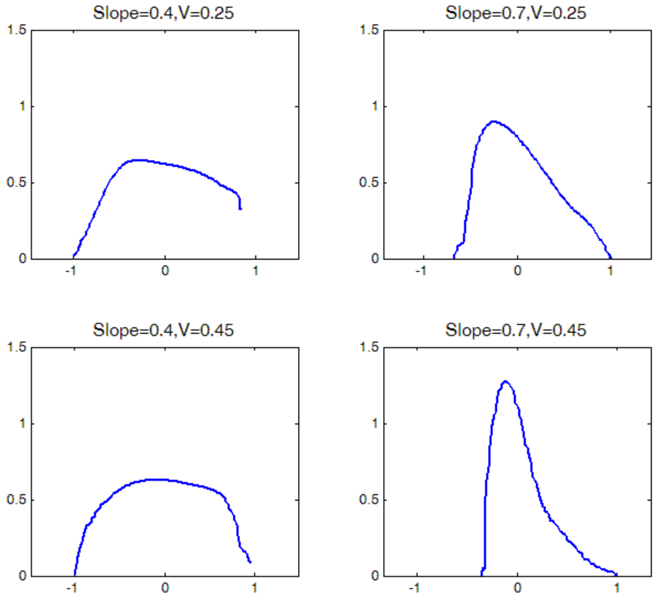
3 Month



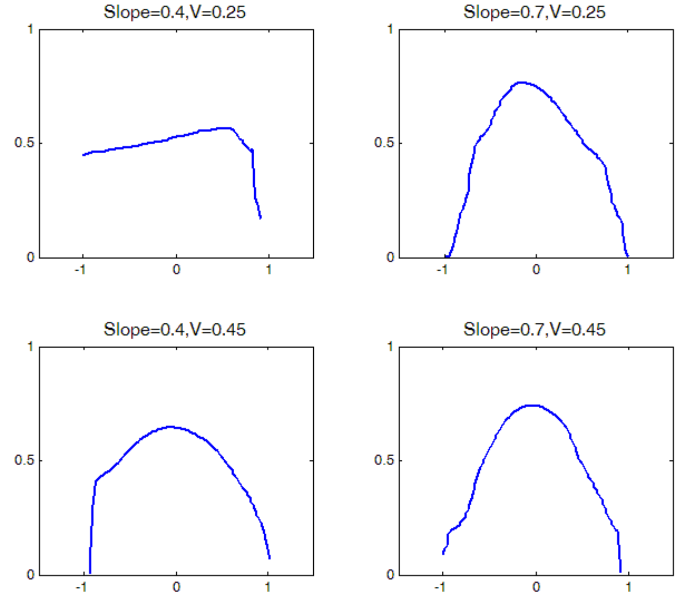
6 Month



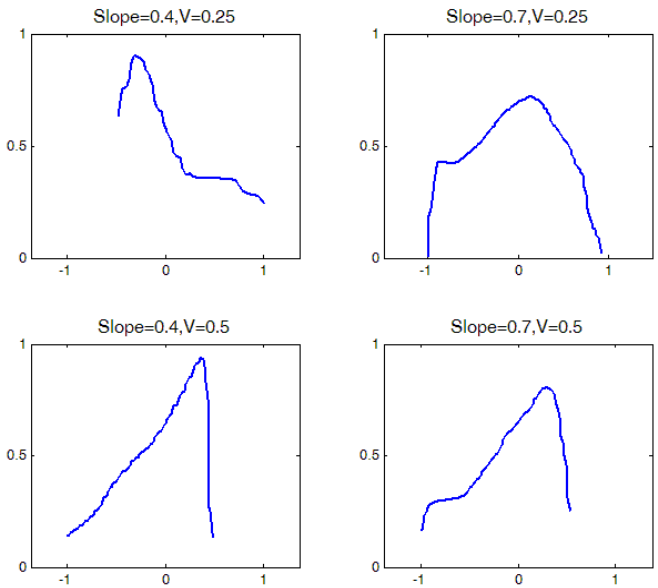
3 Month

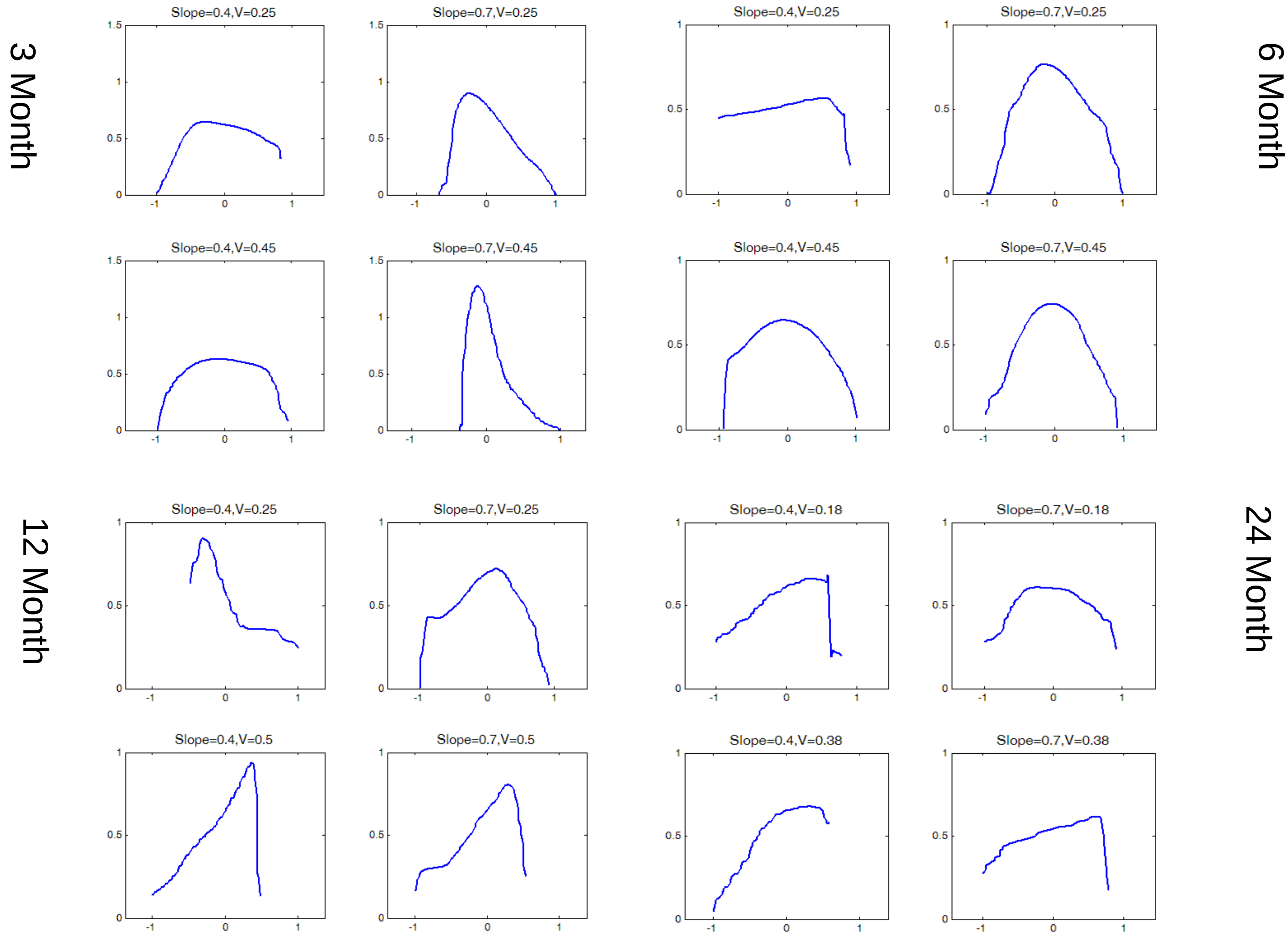


6 Month

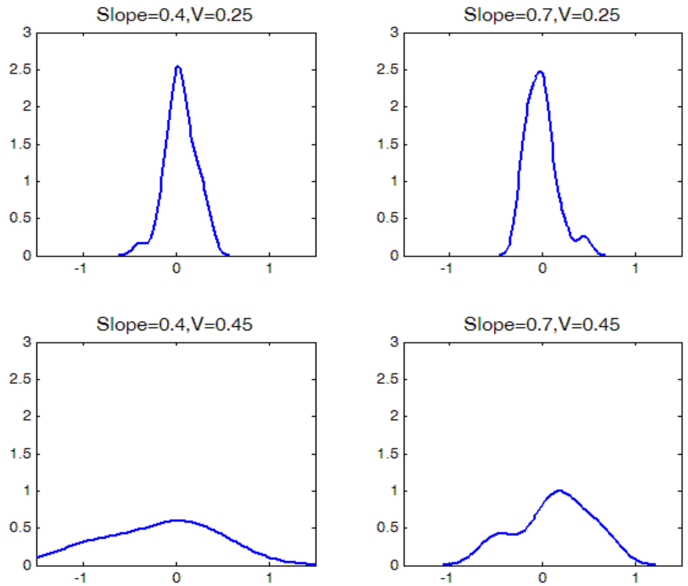


12 Month

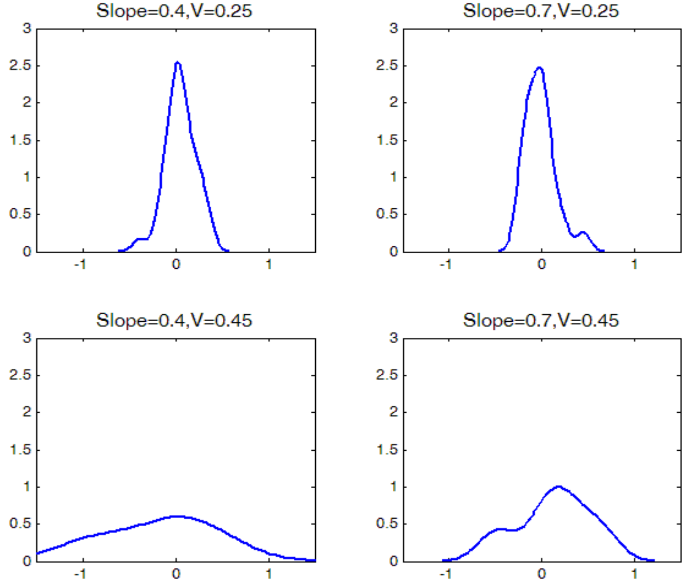


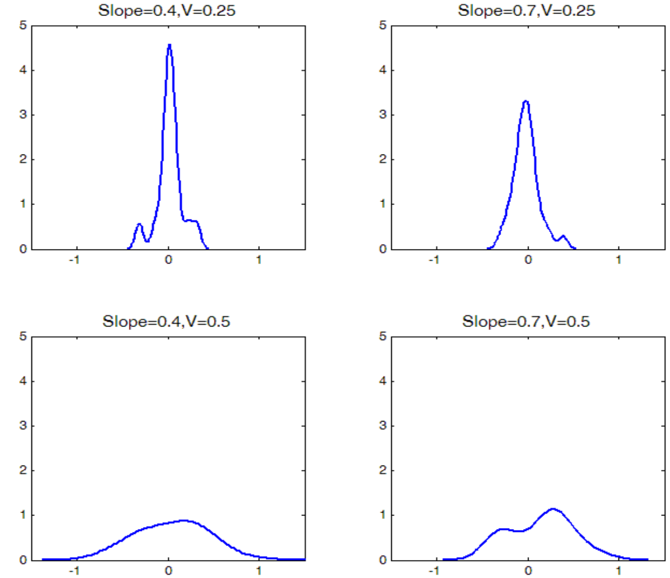

3 Month
Slope=0.4,V=0.25
Slope=0.7,V=0.25
Slope=0.4,V=0.45
Slope=0.7,V=0.45
6 Month
Slope=0.4,V=0.25
Slope=0.7,V=0.25
Slope=0.4,V=0.45
Slope=0.7,V=0.45
12 Month
Slope=0.4,V=0.25
Slope=0.7,V=0.25
Slope=0.4,V=0.5
Slope=0.7,V=0.5
24 Month
Slope=0.4,V=0.18
Slope=0.7,V=0.18
Slope=0.4,V=0.38
Slope=0.7,V=0.38

3 Month



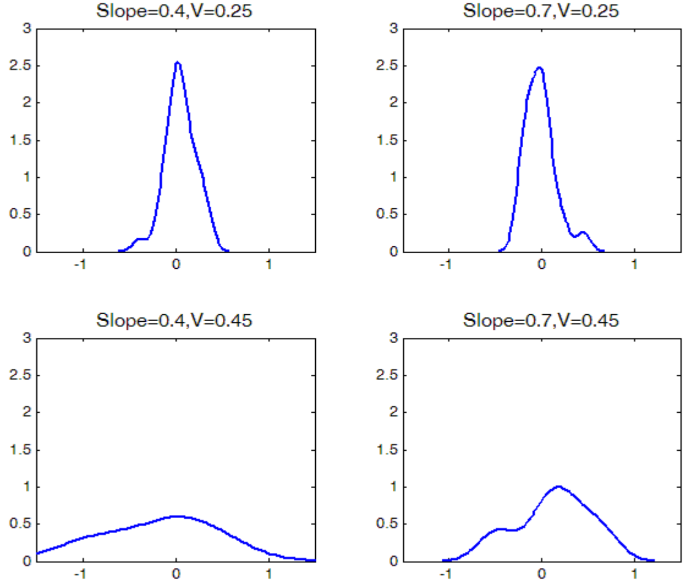
3 Month



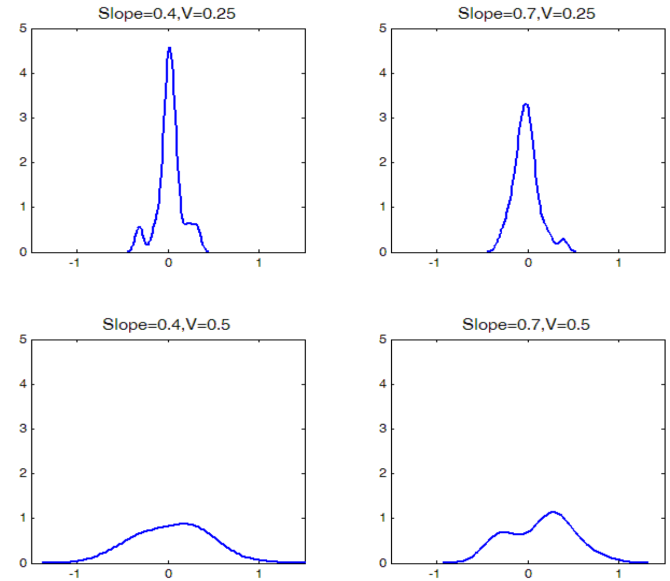


6 Month

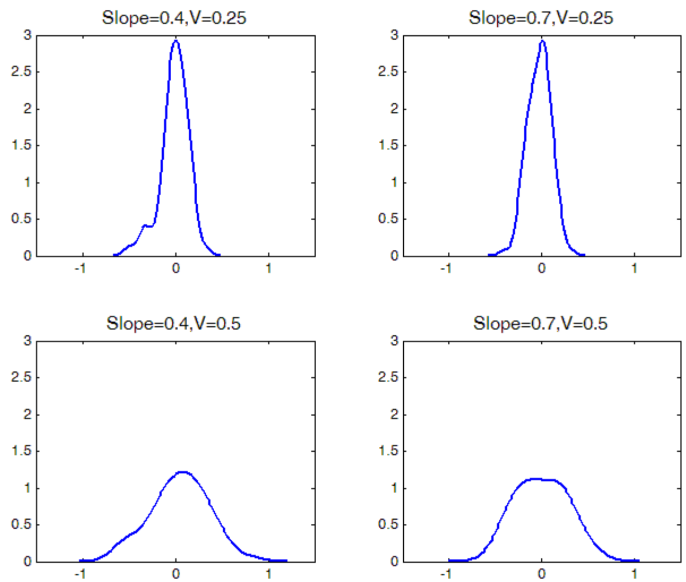
3 Month

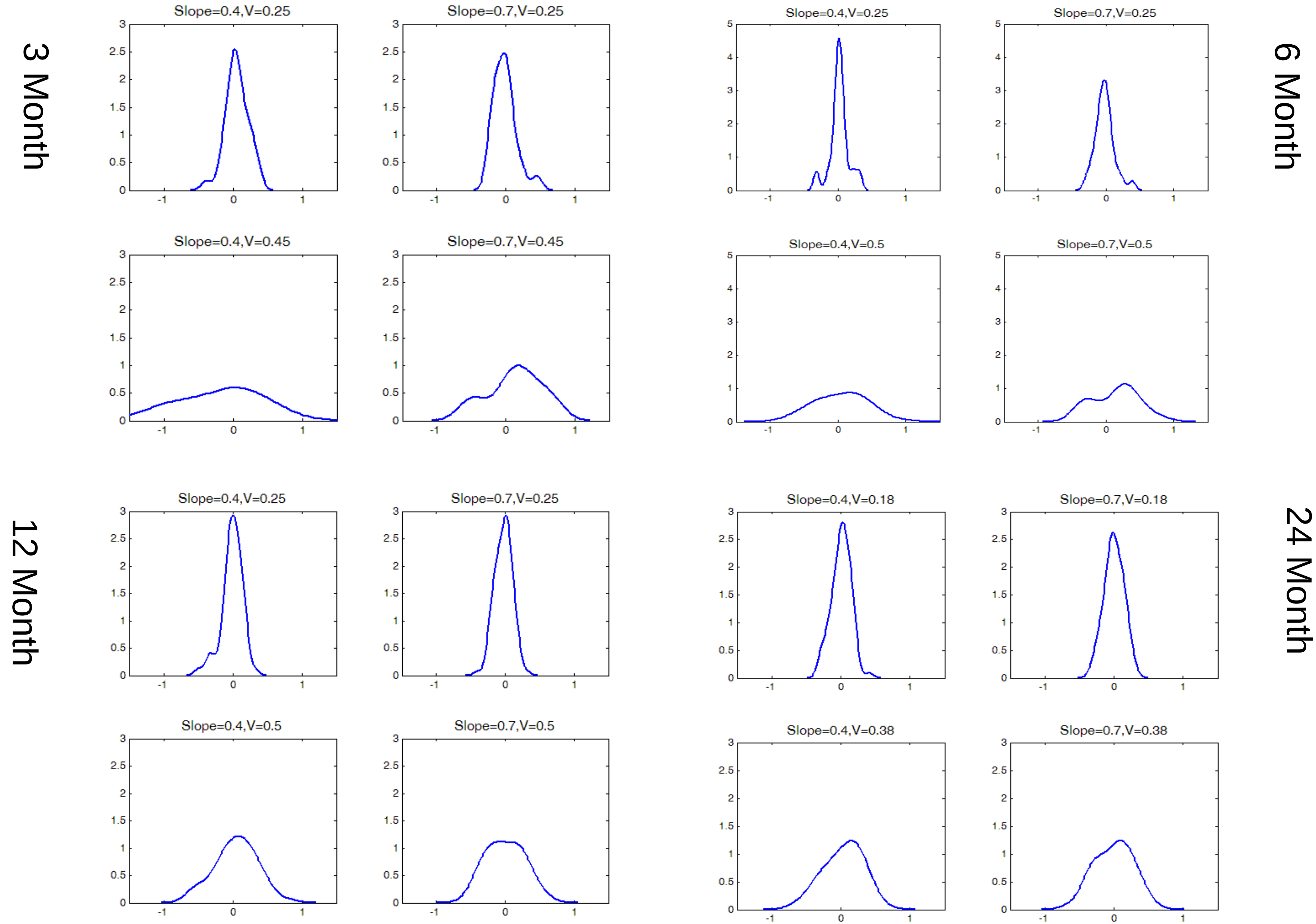


6 Month



12 Month




3 Month
Slope=0.4,V=0.25
Slope=0.7,V=0.25
Slope=0.4,V=0.45
Slope=0.7,V=0.45
6 Month
Slope=0.4,V=0.25
Slope=0.7,V=0.25
Slope=0.4,V=0.5
Slope=0.7,V=0.5
12 Month
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Slope=0.7,V=0.18
Slope=0.4,V=0.38
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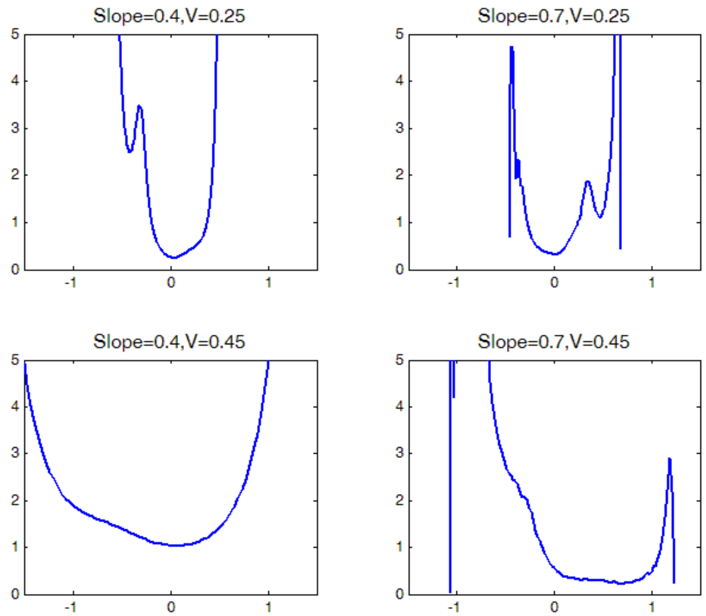
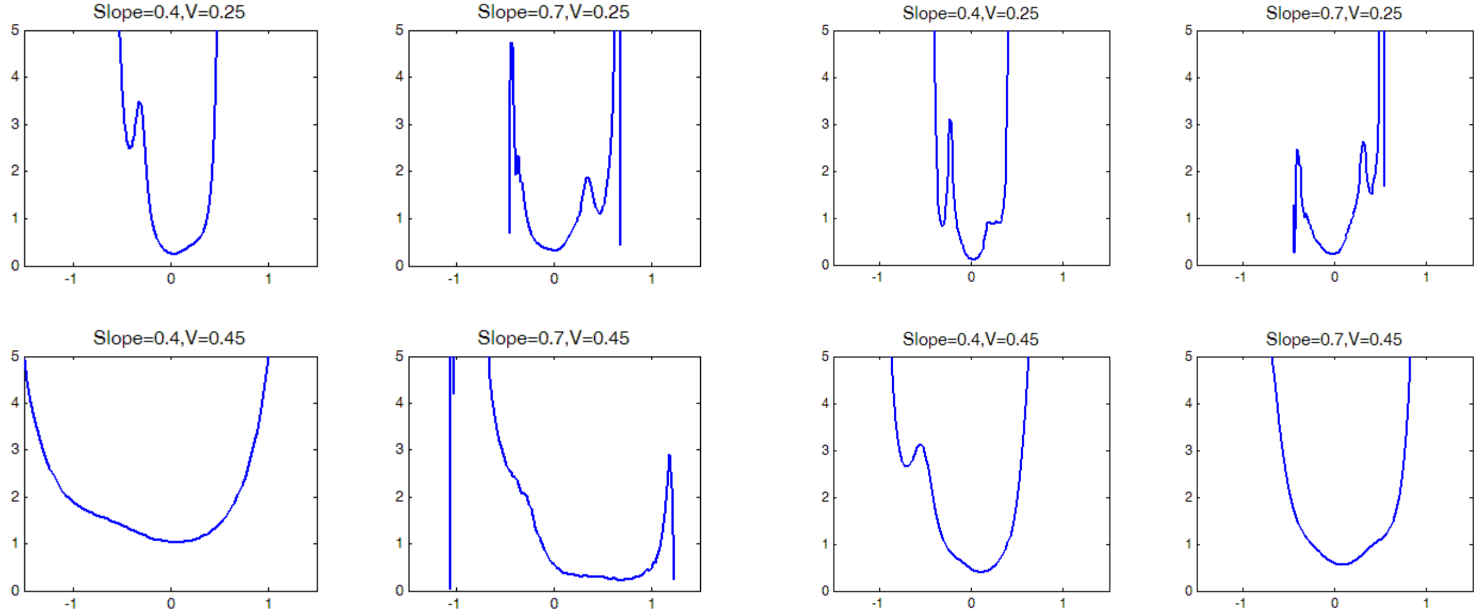
3 Month



3 Month

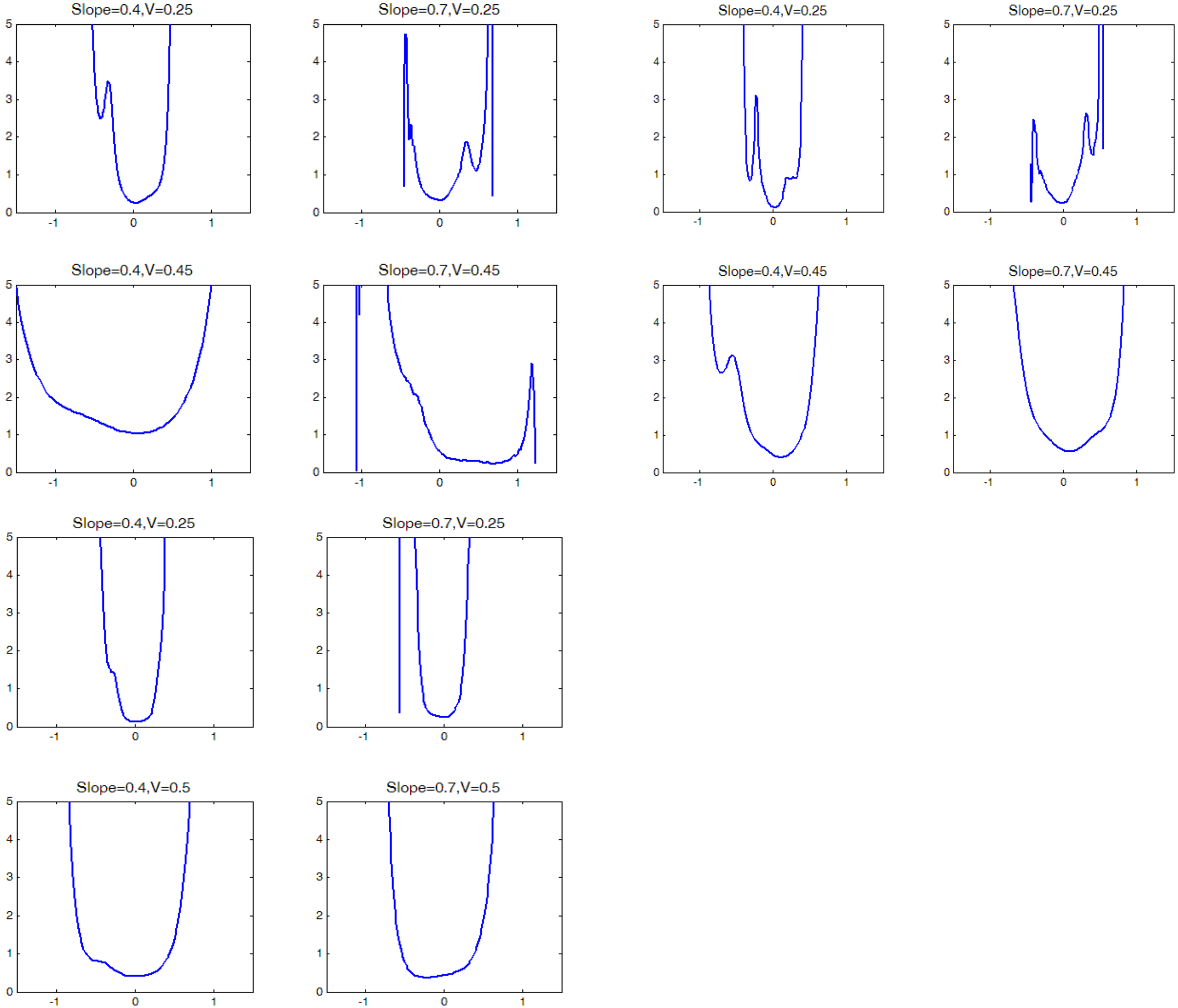


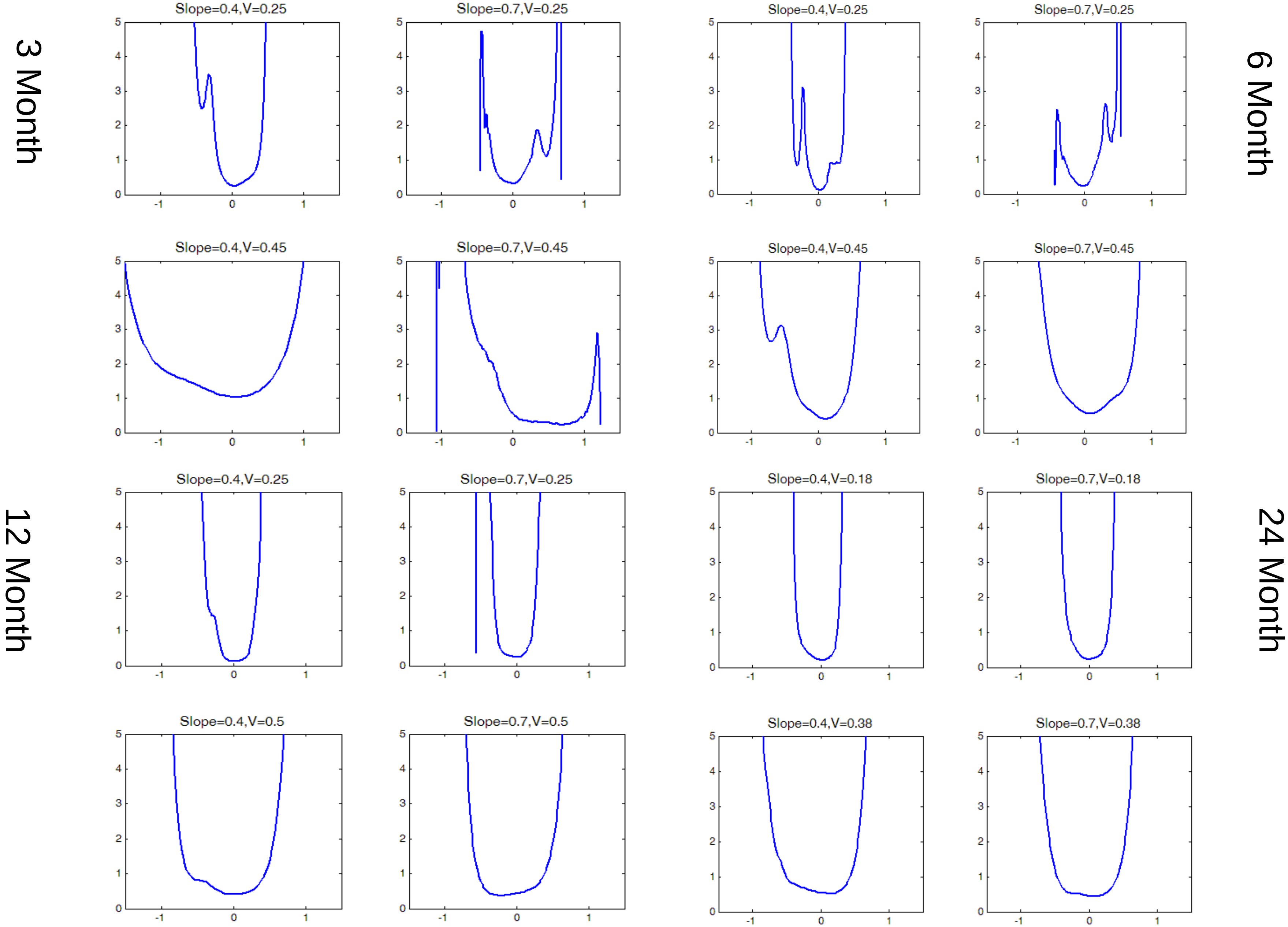
6 Month

3 Month

6 Month

12 Month




3 Month
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Slope=0.7,V=0.25
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Slope=0.7,V=0.18
Slope=0.4,V=0.5
Slope=0.7,V=0.5
Slope=0.4,V=0.38
Slope=0.7,V=0.38

Discussion

- Estimated SPDs are not smooth for many cases, especially for some short maturity contracts;
- Comparison with parametric method.

- Compare the risk neutral densities and SPDs during 2007-2008 and other periods.
- Infer investors' expectation and preference by examining the time evolution of risk neutral densities (dynamics of mean and variance), especially for the period of 2007-2008.

Other economic implications to explore?