Improved Modeling of Double Default Effects in Basel II - An Endogenous Asset Drop Model without Additional Correlation

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Agenda

Double Default Effects and Basel II

IRB Treatment of Double Default Effects

Asset Drop Model

Summary

Sebastian Ebert (Bonn): Improved Modeling of Double Default Effects in Basel II
Credit Risk in Basel II

Internal Ratings Based (IRB) approach under Pillar 1 of Basel II:

Benchmark model to quantify minimal capital requirements for portfolio credit risk

- since 2007 binding for all banks in the European Union
- minimal capital requirement is the 99.9% value-at-risk of the credit portfolio loss distribution
- based on a conditional independence framework: Asymptotic Single Risk Factor (ASRF) model by Gordy (2003) where all idiosyncratic risk is assumed to be diversified away
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Double Default

Hedged exposures are lost if

1. the obligor defaults AND
2. the guarantor defaults. Thus: “double default”

Hedging Instruments: Credit Derivatives such as CDS, collateral securitization, guarantees...

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We

1. reveal structural weaknesses of the IRB treatment of double default effects and any additional correlation approach,

2. propose a new asset drop model that addresses all mentioned weaknesses and which is

3. just as easily applicable as it does not pose extensive data requirements and economic capital can still be computed analytically.
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Additional Correlation Approach under Basel II

The normally distributed asset returns $r_n$ and $r_{gn}$ of obligor $n$ and its guarantor are no more conditionally independent on the systematic risk factor $X$ but

$$r_n = \sqrt{\rho_n}X + \sqrt{1-\rho_n}\left( \sqrt{\psi_{n,gn}}Z_{n,gn} + \sqrt{1-\psi_{n,gn}}\epsilon_n \right)$$

$\rho_n$: asset correlation of obligor $n$

$\psi_{n,gn}$: sensitivity of both $n$ and $g_n$ to stochastic factor $Z_{n,gn}$

$\epsilon_n$: idiosyncratic risk factor of obligor $n$.

This implies the double default probability

$$\mathbb{P}(DD) := \mathbb{P}\left(\{\text{default of obligor } n\} \cap \{\text{default of guarantor } g_n\}\right)$$

$$= \Phi_2 \left( \Phi^{-1}(PD_n), \Phi^{-1}(PD_{g_n}); \rho_{n,g_n} \right).$$

$\rho_{n,g_n}$: additional correlation parameter
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$\rho_{n,g_n}$ : additional correlation parameter
Criticism of the additional correlation approach

1. Correlation, a symmetric measure of dependency, is used to describe an *asymmetric relationship*

2. What is an appropriate value for $\rho_{n,g_n}$?
   \[ \rightarrow \text{In Basel II set } \rho_{n,g_n} \equiv 0.5 \text{ for all } n \text{ and } g_n. \]
   \[ \rightarrow \text{Grundke (2008) empirically evaluates this assumption} \]

3. *Additional* correlation directly violates the conditional independence assumption of the ASRF model

4. Assumes that all guarantors are a) *distinct* and b) *external* to the portfolio
   \[ \rightarrow \text{no reflection of overly excessive contracting of the same guarantor} \]

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Motivation for Asset Drop Model: Merton Model

\[ \ln V_t \]

\[ \ln [B] \]

Default Probability

\[ \text{E} [\ln V_T] \]

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Asset Drop Model

Idea: Adjust $PD_{gn}$ appropriately to *effective default probability* $PD'_{gn}$.

Within a structural model of default:

$$PD_{gn} = P(V_{gn}(T) < B_{gn}),$$

$V_{gn}(t)$: total asset value of $g_n$ in period $t$, $B_{gn}$: default threshold.

Denote with $\hat{E}_{n,gn}$ the nominal $g_n$ guarantees for $n$. Then

$$PD'_{gn} = P(V_{gn}(T) - \hat{E}_{n,gn} < B_{gn}) = P(V_{gn}(T) < B_{gn} + \hat{E}_{n,gn}) \quad (1)$$

→ Within Merton’s model:

$$PD'_{gn} = 1 - \Phi \left( \ln \left( \frac{V_{gn}(0)}{B_{gn} + \hat{E}_{n,gn}} \right) + (r - \frac{n}{2} \sigma_{gn}^2) T \right) \sigma_{gn} \sqrt{T}. \quad (2)$$

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Asset Drop Model

\[ \ln V_t \]

- Increased Guarantor PD if Obligor Defaults

- \[ \ln [B + \hat{E}_{1,g1}] \]

- \[ \ln [B] \]

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Example 1: PD increase

Consider two guarantors $g_1$ ("big bank") and $g_2$ ("small bank").

Here: $V_{g_1}(0) = 50$ and $V_{g_2}(0) = 10$ billion Euros, respectively, $\sigma^2_{g_1} = \sigma^2_{g_2} = 30\%$, $T = 1$, $r = 0.02\%$ and $PD_{g_1} = PD_{g_2} = 0.5\%$ (implies $B_{g_1} = 22.5$ and $B_{g_2} = 4.5$ billion Euros.)
Treatment of Different Hedging Constellations

→ Convexity punishes overly excessive contracting of the same guarantor

→ Treatment of guarantor within the portfolio: Joint loss distribution $L_{1,g_1}$ of obligor 1 and its guarantor $g_1$:

$$
P(L_{1,g_1} = l) = \begin{cases} 
  \text{PD}^\prime_{g_1} \text{PD}_1 & \text{for } l = s_1 \text{ ELGD}_1 \text{ ELGD}_{g_1} \\
  \text{PD}_{g_1} (1 - \text{PD}_1) & \text{for } l = s_{g_1} \text{ ELGD}_{g_1} \\
  (1 - \text{PD}^\prime_{g_1}) \text{PD}_1 + (1 - \text{PD}_{g_1})(1 - \text{PD}_1) & \text{for } l = 0.
\end{cases}
$$

implies

$$
E[L_{1,g_1}] = s_{g_1} \text{ ELGD}_{g_1} \left( \frac{\text{PD}_{g_1} (1 + \text{PD}_1 \lambda_{1,g_1})}{\text{PD}_1 \text{PD}^\prime_{g_1}} \right) + s_1 \text{ ELGD}_1 \text{ ELGD}_{g_1} \frac{\text{PD}_1 \text{PD}^\prime_{g_1}}{\text{PD}_1 \text{PD}^\prime_{g_1}}.
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Treatment of Different Hedging Constellations

→ Convexity punishes **overly excessive contracting** of the same guarantor

→ Treatment of **guarantor within the portfolio**: Joint loss distribution $L_{1,g_1}$ of obligor 1 and its guarantor $g_1$:

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    PD'_{g_1} PD_1 & \text{for } l = s_1 \text{ ELGD}_1 \text{ ELGD}_{g_1} + s_{g_1} \text{ ELGD}_{g_1} \\
    PD_{g_1} (1 - PD_1) & \text{for } l = s_{g_1} \text{ ELGD}_{g_1} \\
    (1 - PD'_{g_1}) PD_1 + (1 - PD_{g_1})(1 - PD_1) & \text{for } l = 0.
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\]
Example 2: Economic Capital (EC)

With IRB treatment of double default effects: 5.40% of total exposure (99.9% VaR) level. With asset drop technique:

Portfolio with 110 obligors, each has exposure 1, maturity 1 year. The first ten are hedged by the last ten (guarantors are in the portfolio). For obligors PD = 1%, LGD = 45%. For guarantors PD = 0.1%, LGD = 100%.
Summary

We criticize the IRB double default treatment for

1. using correlation to model an *asymmetric relationship*
2. not reflecting important characteristics of obligors and guarantors: \( \rho_{n,g_n} \equiv 0.5 \forall n \forall g_n. \)
3. violating the conditional independence assumption of the ASRF model
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We propose a novel *asset drop model* that

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